USE OF THE PLANAR FIT METHOD IN TILT AND DISTORTION ANALYSIS OF THE MEAN FLOW

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Abstract: The Planar Fit Method (PFM) is being used more and more to perform turbulent fluxes calculations in a coordinate system that minimizes the vertical component of wind velocity. It can be applied only after having gathered sonic anemometer data over a long time during which the anemometer position did not change. This implies that PFM has never been used in real time. Coordinate transformations are proposed to convert into a single, time-independent coordinate system the velocity components measured by a set of anemometers with time-dependent tilt fluctuations. By applying the PFM to each anemometer data set it is then possible to check the flow planarity, to detect flow distorsions and to compare the mean streamlines tilt with the terrain slope. Tilt and orientation of each plane become in fact a characteristic of the measurement point, independent from the anemometer that has been or that will be deployed. The use of PFM in real time applications becomes therefore feasible, after a preliminary long period of measurements aimed at locating the planes.

Keywords: *Coordinate systems, Planar fit method, Sonic anemometers*

1. INTRODUCTION

Sonic anemometers measurements are often taken on uniform slopes which can be approximated by an inclined plane. This plane can be identified by employing a method that was proposed in 1977 by Steve Stage (Wilczak et al. (2001)) to calculate turbulent fluxes : the so-called planar fit method (PFM, hereinafter). It consists in taking a two-dimensional linear regression of the vertical velocity component versus the horizontal ones. Wind data from different directions are needed, and therefore the PFM can only be used with data gathered over long periods (weeks or longer) during which the anemometer position does not change or (in principle) whose movements are known (Lee et al. (2004)). The PFM plane position can only be calculated at the end of the measurement period, and this implies that PFM has never been used to evaluate turbulent fluxes in real time.

In field experiments where multiple anemometers are deployed, the orientation of each PFM plane is usually calculated in the coordinate system of each instrument. It is implicitly assumed that each PFM plane is parallel to the underlying surface, its incidental tilt beeing caused by a tilt of the instrument. However, in order to compare measurements from different sensors, it is better to obtain the plane position relative to a single, time-independent coordinate system (Finnigan (2004)) instead of a system which is anemometer-dependent and is incidentally influenced by the time-dependent tilt of the mast. Provided the orientation of the instrument is known with a good accuracy and the tilt fluctuations are measured, the orientation of the PFM plane can be calculated in a fixed system whose z-axis is vertical and the orientation of the x-axis is known. The tilt of the PFM plane can, in this case, be compared to the surface slope to check wether the mean flow is really parallel to the ground. Flow distortions due to neighbouring structures or terrain obstacles can be evaluated precisely by analyzing the angular distribution of the velocity vector tilt above the PFM plane.

Another usefulness of calculating the plane position in an anemometer-independent system is that it also makes it possible to use the PFM in real time applications. The PFM plane position in fact becomes a characteristic of the measurements site. After a preliminary long period of measurements aimed at locating the plane the real time calculations can start, provided the orientation of the instrument is known with a good accuracy.

2. METHOD

As previously mentioned, attention is focused on a single, fixed, z-axis vertically aligned coordinate system in which the measurements of all the anemometers should be transformed before calculating the PFM plane orientation. Let this system be labeled as (x_4, y_4, z_4) : the tilt of the PFM plane is calculated relative to the (x_4, y_4) plane, i.e. the horizontal (Figure 1). Planes (x_i, y_i) are labeled as π_i hereinafter. The orientation of

the (x_4, y_4, z_4) system is determined by μ, the angle between the North direction and the y_4 axis in the horizontal plane. The angle μ is measured counterclockwise from the y_4 axis. Transformation from (x_0, y_0, z_0), the coordinate system used in each anemometer output, to the (x_4, y_4, z_4) system depends on each anemometer orientation. This transformation is performed in subsequent steps through (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3). These systems are now introduced following instrument deployment, i.e. in reverse order starting from (x_4, y_4, z_4).
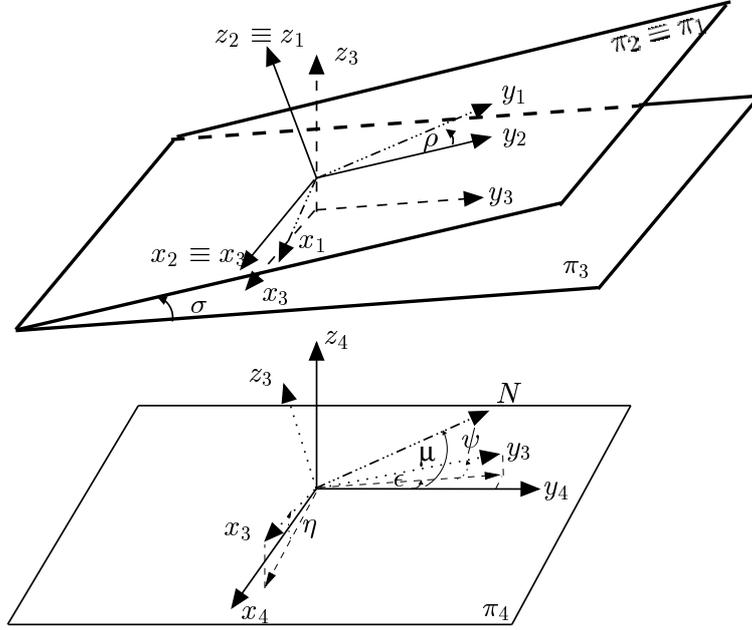

