# Documentation of the TKE-CBR turbulent scheme in the CLIMATE version 5.0 of ARPEGE (based on the cycle 32t0_op1v2_13 + CNRM/GMGEC modset)

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# 1 Part A : Motivations

## 1.1 A1 : Objectives

Among all the different processes which need to be represented in the numerical prediction model for the atmosphere, valid from the short term range forecasting to the simulation of the general climate, one of the most important one is the decreasing of the vertical gradients or heterogeneous feature in potential temperature, wind or humidity. These processes correspond to "vertical transport" or "dissipation", represented either by what is usually called "turbulence" or "convection".

The splitting of the mixing processes between "turbulent" or "convective" ones are made in terms of asymmetric buoyancy effects for the convection, whereas the turbulence is think in terms of symmetric and horizontal rolls, mainly located within the dry PBL, bellow the cloudy part of the atmosphere.

This separation was clear until the 80's and 90's. Since then, more and more moist effects are taken into account in the turbulent schemes, and more and more dry thermal convective processes are associated to the shallow convection (inside the cloud) and in the dry PBL (bellow the cloud base).

Nowadays, several unified "turbulent + mass-flux" schemes exist. They are often called "EDMF" (or "EDKF"), for "Eddy Diffusivity Mass-Flux". The aim of the scheme described in this documentation is to described only the vertical mixing of atmospheric variables due to turbulent processes, with the vapor and condensed waters taken into account. The dry and moist convection processes are not explicitly managed, though they impact in some way on the turbulent scheme via the two "turbulent" and "convective" tendencies which are fully combined for computing the next time-step, with possible covering of the two schemes on the vertical.

The aim the the parameterization is to compute the exchange coefficients $K_m$ (valid for the momentum, i.e. the 2 wind components $u$ and $v$) and $K_h$ (valid for the potential temperature and the specific humidity). These exchange coefficients are used to compute the turbulent fluxes, used in the implicit solver for the vertical mixing and obtained by the inversion of a tri-diagonal matrix.

The choice of the present parameterization, implemented both in the NWP and the GCM versions of ARPEGE, is mainly motivated by the hope to better represent the marine Strato-cumulus in the model. Indeed, there is a lack of strato-cumulus in most of the GCM and NWP models in the Eastern part of the tropical oceans, close to the coast. These low-level clouds are important features for determining even the sign of the local impacts associated with the Climate Change. The control of these marine Strato-cumulus is realized via a special refinement of the scheme, i.e. the "Top-PBL (vertical) entrainment scheme".

## 1.2 A2 : The Physical Proceses

Different methods have been used in the numerical models to compute the exchange coefficients : $K_m$ for the momentum (the wind components) and $K_h$ for the potential temperature and the water species (also for passive scalar variables). The different methods presently available in the Climate and the NWP versions of ARPEGE are described in the following.

**First Step :**

Starting from the old NWP "EMERAUDE" model (operational in France until 1991), the Louis (1979, 1982) scheme is available since the very first version of ARPEGE (from 1988-90). Due to a too expensive computational cost, the iterative methods using the Monin-Obokhov theories have been simplified by Louis, by fitting the results of Monin-Obokhov iterative processes with some analytical functions depending on the dry and local Richardson number, with no iterative process.

An improvement has been made by Geleyn (1986), by taking into account the vertical profile of the water vapour in a modified and moist Richardson number, allowing in some way the parameterization of the non-precipitating shallow convection.

Since the 90's, the parameterization of Louis (79, 82) and Geleyn (86) has been continuously improved in the ARPEGE model : the mixing lengths are defined differently for the "dynamic" and the "thermal" parts ; the PBL height is no longer a fixed value, it vary with time as a diagnostic value made at each grid point and at each time step ; an "anti-fibrillation" method has been implemented in order to fix some problems of time oscillations, as seen first in a BOMEX SCM inter-comparison ; the Louis (79, 82) and Geleyn (86) formulas has been tuned, in order to obtain more realistic shallow convection clouds (SCM and NWP tunings)...

**Second Step :**

Since 1999, another turbulent scheme is used in the Climate version of ARPEGE. It is based on the stationary equation for the turbulent kinetic energy, using the 2.0 order scheme of Mellor and Yamada (1977, 1982).

The numerical computations of the exchange coefficients $K_m$ and $K_h$ are described more precisely in Ricard and Royer (1993). The moist processes are taken into account in the Mellor-Yamada set of equations, by using the subgrid variance of atmospheric humidity, following the ideas of Sommeria, Deardorff, Bougeault (1976, 1977, 1982) where a certain

statistical law is prescribed (an asymmetric and exponential fixed PDF).

The turbulent kinetic energy variable is defined on the full-levels of ARPEGE (same location as for the wind or the temperature). As a consequence the advective processes could be activated, but they are switch-off since it is a stationary (diagnostic) turbulent kinetic energy equation. A lot of half-summations are made to go from the full-levels to the half-levels, and vice versa. With the crude spacing of the vertical levels, such half-summations processes create an artificial and strong mixing at the top of the PBL and at the inversion. It is probably the reason why the shallow convection processes are - in some way - represented by this moist diagnostic turbulent scheme, via a synergy between the moist subgrid representation and the (too) strong vertical mixing.

**Third Step :**

The present prognostic scheme for the turbulent kinetic energy corresponds to a continuous series of developments, starting in 1998-99 from the prognostic TKE scheme implemented in the old LAM model "PERIDOT". This code of "PERIDOT" has been rewritten for ARPEGE by P. Lacarrère, then tested in climate mode during the PhD of C. Bossuet.

