

# A GENERALIZED SLEVE COORDINATE ALLOWING FOR THIN NEAR-SURFACE LEVELS

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**Abstract:** Smooth vertical coordinate levels in atmospheric models can considerably reduce numerical errors in the simulation of flow past complex topography. In this paper we propose a new Smooth LEvel VERTICAL (SLEVE) coordinate formulation with a modified vertical decay of topography, leading to a more uniform level thickness near the ground while preserving the fast transition to smooth levels in the upper atmosphere. Using numerical simulations of flow past complex topography, we demonstrate the feasibility of this approach. As a result, the new formulation allows for generally thinner near-surface levels of almost constant thickness and thus a better resolution of the boundary layer compared to the SLEVE coordinate, while retaining its advantages over classical sigma-coordinates at upper levels.

**Keywords:** *Vertical coordinate, non-hydrostatic models, complex terrain*

## 1. INTRODUCTION

The simulation of atmospheric flow over complex terrain using high-resolution atmospheric numerical models can be affected by numerical errors due to fine-scale variations in the height of the terrain-following model levels. The recently proposed Smooth Level Vertical (SLEVE) coordinate (Schär et al., 2002) is able to reduce such errors by using a scale-dependent, exponential vertical decay of the model topography in the calculation of the level height, resulting in much smoother model levels in the middle and upper troposphere, as compared to the classical Gal-Chen coordinate (Gal-Chen and Sommerville, 1975) or hybrid coordinate systems. The SLEVE formulation has the ability to improve simulations of flow past complex topography, e.g. in terms of precipitation (Zängl, 2004) or potential vorticity (Hoinka and Zängl, 2004).

A drawback of the SLEVE formulation is that the level thickness of the lowest model level can be substantially reduced above high mountain peaks. This can potentially lead to numerical stability problems and prevents the use of thin levels in the boundary layer, where a high vertical resolution is crucial.

This paper proposes a modified SLEVE formulation which reduces the compression of the lowest levels to a large extent, yet retaining the attractive properties of the SLEVE coordinate in the upper atmosphere.

## 2. DEFINITION OF THE COORDINATE

The classical, height-based Gal-Chen coordinate is defined by  $z(x, y, Z) = Z + h(x, y) \cdot b(Z)$ , where  $z$  is the height of the model level,  $x$  and  $y$  are the horizontal coordinates and  $Z$  is a height-based vertical coordinate, which is generally chosen to be non-aequidistant, with many thin levels in the boundary layer and thicker levels aloft.  $h(x, y)$  is the model orography and  $b(Z) = 1 - Z / Z_T$  may be interpreted as a vertical decay function, which determines the vertical decay of topographic features with height.  $Z_T$  is the height of the model top. In the Gal-Chen formulation this decay is linear, leading to noisy level heights above complex topography in the mid-to-upper model domain.

The more generalized, height-based, terrain-following coordinate formulation proposed by Schär et al. (2002) reads

$$z(x, y, Z) = Z + h_1(x, y) \cdot b_1(Z) + h_2(x, y) \cdot b_2(Z). \quad (1)$$

Here the decay of topography is done separately for the smooth large-scale contribution of the model topography  $h_1(x, y)$ , which is usually obtained from  $h(x, y)$  with a digital filter, and its small-scale counterpart  $h_2(x, y)$  satisfying  $h = h_1 + h_2$ .  $b_i(Z)$  are the vertical decay functions corresponding to the two topographic contributions  $h_i$ .

The new formulation proposed in this paper retains (1) but uses modified decay functions  $b_i(Z)$ :

$$b_i(Z) = \frac{\sinh\left(\left(\frac{Z_T}{s_i}\right)^n - \left(\frac{Z}{s_i}\right)^n\right)}{\sinh\left(\left(\frac{Z_T}{s_i}\right)^n\right)} \quad (2)$$

with  $s_i$  being the decay constants of the respective topographic contributions. The original SLEVE formulation is represented by  $n=1$ . To illustrate its difficulties with the thin near-surface model layers, it is helpful to consider the vertical derivative of (1):

$$\frac{\partial z}{\partial Z} = 1 + h_1(x, y) \cdot \frac{\partial b_1}{\partial Z} + h_2(x, y) \cdot \frac{\partial b_2}{\partial Z} \quad (3)$$