Figure 1: 1st to 4th coordinate systems. π_4 is the horizontal plane, π_3 the inclinometer plane and $\pi_2 \equiv \pi_1$ the plane perpendicular to the vertical axis of the instrument. N indicates the North direction in the horizontal plane π_4. The untagged dashed lines indicate the projections of the x_3 and y_3 axes on the horizontal plane.

If a couple of inclinometers are used to measure the small fluctuations of the anemometer bearing, let x_3 and y_3 be the orthogonal axes whose tilts are measured above the horizontal (η and ψ respectively). These axes identify the inclinometer coordinate system (x_3, y_3, z_3) (Figure 1). Angles η and ψ are usually small, so that we can assume, without loosing any generality, that if they are null or inclinometers are not used, (x_3, y_3, z_3) and (x_4, y_4, z_4) coincide.
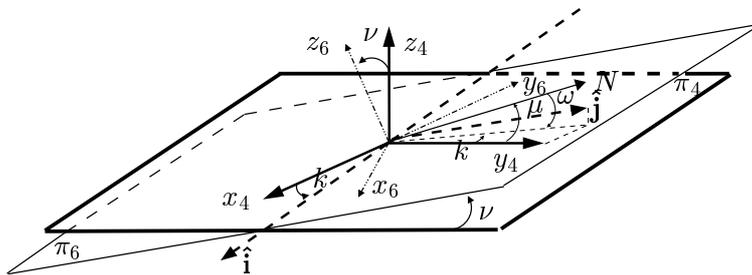

Figure 2: The tilt angle ν of the PFM plane π_6 and the direction ω where the tilt is measured (clockwise from the North direction in the horizontal plane π_4). The unit vector $\hat{\mathbf{i}}$ indicates the intersection of the π_6 and π_4 planes. The unit vector $\hat{\mathbf{j}}$ indicates the direction of the tilt in the plane π_6.

System (x_2, y_2, z_2) is obtained from (x_3, y_3, z_3) from a counterclockwise rotation σ around the x_3 axis (Figure 1). This angle is introduced to allow the installation of the anemometer in a surface-normal position when measurements are taken on mountain slopes. The z_2 axis therefore coincides with the vertical axis of the anemometer. In uniform slopes, a surface-normal setup of a sonic anemometer is in fact preferred to a vertical

setup because it minimizes the flow distortion by the instrument itself (Geissbühler et al. (2000)). System (x_1, y_1, z_1) is obtained from (x_2, y_2, z_2) from a counterclockwise rotation ρ around the z_2 axis (Figure 1). This angle is introduced to allow a rotation of the anemometer around its vertical axis when it is fastened to its installation bracket. The y_1 axis is defined as the nominal North direction of the anemometer. Unless there are mechanical reasons, it is suggested using $\rho = 0$, i.e. to align the nominal North with the y_2 axis.

After having expressed the measured velocities in the fixed system (x_4, y_4, z_4), the PFM can be applied. This gives the matrix $\mathbf{P}$ (Wilczak et al. (2001)) that can be used to transform the velocities into a new system (x_5, y_5, z_5), where now (x_5, y_5) is the PFM plane. System (x_6, y_6, z_6) is defined as having the (x_6, y_6) plane coinciding with the PFM plane (x_5, y_5) and with y_6 axis lying in the vertical plane that contains the y_4 axis, i.e. (y_4, z_4). It can be demonstrated that $(x_6, y_6, z_6) \equiv (x_5, y_5, z_5)$, and therefore, we never use the (x_5, y_5, z_5) system. The position of the PFM plane π_6 can be defined by 2 parameters : its tilt ν (the angle with the horizontal plane) and the azimuth direction ω where the tilt is measured (the clockwise angle from North in the horizontal plane) (Figure 2).

3. RESULTS

The coordinate transformations described in the previous Section were applied to wind data collected continuously for 60 days on a flat area of the Hells Gate ice shelf in Antarctica during the XVII PNRA expedition (southern-hemisphere summer 2001-02).

Three sonic anemometers were placed at different heights (5 m, 7.5 m and 10 m) on a mast. Two inclinometers were placed at a 5 m height to measure the eastward and northward tilts of the mast. The inclinometer data showed that tilt was not constant, being influenced by the strong katabatic wind (highest speed $30 \ ms^{-1}$) and by ice melting near the mast base. The amplitude range of the tilt fluctuations was nevertheless limited to $\pm 0.60°$ by occasionally resorting to straightening the mast.