The different processes represented in this new set of parameterizations are

- the turbulent kinetic energy ($e$) is computed with a prognostic equation where the horizontal and vertical advections are switched-off, with the vertical mixing of $e$, the dissipation, the shear (dynamical) production and the buoyancy (thermal) production occur ;
- the turbulent kinetic energy ($e$) is computed on the "half-levels", the levels where the exchange coefficients $K_m$ and $K_T$, also the vertical velocity, are computed (in between the "half-levels" where are computed the wind components, the temperature and the specific humidity) ;
- the turbulent kinetic energy ($e$) has a minimum value (typically of $10^{-6}\ m^2\ s^{-2}$ ) ;
- the formulations for the turbulent fluxes are given in Redelsperger and Sommeria (1981) and Cuxart, Bougeault, Redelsperger (2000) ;
- the "moist" versions of the fluxes are computed by using the Lilly (1968) potential temperature (i.e. $\theta_{vl}$) and the Betts (1973) variables (i.e. for $\theta_l$ and $q_t$), for the vertical mixing of the conservative variables ;
- the computations of the sub-grid variance of cloud liquid water are made with the hypotheses of Bougeault (1982) and Bechtold (1995), by using some mixed symmetric (Gaussian) and asymmetric (Exponential), in order to represent the Cumulus and the Strato-Cumulus, respectively ;
- the mixing and dissipation lengthes are given by the non-local formulation of Bougeault and Lacarrère (1989) ;
- the surface layer value for the turbulent kinetic energy is given by André et al.(1978) ;
- the surface layer values for the exchange coefficients are given by Louis (1979, 1982) ;
- the vertical mixing of the turbulent kinetic energy is made in an implicit way, by taking into account a linearized version of the dissipation term, i.e. the non-linear

term $(\overline{e})^{3/2}$, leading to the weighting factors $\alpha_{exp} = -0.5$ and $\alpha_{imp} = 1.5$ ;

- the top-PBL vertical entrainment is parameterized following the ideas of Grenier and Bretherton (2001) and Grenier (2002).

## 1.3 A3 : The theory : equations and hypotheses

### 1.3.1 A3-a : the turbulent kinetic energy equation

The turbulent kinetic energy equation gives the change in time of the grid-cell mean value ($\overline{e}$).

$$\frac{\partial \overline{e}}{\partial t} = [\,\text{Advect.}\,] + \text{Diff}_{\text{vert}} + P_{\text{dyn.}} + P_{\text{ther.}} - D\text{iss}\,, \tag{1}$$

$$\text{Diff}_{\text{vert}} = -\frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho\,\overline{e'w'}\right)\,, \tag{2}$$

$$P_{\text{dyn.}} = -\left[\overline{u'w'}\,\frac{\partial\,\overline{u}}{\partial z} + \overline{v'w'}\,\frac{\partial\,\overline{v}}{\partial z}\right]\,, \tag{3}$$

$$P_{\text{ther.}} = \beta\,\overline{w'\theta'_{vl}}\,, \tag{4}$$

$$D\text{iss} = C_\epsilon\,\frac{\overline{e}\,\sqrt{\overline{e}}}{L_\epsilon}\,. \tag{5}$$

Except the (neglected) advective part (Advect.), it is the sum of 4 terms. The first term ($\text{Diff}_{\text{vert}}$) is the vertical mixing (or diffusion) of $\overline{e}$. It represents the change of $\overline{e}$ by the turbulent processes plus, in some way, the impact of the (unknown) presso-correlation term. There is two production terms. The dynamical production $P_{\text{dyn.}}$ is always positive and it represents the impact of the shear of the wind components. The thermal production $P_{\text{ther.}}$ can be positive or negative, depending on the vertical fluxes of the Lilly (1968) potential temperature $\theta_{vl}$, computed in a complex way by using the fluxes of the conservative Betts (1973) variables $\theta_l$ and $q_t$. The last dissipation term ($D$iss) depends on a constant $C_\epsilon$ and on a dissipation length $L_\epsilon$.

For the thermal production, the formulation for $\beta$ and $\overline{\theta}_{vl}$ writes

$$\beta = \frac{g}{\overline{\theta}}\,, \tag{6}$$

$$\overline{\theta}_{vl} = \overline{\theta}\,(1 + 0.608\,\overline{q}_v - \overline{q}_c)\,. \tag{7}$$

The potential temperature of Lilly depends on the grid-cell average of both the water vapor $\overline{q}_v$ and the condensed water $\overline{q}_c$ (either liquid or solid). The main difficulty is to compute the fluxes of these quantities, whereas the moist fluxes are only known for the Betts variables $\overline{w'\theta'_l}$ and $\overline{w'q'_t}$.

The conservative variables of Betts write in terms of the mean water vapor ($\overline{q}_v$), the mean liquid cloud water ($\overline{q}_l$), the mean solid cloud water ($\overline{q}_i$) and the mean total condensed cloud water ($\overline{q}_c = \overline{q}_l + \overline{q}_i$) :

$$\overline{\theta}_l = \overline{\theta}\left(1 - \frac{L_v\,\overline{q}_l + L_f\,\overline{q}_i}{c_p\,T}\right)\,, \tag{8}$$

$$\overline{q}_t = \overline{q}_v + \overline{q}_c\,. \tag{9}$$

The two terms $L_v$ and $L_f$ are the latent heats of vaporization and fusion, respectively.