The thickness of the  $k$ -th model level can then be written as  $\Delta z_k \approx \Delta Z_k \cdot \partial z / \partial Z$ , where  $\Delta Z_k$  is the distance between the  $k$ -th and the  $(k-1)$ -th level in absence of topography. From (3) it is evident that the level compression due to underlying topography is determined by the minima of

$$\frac{\partial b_i}{\partial Z} = -\frac{n}{s_i^n} Z^{(n-1)} \cdot \frac{\cosh\left(\left(\frac{Z_T}{s_i}\right)^n - \left(\frac{Z}{s_i}\right)^n\right)}{\sinh\left(\left(\frac{Z_T}{s_i}\right)^n\right)} \quad (4)$$

and the local heights of the two topographic contributions  $h_1$  and  $h_2$ . Thus the model levels are most compressed above high mountain tops where the horizontal wind is usually strong; two factors that can lead to stability problems.

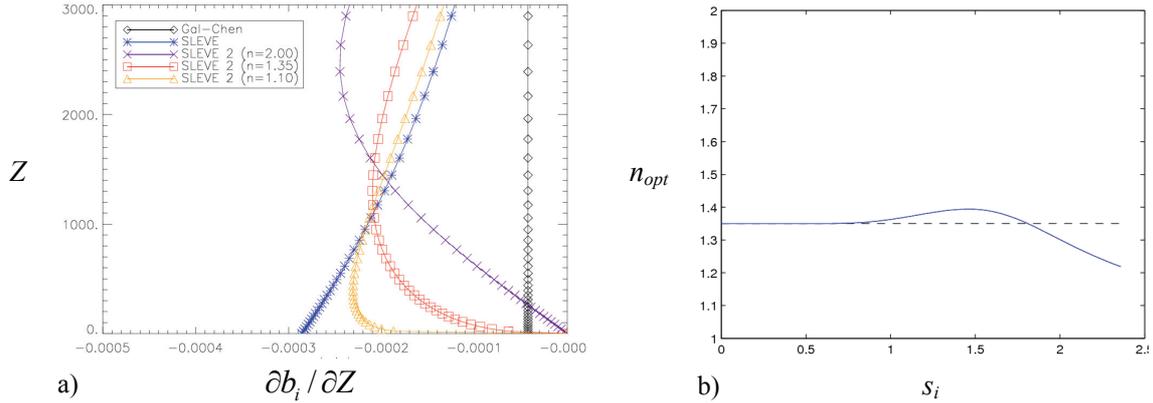
For the original SLEVE formulation ( $n=1$ ) the minima of (4) amount to  $-\coth(Z_T/s_i)/s_i$  at  $Z=0$ , resulting in a strong compression of the lowest model level, particularly for small  $s_i$ , which are attractive to promote a rapid vertical decay of the topography in the calculation of the level heights.

For  $n > 1$  the term  $\partial b_i / \partial Z$  vanishes at  $Z=0$ , and thereby dramatically reduces the compression of the lowest model level, while retaining some compression at the (less critical) higher levels. Figure 1a illustrates  $\partial b_i / \partial Z$  as a function of  $Z$  for the Gal-Chen coordinate, the original SLEVE coordinate and the new SLEVE2 coordinate for three different values of  $n$ . In comparison to SLEVE ( $n=1$ ), the vertical minimum of  $\partial b_i / \partial Z$  is larger for all three examples of SLEVE2, and is largest for  $n=1.35$ . Thus there must be an optimal choice of  $n$ , which minimizes the magnitude of  $\partial b_i / \partial Z$ . This optimum is derived in section 3.

Since the new formulation reduces the minimum of  $\partial b_i / \partial Z$ , it also influences favourably the invertibility condition of (1) (see Schär et al., 2002), i.e. the vertical decay parameters  $s_i$  can be chosen smaller compared to the original SLEVE formulation, leading to a faster transition to smooth and eventually horizontal model levels.