Table 1: PFM plane tilt(ν) and orientation (ω) during the Antarctic experiment.Standard deviations were derived from those of the planar fit coefficients.

Height	Anemometer	ν (degrees)	ω (degrees)
10 m	161	2.65 ± 0.01	51.5 ± 1.0
7.5 m	160	2.90 ± 0.01	59.9 ± 1.0
5 m	162	2.61 ± 0.01	65.7 ± 1.0

The 5 minute averaged velocity components were converted from each anemometer (x_0, y_0, z_0) output coordinate system to a single (x_4, y_4, z_4) system. Planar fit was then applied and tilt ν and its orientation ω were calculated. Tilt and orientations at the three levels are about the same, as expected above a flat surface (Tab. 1). The orientations are consistent with the direction of the gentle slope of the glacier, but the tilts are about five times that of the slope. This discrepancy is probably due to a positioning error as a consequence of having mounted the inclinometers over a bracket that is separated from the anemometer stand. Before raising the mast, the anemometers were installed on booms and it was checked that they were aligned along the same line. The mast was then raised and braced, checking with a spirit-level that the mast was vertical. By standing on the mast (an awkward position that could have made it difficult to obtain a good accuracy) the inclinometers were then adjusted to measure zero tilt in both orthogonal directions. This procedure resulted in the alignment of the anemometer pedestals (the PFM planes were approximately parallel), but the pedestals were probably not made parallel to the mast, i.e. vertical, with a good accuracy.

In Figure 3 the tilt of each $10°$ wide-sector mean velocity is plotted versus the sector direction, and the sinusoidal curves show the expected results in the case where the mean velocities lie exactly on the PFM plane. In that case their tilt above the horizontal would reach the maximum value ν at direction ω, and would be null at right angles, where the vectors would become parallel to the horizontal plane. Each mean vector was calculated from the 5 minute averaged velocity vectors whose direction fell inside the sector. Only vectors with a modulus
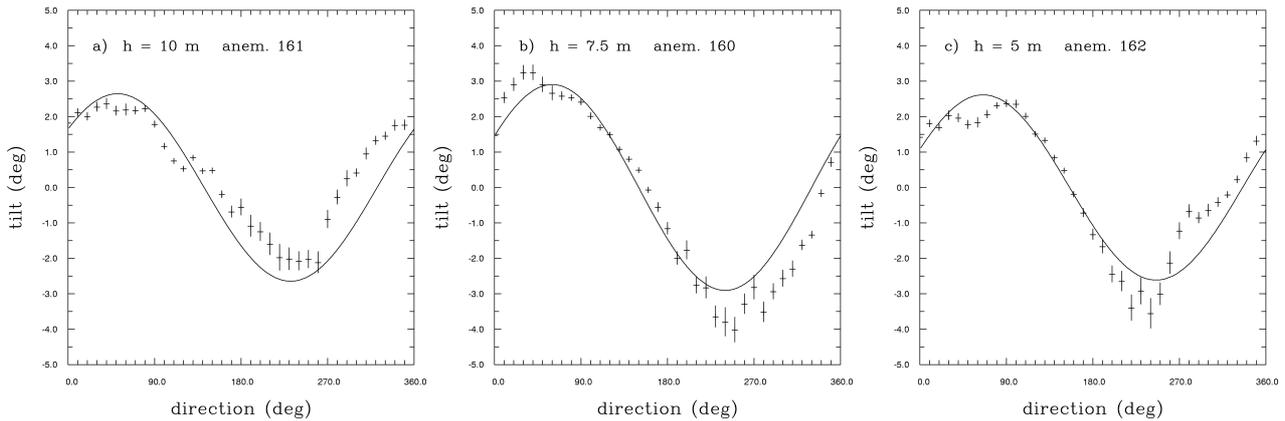
Figure 3: Azimuthal distribution of the $10°$ wide-sector mean velocity vector tilt above the horizontal plane π_4 at 10 m (a), 7.5 m (b) and 5 m (c). The continuous curves indicate the predicted values if the velocity vectors lie exactly on the PFM plane. The vector directions are clockwise from North.

greater than 0.5 ms^{-1} and a time distance greater than 30 minutes were considered, in order to discard the calms and assure that the tilt error of the mean velocity vector was estimated from independent samples.

Figure 3 shows a good agreement between the data and the PFM theoretical curves when the velocity vectors were directed towards the directions ranging from $0°$ to $180°$, indicating that the flow was planar at all levels when the winds were westerly. A deviation from planarity is observed in the range of velocity vectors from $180°$ to $360°$. It is probably related to the flow distortion induced by the mast and by the krypton igrometers that were placed at a short distance from the anemometers. In fact, anemometers were installed to be upwind to the mast during the (westerly) katabatic wind. The flow distortion is smaller when the anemometer is mast-upwind ($85°$) and increases as the mean wind velocity departs from this direction. However, the maximum distortion is not reached when the anemometer is exactly mast-downwind ($265°$), but occurs instead nearby. In this direction, the distortion has a local minimum and is very sensitive to wind direction.

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