The first order turbulent fluxes are written in terms of the vertical gradients of the mean variables, following the formulations of Redelsperger and Sommeria (1981 ; or RS81), also of Cuxart, Bougeault and Redelsperger (2000 ; or CBR00). They correspond to the option "TURB1D" of the Meso-NH and AROME LAM.

$$\overline{u'w'} = -C_m\, L_m\, \sqrt{\overline{e}}\, \frac{\partial\, \overline{u}}{\partial z} \quad ;\; K_m = C_m\, L_m\, \sqrt{\overline{e}}\,, \tag{10}$$

$$\overline{v'w'} = -C_m\, L_m\, \sqrt{\overline{e}}\, \frac{\partial\, \overline{v}}{\partial z} \quad ;\; K_m = C_m\, L_m\, \sqrt{\overline{e}}\,, \tag{11}$$

$$\overline{e'w'} = -C_e\, L_m\, \sqrt{\overline{e}}\, \frac{\partial\, \overline{e}}{\partial z} \quad ;\; K_e = C_e\, L_m\, \sqrt{\overline{e}}\,, \tag{12}$$

$$\overline{w'\theta'_l} = -C_\theta\, L_m\, \sqrt{\overline{e}}\, \frac{\partial\, \overline{\theta}_l}{\partial z}\, \phi_3 \quad ;\; K_T = C_\theta\, L_m\, \sqrt{\overline{e}}\, \phi_3\,, \tag{13}$$

$$\overline{w'q'_t} = -C_q\, L_m\, \sqrt{\overline{e}}\, \frac{\partial\, \overline{q}_t}{\partial z}\, \psi_3 \quad ;\; K_q = C_q\, L_m\, \sqrt{\overline{e}}\, \psi_3\,. \tag{14}$$

The first order turbulent fluxes and the associated exchange coefficients (10) to (14) depend on four unknown constants ($C_m$, $C_e$, $C_\theta$, $C_q$), with the mixing length denoted by $L_m$ and the dissipation length by $L_\epsilon$.

This "CBR00" turbulent scheme includes two stability functions ($\phi_3$ and $\psi_3$), which are found to be equal in that case of an "1D-vertical" turbulence, leading to

$$\phi_3 = \psi_3 = \frac{1}{1 + \mathrm{C}\,(R_\theta + R_q)}\,. \tag{15}$$

The term $\mathrm{C} = C_\theta C_{\epsilon_\theta}$ is another unknown constant.

**Hypothesis 1 :** In the Meso-NH and AROME LAM, the true computations of $R_\theta + R_q$ need to keep some pseudo-prognostic variables (kept in memory from one time step for the next one). In the CBR00 scheme implemented in ARPEGE, the computation of (15) are made with a more straightforward method, without pseudo-prognostic variables but with a direct computation of the vertical gradient of $\overline{\theta}_{vl}$ instead, with the assumption

$$R_\theta + R_q \approx \beta\, \frac{L_m\, L_\epsilon}{\overline{e}}\, \frac{\partial \overline{\theta}_{vl}}{\partial z}\,. \tag{16}$$

The vertical gradient is computed from (7), with no hypothesis concerning the sub-grid variability of the humidity.

The stability function $\phi_3 = \psi_3$ varies from 0.78 (for the stables cases) to possible large or even infinite values for unstable cases, for instance if $\mathrm{C}\,(R_\theta + R_q) = -1$. As a practical rule, from CBR00, values for $\phi_3 = \psi_3$ are limited to the maximum value of ACBRPHIM = 2.2 (or possibly less), available in the NAMELIST of ARPEGE.

### 1.3.2 A3-b : The constants

The set of constants used in ARPEGE are different from the one used in Meso-NH and AROME. Three of the independent constants defined in Meso-NH and AROME are $C_{pv}$,

$C_{p\theta}$ and $C_{\epsilon_\theta}$, leading to the four ARPEGE constants $C_m$, $C_\theta$, $C_q$ and C, according to

$$C_m = \frac{4}{15\, C_{pv}}\,, \tag{17}$$

$$C_\theta = C_q = \frac{2}{3\, C_{p\theta}}\,, \tag{18}$$

$$\mathrm{C} = \frac{C_\theta}{C_{\epsilon_\theta}} = \frac{2}{3\, C_{p\theta}\, C_{\epsilon_\theta}}\,. \tag{19}$$

In addition to $C_{pv}$, $C_{p\theta}$ and $C_{\epsilon_\theta}$, the other independent constants are $C_\epsilon$ in (5) and $C_e$ in (12).

There exist different sets of values for these five independent constants, depending on different papers (like RS81, CBR00 or CCH02, see the table bellow) or different tuning of the models (like for the old PERIDOT).

| | |
|---|---|
| PERIDOT | the tuning of the old PERIDOT LAM |
| RS81 | Redelsperger and Someria (1981) |
| CBR00 | Cuxart, Bougeault and Redelsperger (2000) |
| CCH02 | Cheng, Canuto and Howard (2002) |

The choice of the PERIDOT values for the constants must be made in accordance with the choice of the mixing and dissipation lengths given by the Eqs.(56) and (57). Similarly, the choice of the RS81 and CBR00 set of constants is associated with the mixing and dissipation lengths given by the Eq.(55).

The use of the CCH02 set of constant gave rise from common diagnostics made in Climate and Meso-scale runs, from which the RS81 and CBR00 tuning lead to a too weak mixing of the wind in the PBL. The CCH02 tunings partly solve this problem, with an enhancement of the momentum fluxes (a decrease in $C_{pv}$), an increase of the dissipation (a decrease in $C_\epsilon$) and with almost the same thermal and moisture fluxes (not so different values for $C_e$, $C_{p\theta}$ and $C_{\epsilon_\theta}$).

| | PERIDOT | RS81 CBR00 | CCH02 |
|---|---|---|---|
| $C_\epsilon$ | 0.70 | 0.7 | 0.845 |
| $C_e$ | 0.20 | 0.40 | 0.34 |
| $C_{pv}$ | 1.33 | 4.0 | 2.11 |
| $C_{p\theta}$ | 3.33 | 4.0 | 4.65 |
| $C_{\epsilon_\theta}$ | (-) | 1.2 | 1.01 |

The corresponding values for C, $C_m$ and $C_\theta$ write

| | PERIDOT | RS81 CBR00 | CCH02 |
|---|---|---|---|
| $\mathrm{C} = 2/(3\, C_{p\theta} C_{\epsilon_\theta})$ | (-) | 0.139 | 0.143 |
| $C_m = 4/(15\, C_{pv})$ | 0.20 | 0.0667 | 0.126 |
| $C_\theta = C_q = 2/(3\, C_{p\theta})$ | 0.20 | 0.167 | 0.143 |