### 3. DETERMINATION OF THE OPTIMAL $n$

The optimal  $n$  of (4) minimizes the magnitude of  $\partial b_i / \partial Z$ , leading to a minimal compression of the model levels and a better invertibility property. Thus this optimal  $n$  is defined by  $n_{opt} = \arg \max_n (\min_Z (\partial b_i(Z, n, s_i) / \partial Z))$ . For a given  $s_i$ ,  $n_{opt}$  can be obtained by finding the saddle point of the two-dimensional function  $\partial b_i(Z, n) / \partial Z$ . Figure 1b shows the numerically derived  $n_{opt}$  as a function of  $s_i$ . Fortunately  $n_{opt}$  only slightly depends upon  $s_i$ , at least within the typical range of parameters under consideration ( $s_1$  is typically chosen smaller than 10,000m, and  $s_2$  even smaller). Thus we propose to set  $n$  in eq. (2) to the value of  $n_{opt}=1.35$ .



**Figure 1:** Panel a): Vertical derivatives of the decay functions  $b_i$  for the Gal-Chen (black diamonds), SLEVE (blue stars) and the new SLEVE2 coordinate with different values of  $n$  (crosses, squares and triangles). Panel b): Dependence of the optimal parameter  $n_{opt}$  on the vertical decay rate  $s_i$ .

#### 4. APPLICATION IN AN NWP MODEL

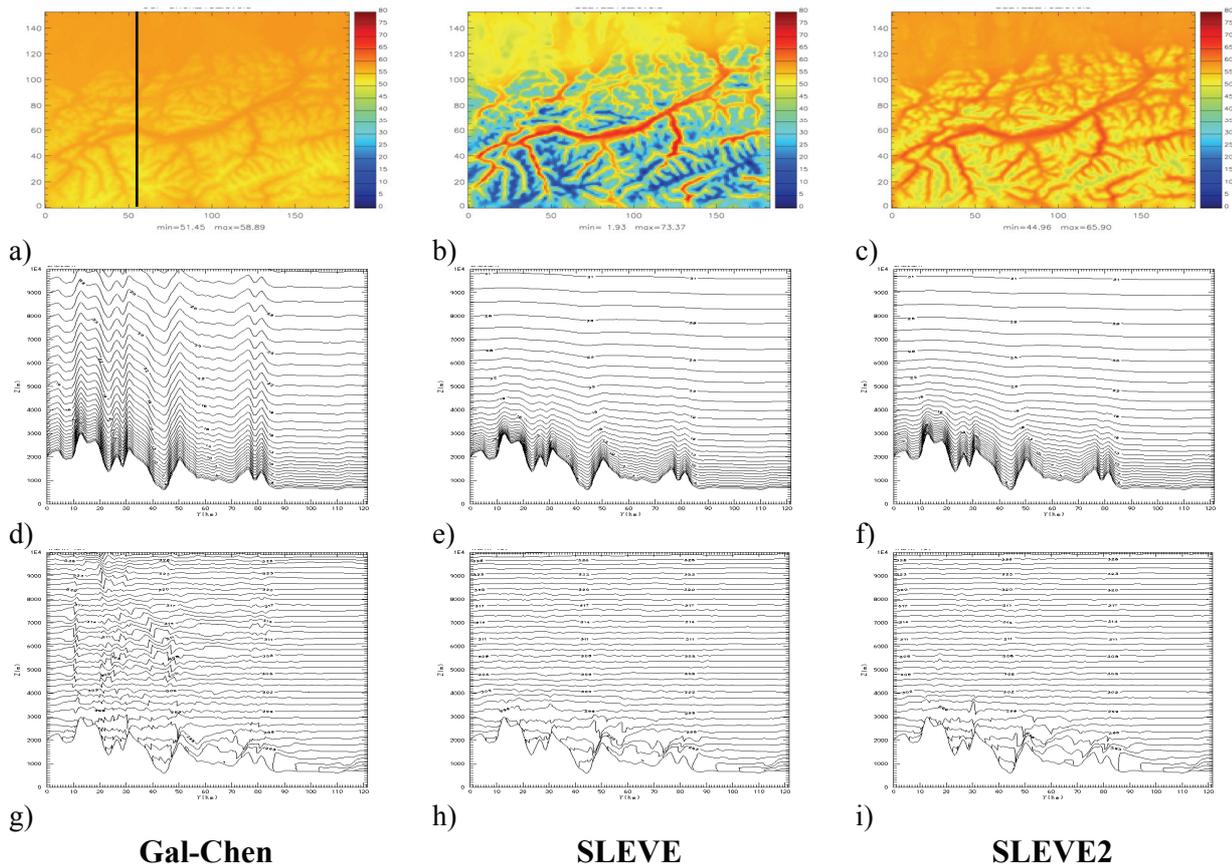
The new SLEVE2 formulation is tested in the non-hydrostatic NWP model of the Consortium for small-scale modelling (COSMO) in a semi-idealized case study. The COSMO model is used operationally at MeteoSwiss and is currently prepared for its application at a resolution of 2.2km covering the Alpine region. The test case comprises the simulation of the valley wind system of the Austrian Inn valley as described in Zängl (2002). The simulation is started on a fictional 15. October at 00UTC and is integrated out to 30h. The lateral boundary forcing employs a vertical temperature and humidity profile, and absence of large-scale flow. The simulation thus describes the radiation-driven heating of the mountains during the day, resulting in an upward mass flux from the Alpine foreland into the valley and vice versa during the night.