TAB. 1 – Value of the constants to be used in the ARPEGE NAMELIST

| | PERIDOT | RS81<br>CBR00 | CCH02 |
|---|---|---|---|
| AKN $= C_m = 4/(15\,C_{pv})$ | 0.20 | 0.0667 | 0.126 |
| ALPHAT $= \alpha_T = (5\,C_{pv})\,/(2\,C_{p\theta})$ | 1.0 | 2.5 | 1.13 |
| ALD $= 1/C_\epsilon$ | 1.43 | 1.43 | 1.18 |
| ALPHAE $= \alpha_e = C_e/C_m$ | 1.0 | 6.0 | 2.7 |
| $\phi_3 = \psi_3$ (see ACBRPHIM) | $\equiv 1$ | $\in [0.78; 2.2]$ | ( ?) |

In ARPEGE, there is five constants which are all set in the NAMELIST of the model. In addition to the same constant C used in (15) for the definition of $\phi_3 = \psi_3$, the four other constants are AKN, $\alpha_T$, $\alpha_e$ and ALD, corresponding to

$$K_m \;=\; \text{AKN}\;\; L_m\,\sqrt{\overline{e}} \qquad \Rightarrow \quad \text{AKN} \;=\; C_m \;=\; \frac{4}{15\,C_{pv}} \quad ; \quad C_{pv} \;=\; \frac{4}{15\;\text{AKN}}\,, \tag{20}$$

$$K_T \;=\; K_q \;=\; \alpha_T\,K_m\,\phi_3 \;\Rightarrow \quad \alpha_T \;=\; \frac{C_\theta}{C_m} \;=\; \frac{5}{2}\,\frac{C_{pv}}{C_{p\theta}} \quad ; \quad C_{p\theta} \;=\; \frac{2}{3\;\text{AKN}\;\alpha_T}\,, \tag{21}$$

$$K_e \;=\; \alpha_e\,K_m \qquad \Rightarrow \quad \alpha_e \;=\; \frac{C_e}{C_m} \;=\; \frac{15}{4}\,C_{pv}\,C_e \quad ; \quad C_e \;=\; \alpha_e\;\;\text{AKN}\,, \tag{22}$$

$$D\text{iss} \;=\; \frac{\overline{e}\,\sqrt{\overline{e}}}{\text{ALD}\;\;L_\epsilon} \qquad \Rightarrow \quad \text{ALD} \;=\; \frac{1}{C_\epsilon} \qquad ; \quad C_\epsilon \;=\; \frac{1}{\text{ALD}}\,. \tag{23}$$

As a consequence, the different sets of constants for ARPEGE are given by the Table(1). The large decrease in $\alpha_e$ from 6.0 to 2.7 when passing from CBR00 to CCH02 is associated with a large increase in AKN, with a resulting impact on $K_e$ and values of $C_e$ which remains nearly the same (0.40 versus 0.34).

### 1.3.3 A3-c : The vertical mixing and the (dynamic+thermal) productions

The dissipation term is given directly by (5). The vertical mixing term and the dynamical production term are computed by putting (10) and (12) into (2) and (3), with $dp = -\,\rho\,g\,dz \;=\; -\,\rho\,d\phi$, leading to

$$\text{Diff}_{\text{vert}} \;=\; -\,g\,\frac{\partial}{\partial p}\left(\rho\;g\;K_e\,\frac{\partial \overline{e}}{\partial \phi}\right) \;=\; -\,g\,\frac{\partial}{\partial p}\left(\rho\;g\;C_e\,L_m\,\sqrt{\overline{e}}\,\frac{\partial \overline{e}}{\partial \phi}\right)\,, \tag{24}$$

$$P_{\text{dyn.}} \;=\; K_m\left[\left(\frac{\partial \overline{u}}{\partial z}\right)^2 + \left(\frac{\partial \overline{v}}{\partial z}\right)^2\right] \;=\; C_m\,L_m\,\sqrt{\overline{e}}\left[\left(\frac{\partial \overline{u}}{\partial z}\right)^2 + \left(\frac{\partial \overline{v}}{\partial z}\right)^2\right]\,. \tag{25}$$

The thermal production term (4) is computed differently. It is not computed as the product of an exchange coefficient with the associated gradient, i.e.

$$\overline{w'\theta'_{vl}} \;\neq\; -\,C_\theta\,L_m\,\sqrt{\overline{e}}\,\frac{\partial\,\overline{\theta}_{vl}}{\partial z}\;\phi_3\,. \tag{26}$$

The usual method is rather to express $P_{\text{ther.}}$ in terms of the conservative Betts variables, given by (8) and (9), leading to a formulation

$$P_{\text{ther.}} \equiv \beta \, \overline{w'\theta'_{vl}} = \beta \, E_\theta \, \overline{w'\theta'_l} + \beta \, E_q \, \overline{w'q'_t} \, , \tag{27}$$

where the two terms $E_\theta$ and $E_q$ are two non-trivial coefficients to be determined, which depends on the normalized saturation deficit and to the the sub-grid variance of the humidity (cloud water).

The two following additional hypotheses are made in (27)

$$\overline{w'\theta'_l} = - C_\theta \, L_m \, \sqrt{\overline{e}} \, \frac{\partial \, \overline{\theta}_l}{\partial z} \, \phi_3 \, , \tag{28}$$

$$\overline{w'q'_t} = - C_q \, L_m \, \sqrt{\overline{e}} \, \frac{\partial \, \overline{q_t}}{\partial z} \, \psi_3 \, , \tag{29}$$

with $\phi_3 = \psi_3$ given by (15) and (16).