The COSMO model is set up with the topography of the Inn valley and its adjacent Alpine foreland with a horizontal grid-size of 800m. The model parameterizations include turbulence, surface transfer scheme and radiation. Microphysics and convection schemes and the soil model are switched off. No explicit numerical diffusion along model surfaces is used, which allows for the successful simulation of the valley wind system (Zängl, 2002). 40 stretched vertical levels with level spacings from 60m near the ground to 2000m at the model top at 23,000m are used. Three simulations using the Gal-Chen coordinate, the SLEVE coordinate with  $s_1=5,000\text{m}$  and  $s_2=1,800\text{m}$  and the new SLEVE2 formulation with the same settings for  $s_i$  but  $n=1.35$  are performed. Figure 3 shows the geographical distribution of the lowest model level thickness for the three configurations along with its minimum value. The level thickness of  $Z_l=60\text{m}$  is compressed to a minimum value of 51m for the Gal-Chen coordinate (Fig. 2a) whereas in the SLEVE setting a minimum level thickness of 1.9m is obtained (Fig. 2b). The new SLEVE2 formulation is able to increase this value to 45m (Fig. 2c). Figure 2d-f show the vertical level distribution along the North-South cross-section at  $x=55\text{km}$  for the three coordinate settings. At upper levels, SLEVE and SLEVE2 yield a very similar distribution of the smooth model levels.

With a time step of 6s, all three simulations are able to simulate the valley-wind system as described by Zängl (2002). Even the SLEVE simulation remains stable, despite a minimum level thickness of 1.9m. This can be explained with the very low horizontal wind speed at mountain tops. Large wind speeds do only occur in the valley, where the levels are much thicker.

Both SLEVE coordinate versions substantially reduce the numerical noise above the mountains, as can be seen in the vertical cross-section of potential temperature in Figure 2g-i.

With an increased time step of 18s, only the new SLEVE2 coordinate is able to stably integrate the simulation. Both, the Gal-Chen and the SLEVE simulations become unstable, with the instability growing at the lowest levels. This indicates that the SLEVE2 levels are superior to the other formulations and can indeed improve the stability of numerical simulations above complex topography.



**Figure 3:** Simulation of valley winds with a time step of 6s for three different coordinate formulations valid at +15h. Panels (a,d,g) depict the simulation with the Gal-Chen coordinate, panels (b,e,h) with the SLEVE coordinate and panels (c,f,i) with the SLEVE2 coordinate. Panels (a-c): Thickness of lowest model level (in m). The black line in panel (a) marks the position of the vertical cross-section of panels (d-i). Panels (d-f): Vertical cross-sections of the coordinate levels at  $x=55\text{km}$ . Panels (g-i): Vertical cross-sections of potential temperature (K) at  $x=55\text{km}$ .

## 5. CONCLUSIONS

A generalized SLEVE vertical coordinate formulation is proposed with a modified vertical decay of topographic variations with height, leading to a more uniform level thickness near the ground while preserving the fast transition to smooth levels in the upper atmosphere. The new formulation improves the numerical stability of the model and allows for thinner near-surface levels and thus a better resolution of the boundary layer compared to the SLEVE coordinate. The new formulation has been shown to be superior to the original SLEVE formulation in an idealized simulation of flow above complex topography. The new SLEVE2 coordinate is intended to be introduced in the future high-resolution NWP model of MeteoSwiss.

**Acknowledgements:** The numerical simulations of this study have been carried out at the Swiss National Supercomputing Centre (CSCS) in Manno, Switzerland.

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