### 1.3.4 A3-d : The sub-grid variability of cloud water

The two coefficients $E_\theta$ and $E_q$ are to be determined in order to allow the computation of the moist thermal production (27). To do so, some additional hypotheses must be done, concerning the sub-grid variability of the humidity. It is the reason why the moist turbulent scheme is so dependent on the way to represent the moist processes.

We will describe the same method that is already used in the old version 3, also in the actual version 4, of the ARPEGE-GCM (see Ricard and Royer, 1993). Let us define the parameter "$s$" which depends on the sub-grid departure terms $\theta'_l$ and $q'_t$ of the conservative Betts variables :

$$s = \frac{a}{2} \, (q'_t - \alpha_1 \, \theta'_l) \, , \tag{30}$$

$$a = \left[ 1 + \frac{L_{v/f}}{c_p} \left( \frac{\partial \, q_{sat}}{\partial \, T} \right)_{(T=T_l)} \right]^{-1} \, , \tag{31}$$

$$\alpha_1 = \frac{T}{\theta} \left( \frac{\partial \, q_{sat}}{\partial \, T} \right)_{(T=T_l)} \, . \tag{32}$$

Both $a$ and $\alpha_1$ depend on the derivative with respect to $T$ of the saturating water vapor $q_{sat}$, where the derivative is computed at the value $T_l$.

Let us denote by $Q_1$ the normalized saturation deficit, which depends both on $a$ and on the standard deviation of $s$, denoted by $\sigma_s$, leading to

$$Q_1 = a \left[ \frac{q_t - q_{sat}(T_l)}{2 \, \sigma_s} \right] \, . \tag{33}$$

The standard deviation of $s$, i.e. $\sigma_s$, writes in terms of $a$, $\alpha_1$ and the second order fluxes, giving

$$\sigma_s = \frac{a}{2} \left[ \, \overline{(q'_t)^2} - 2 \, \alpha_1 \, \overline{(q'_t \, \theta'_l)} + (\alpha_1)^2 \, \overline{(\theta'_l)^2} \, \right]^{1/2} \, . \tag{34}$$

From CBR00, the second order fluxes write

$$\overline{(q_t')^2} = \mathrm{C}\,(L_m)^2\,\psi_3\,\left(\frac{\partial \overline{q_t}}{\partial z}\right)^2 , \tag{35}$$

$$\overline{(\theta_l')^2} = \mathrm{C}\,(L_m)^2\,\phi_3\,\left(\frac{\partial \overline{\theta}_l}{\partial z}\right)^2 , \tag{36}$$

$$\overline{(q_t'\,\theta_l')} = \mathrm{C}\,(L_m)^2\,\frac{(\phi_3+\psi_3)}{2}\,\left(\frac{\partial \overline{q_t}}{\partial z}\right)\left(\frac{\partial \overline{\theta}_l}{\partial z}\right) . \tag{37}$$

Let us replace the second order fluxes (35) to (37) into (34), with the property $\phi_3 = \psi_3$ valid for the present "1D-column" version of the scheme. The result can be factorized into the square of a quantity expressed in terms of the vertical gradients of $\overline{\theta}_l$ and $\overline{q_t}$, leading to

$$\sigma_s = \frac{a}{2}\,\sqrt{\mathrm{C}\,(L_m)^2\,\phi_3}\,\left|\,\frac{\partial \overline{q_t}}{\partial z} - \alpha_1\,\frac{\partial \overline{\theta}_l}{\partial z}\,\right| . \tag{38}$$

It is possible to write differently the term $\sqrt{\mathrm{C}\,(L_m)^2\,\phi_3}$, in order to express the result in terms of the exchange coefficient $K_T$ given by (13), the constant $C_{\epsilon_\theta}$ given by (19), the mixing length $L_m$ and the turbulent kinetic energy $\overline{e}$, leading to

$$\sqrt{\mathrm{C}\,(L_m)^2\,\phi_3} = \frac{1}{\sqrt{C_{\epsilon_\theta}}}\,\sqrt{\frac{L_m\,K_T}{\sqrt{\overline{e}}}} . \tag{39}$$

The coefficient $1/\sqrt{C_{\epsilon_\theta}}$ is close to 1 in RS81, CBR00 and CCH02 papers

| | PERIDOT | RS81<br>CBR00 | CCH02 |
|---|---|---|---|
| $1/\sqrt{C_{\epsilon_\theta}}$ | (-) | 0.833 | 0.99 |

It is possible to find the equivalent of $1/\sqrt{C_{\epsilon_\theta}}$, denoted by $B_2$ in the version 4 of the ARPEGE-GCM, where a moist diagnostic equation is managed to compute $\overline{e}$ following Ricard and Royer (1993). From Mellor and Yamada (1982), $1/\sqrt{C_{\epsilon_\theta}}$ is equivalent to $B_2 = 10.1$, a value more than ten times higher than for RS81, CBR00 or CCH02.

In the papers of Bechtold et al. (1992, 1995), $B_2 = 0.8$ for the marine Strato-Cumulus and $B_2 = 1.54$ for the trade-winds Cumulus. These values are more close to the one suggested in RS81, CBR00 or CCH02.

### 1.3.5 A3-e : The sub-grid variability of cloud water

The method, described in Mellor (1977), start with the definition of (7), (8) and (9) for the temperature of Lilly and the conservative variables of Betts.

In order to arrive at an equation like (27) for the flux of $\theta_{vl}$, let us write $\theta_{vl}$ in terms of the Betts variables $\theta_l$ and $q_t$ :

$$\theta_{vl} = \theta_l + D_1\, q_t + D_2\, q_c\,, \tag{40}$$

$$D_1 = 0.608\,\theta\,, \tag{41}$$

$$D_2 = \left(\frac{L_{v/f}}{c_p\, T} - 1.608\right)\,\theta\,. \tag{42}$$

From (40), the flux of the Lilly temperature writes

$$\overline{w'\theta'_{vl}} = \overline{w'\theta'_l} + D_1\,\overline{w'q'_t} + D_2\,\overline{w'q'_c}\,. \tag{43}$$

The last terms of (40) or (43), i.e. $D_2\, q_c$ or $D_2\overline{w'q'_c}$, cannot be computed without making additional hypotheses. There are 3 such hypotheses, all involving the fluxes, not directly the variables.

**Hypothesis 2 :** established by Mellor (1977) for the Gaussian distributions, it has been extended for non-Gaussian cases by Bougeault (1982). From Bougeault (1982), Cuijpers and Bechtold (1995), Bechtold et al. (1995), it is assumed that the residual term $\overline{w'q'_c}$ in (43) can be expressed in terms of '$s$' defined by (30), leading to

$$\overline{w'q'_c} = \overline{w's'}\;\lambda_3(Q_1, A_S)\;\left\{\frac{\overline{s'q'_c}}{(\sigma_s)^2}\right\}\,, \tag{44}$$

$$\text{où}\quad \overline{w's'} \equiv \frac{a}{2}\,\left[\,\overline{s'q'_t} - \alpha_1\,\overline{s'\theta'_l}\,\right]\,, \tag{45}$$

where $\lambda_3$ is a function of the two variables $(Q_1, A_S)$, function defined hereafter, depending of the normalized saturation deficit $Q_1$ and the asymmetry factor $A_S$.

**Hypothesis 3 :** from Bougeault (1982), it is assumed that the last term of (44) can be written as

$$\frac{\overline{s'q'_c}}{(\sigma_s)^2} = 2\;F_2(Q_1, A_S)\,, \tag{46}$$

où $F_2$, like $\lambda_3$, is a function of the two variables $(Q_1, A_S)$.

Finally, from (43) to (46), the flux of $\theta_{vl}$ can be rewritten as

$$\overline{w'\theta'_{vl}} = \left[\,\overline{w'\theta'_l} + D_1\,\overline{w'q'_t}\,\right] - \{a\,D_2\,F_2\,\lambda_3\}\left[\,\alpha_1\overline{w'\theta'_l} - \overline{w'q'_t}\,\right]. \tag{47}$$

It is the sum of two terms. The first term into brackets do not depends on $F_2$ or the variability in cloud water and moisture. The second term directly depends on the variability in cloud water and moisture, via the product of the four terms $a\,D_2\,F_2\,\lambda_3$.

**Hypothesis 4 :** from Bougeault (1982), it is assumed that the probability density function $G(s)$ is a possibly mixed symetric (Gaussian) and asymmetric (exponential) function. This PDF gives the average values over the grid-cell for the cloud cover $\overline{N}_s$, for the cloud water content $\overline{q_c}_s$ and the normalized second order flux $\overline{s'q'_c}\,/\,[\,2\,(\sigma_s)^2\,]$. It results

$$\overline{N}_s = F_0(Q_1, A_S) = \int_{-Q_1}^{+\infty} G(t)\,dt\,, \tag{48}$$

$$\frac{\overline{q_c}_s}{2\,\sigma_s} = F_1(Q_1, A_S) = \int_{-Q_1}^{+\infty} (Q_1 + t)\,G(t)\,dt\,, \tag{49}$$

$$\frac{\overline{s'q'_c}}{2\,(\sigma_s)^2} = F_2(Q_1, A_S) = \int_{-Q_1}^{+\infty} t\,(Q_1 + t)\,G(t)\,dt\,. \tag{50}$$

TAB. 2 – Value of $A_S$ and $\lambda_3$ in terms of $Q_1$.

| Exp / Asym. (Cu) | Mixed regime ( Cu / Sc) | Gaussian (Sc) |
|---|---|---|
| $Q_1 < -2$ | $-2 < Q_1 < 0$ | $Q_1 > 0$ |
| $A_S = 2$ | $A_S = -Q_1$ | $A_S = 0$ |
| $\lambda_3 = 3$ | $\lambda_3 = 1 - Q_1$ | $\lambda_3 = 1$ |

For the given PDF, i.e. $G(s)$, all the functions $F_0$, $F_1$, $F_2$ and $\lambda_3$ can be determined analytically and computed numerically. They depend on the two variables $(Q_1, A_S)$ and they determine the flux (47), and thus the thermal production term (27).

The Table (2) gives the definitions for $A_S$ and $\lambda_3$ in terms of the normalized saturation deficit $Q_1$. The symmetric Gaussian distributions regime ($Q_1 > 0$) is supposed to represent the Strato-Cumulus. The asymmetric Exponential regime ($Q_1 < -2$) is supposed to represent the Cumulus regime. The regime in between, a mix of the Gaussian and the Exponential PDFs, has been pre-computed and tabulated in the Meso-NH model.

The "large-scale" (or "stratiform") cloud cover $\overline{N}_s$ is computed at each time step and for each grid-point with (48). Similarly, the "large-scale" cloud water content $\overline{q_{cs}}$ is computed with (49), with $2\,\sigma_s$ given by (38) and (39).

These "large-scale" cloud cover and cloud water content are merge with the "convective" quantities, with some overlapping assumptions, in order to transmit them to the radiation code.

Also, the "large-scale" cloud water content is transmitted to the Bulk prognostic scheme of condensation and precipitation.

The identification of (27) with (47) leads to the following formulas for $E_\theta$ et $E_q$ :

$$E_\theta = 1 - \alpha_1 \left\{ a\, D_2\, F_2\, \lambda_3 \right\} , \tag{51}$$

$$E_q = D_1 + \left\{ a\, D_2\, F_2\, \lambda_3 \right\} , \tag{52}$$

### 1.3.6 A3-f : The mixing and dissipation lengths

The mixing length $L_m$ and the dissipation length $L_\epsilon$ are computed in a non-local way, following Bougeault et Lacarrère (1989). Starting from a level at the altitude $z$ where the energy is set to the local turbulent kinetic energy $\overline{e}\,(z)$, the non-local lengths $L_{up}$ and $L_{down}$ are computed as the possible upward and downward displacements, respectively, until the energy $\overline{e}\,(z)$ is equal to the integral of the work of the buoyancy force, expressed in terms of $\overline{\theta}_{vl}$ give by (7).

It results

$$\overline{e}\,(z) = \int_z^{z+L_{up}} \beta \left[ \overline{\theta}_{vl}(z') - \overline{\theta}_{vl}(z) \right] dz' , \tag{53}$$

$$\overline{e}\,(z) = \int_{z-L_{down}}^{z} \beta \left[ \overline{\theta}_{vl}(z) - \overline{\theta}_{vl}(z') \right] dz' . \tag{54}$$

The mixing length $L_m$ and the dissipation length $L_\epsilon$ are the same in the present formulation. Moreover, no difference is made between possible dynamic and thermal versions of $L_m$, as done in other models.

The formulation of Meso-NH is retain in the ARPEGE-GCM :

$$L_m \;=\; L_\epsilon \;=\; \left[ \frac{1}{2} \left\{ \left(L_{up}\right)^{-2/3} + \left(L_{down}\right)^{-2/3} \right\} \right]^{-3/2} . \tag{55}$$

The formulations used in the old PERIDOT model gave distinct values for $L_m$ and $L_\epsilon$

$$L_m \;=\; 2 \,\star\, \mathrm{Min}(\, L_{up},\, L_{down}\,) \,, \tag{56}$$

$$L_\epsilon \;=\; \sqrt{L_{up}\, L_{down}} \,. \tag{57}$$

Close to the ground, the Karman law $L_m \approx 0.4\, z$ is not set. Instead, following the theoretical arguments of Redelsperger, Mahé and Carlotti (2001), it could be normal to approach $L_m \approx 2.8\, z$ in true convective cases (values of $L_m \approx 2\, z$ are simulated in the RICO or ARM-Cu SCM cases).

For the stable regimes, the numerical scheme for the computation of $L_{up}$ and $L_{down}$ are made in a very accurate way, with a second order scheme, in order to be as close as possible to the length of Deardorff

$$L_D \;= \sqrt{\frac{2\,\overline{e}}{\beta\,\left(\partial\overline{\theta}_{vl}/\partial z\right)}} \;=\; \frac{\sqrt{2}}{N}\,\sqrt{\overline{e}}\,, \tag{58}$$

where $N^2 = \beta\,\partial\overline{\theta}_{vl}/\partial z$ is the square of the Brunt Väisälä frequency.

The length of Deardorff can be very small for very stable cases ($L_m \equiv L_\epsilon \approx L_D \ll 1$ m), because in that cases $\overline{e}$ is small, reaching the minimum value of $10^{-4}$ to $10^{-6}$ $m^2\ s^{-2}$, also because $\partial\,\overline{\theta}/\partial z \gg 0$ is large.

As a consequence, the small values for both $L_m$ and $\overline{e}$ in the stable regions lead to small values for the exchange coefficients $K_m$ to $K_q$ given by (10) to (14), which all vary as $L_m\,\sqrt{\overline{e}}$).

The large scale ARPEGE-GCM must be able to manage all the cases, from the surface layer to the mesosphere, including the planetary boundary layer, the troposphere and the stratosphere. In order to maintain a minimum vertical mixing in all stable regions, some modifications have been include after the computation of $L_m$ and $L_\epsilon$ in (55).

$$L_m \;=\; L_\epsilon \;=\; \mathrm{Max}\left[\, L_m \,;\, \mathrm{Min}\left(\lambda_E \,;\, 0.4\, z\right) \right] . \tag{59}$$

As a consequence, the asymptotic value $\lambda_E$ replaces the Deardorff length in the stable regions. Close to the ground, the formulae (59) is a security, leading to $L_m \geq 0.4\, z$ for $z < \lambda_E/0.4$ and $L_m \geq \lambda_E$ for $z > \lambda_E/0.4$. Typically, $\lambda_E = 10$ m and $\lambda_E/0.4 = 25$ m.

### 1.3.7 A3-g : The turbulent kinetic energy in the surface layer

The values of $\overline{e}$ are given by the prognostic equation (1) for each upper-air half-levels, i.e. on the half-levels where the exchange coefficients $K_m$ to $K_q$ are computed, at the

middle points between the full-levels where all the thermodynamics variables are available (temperature, wind, specific humidity, ...)

But the vertical mixing processes represented by (2) needs the knowledge of $\overline{e}$ at the groung level or in the surface layer, denoted by $\overline{e}_S$.

In the present version of the code, $\overline{e}_S \equiv [\overline{(u'_S)^2} + \overline{(v'_S)^2} + \overline{(w')^2}]/2$ is computed following André et al. (1978), see their Eq.(29), p.1866, with the term $(-\zeta)^{2/3}\,(u_*)^2$ dropped in the instable case, leading to :

$$\overline{e}_S \;=\; 3.75\,(u_*)^2 \;+\; 0.3\,(w_*)^2\,(1-\delta_{stab})\,. \tag{60}$$

The first part, valid for both stable and instable conditions, depends on the friction velocity $u_*$. The values of $u_*$ are given by the "Bulk" scheme of Louis (1979, 1982), previously computed in ACHMT in ARPEGE.

The second part, only valid for the instable (convective) cases, i.e. for $1-\delta_{stab}=1$, and including the term $0.3\,(w_*)^2$, depends on the convective velocity $w_*$ (Deardorff, 1980), defined by :

$$w_* \;=\; (\,\beta\,H_{PBL}\,Q_0\,)^{1/3}\;, \tag{61}$$

$$\text{où}\;\; Q_0 \;=\; (\overline{w'\theta'})_{surf} \;=\; -\,C_h\,|u_N|\,(\Delta\theta)_N\,. \tag{62}$$

In stable regime, (60) reduces to $\overline{e}_S \;=\; 3.75\,(u_*)^2$.

The term $H_{PBL}$ in (61) is a diagnostic PBL height. It is computed starting from the ground as the height of the first level where an important decrease in $\overline{e}$ occurs, corresponding to the top of the PBL and the beginning of the stable layers located above the PBL.

The term $C_h$ in (62) is the thermal surface Drag coefficient given by (65). The term $|u_N|$ is the norm of the wind speed at the first upper-air level above the ground. The term $(\Delta\theta)_N$ is the difference in $\theta$ between the ground level and the first upper-air level above the ground.

### 1.3.8 A3-h : The "Bulk" formulations in the surface layer

The neutral, dynamical and thermal versions of the surface Drag are denoted by $C_{dn}$,$C_d$ and $C_h$, respectively. They are computed following the "Bulk" scheme of Louis (1979, 1982) :

$$C_{dn}(z,z_0) \;=\; \left(\frac{0.4}{\log(1+z/z_0)}\right)^2\,, \tag{63}$$

$$C_d(R_i,z,z_0) \;=\; f_d(R_i)\;C_{dn}(z,z_0)\,, \tag{64}$$

$$C_h(R_i,z,z_0) \;=\; f_h(R_i)\;C_{dn}(z,z_0)\,. \tag{65}$$

The functions $f_d(R_i)$ and $f_h(R_i)$ only depend on the Richardson's number $R_i$. The coefficient $C_h$ given by (65) is the one used to define $Q_0$ in (62), in order to compute $w_*$ by (61).

### 1.3.9 A3-i : The vertical mixing and the dissipation term

The dissipation term (5) writes $C_\epsilon \, \overline{e} \, \sqrt{\overline{e}}/L_\epsilon$. It is computed numerically by using either explicit or implicit methods, depending of the value of the two coefficients $\alpha_{exp}$ and $\alpha_{imp}$ (possible control in the NAMELIST of ARPEGE) :

| | explicit | implicit-I | implicit-II |
|---|---|---|---|
| $\alpha_{exp}$ | 1.0 | 0.0 | -0.5 |
| $\alpha_{imp}$ | 0.0 | 1.0 | 1.5 |

The term $\sqrt{\overline{e}}$ in the numerator of $C_\epsilon \, \overline{e} \, \sqrt{\overline{e}}/L_\epsilon$ always appears in the explicit form $\sqrt{\overline{e}(t)}$ . The two coefficients $\alpha_{exp}$ and $\alpha_{imp}$ determine how the other term $\overline{e}$ is computed, leading to the following discretization for the term $(\overline{e})^{3/2}$, in terms of the time steps $t$ and $t + dt$ :

$$\sqrt{\overline{e}(t)} \;\; [\, \alpha_{imp} \, \overline{e}(t + dt) \;+\; \alpha_{exp} \, \overline{e}(t) \,] \;. \tag{66}$$

The three methods (explicit, implicit-I or implicit- II) correspond to :

$$\text{explicit :} \quad \sqrt{\overline{e}(t)} \;\; [\;\; \overline{e}(t) \,] \;, \tag{67}$$

$$\text{implicit (I) :} \quad \sqrt{\overline{e}(t)} \;\; [\;\; \overline{e}(t + dt) \,] \;, \tag{68}$$

$$\text{implicit (II) :} \quad \sqrt{\overline{e}(t)} \;\; [\; 1.5 \, \overline{e}(t + dt) \;-\; 0.5 \, \overline{e}(t) \,] \;. \tag{69}$$

The last formulation is obtained when the term $X^{3/2} = (\overline{e})^{3/2}$ is written as the following Taylor expansion (personal communication of V. Masson)

$$X^{3/2} \;=\; (X_0)^{3/2} \;+\; 1.5 \, (X_0)^{1/2} \, (X - X_0) \,, \tag{70}$$

$$(\overline{e})^{3/2}(t + dt) \;=\; \overline{e}(t) \, \sqrt{\overline{e}(t)} \;\; +\; 1.5 \, \sqrt{\overline{e}(t)} \;\; [\, \overline{e}(t + dt) - \overline{e}(t) \,] \,, \tag{71}$$

where $1.5 \, (X_0)^{1/2}$ is the derivative of $X^{3/2}$ at $X_0$, leading indeed to (69).

The choice of any of (67) to (69) for the explicit or implicit terms $\overline{e}(t)$ and $\overline{e}(t + dt)$ leads to important changes in the core of the tri-diagonal solver of the vertical mixing (2), with the possible use of provisional variables during the algorithm.

### 1.3.10 A3-j : The Top-PBL vertical entrainment

The vertical resolution of the large scale models, either in NWP or in GCM mode, are often very coarse. Above an altitude of 1000 to 1500 m height, the layer depth are typically as high as $\Delta z > 200$ to 300 m.

With such coarse vertical resolutions (above the critical value of $(\Delta z)_{crit} \approx 20$ m to 50 m) it is not possible to represent in a realistic way the vertical entrainment in the Stratocumulus. This vertical entrainment occurs at the very top of the PBL, where the dry air above the Top-PBL is mixed into the underlying cloudy air, located below the Top-PBL height.

In order to parameterize this sub-grid process (sub-grid on the vertical), the old ideas of Tenekes (1973), revisited by Nicholls and Turton (1986) or Grenier et Bretherton (2001), and summarize in the note of Grenier (2002), has been implemented in the CBR00 version of ARPEGE. Some additional modifications has been tested, with SCM and 3D (NWP and GCM) validations.

The approach of Tenekes (1973) was implemented at the end of the subroutine VDFEXCU < VDFMAIN < VDFOUTER < CALLPAR in the older version of IFS (see for instance the cycle 24T1, the one corresponding to the Climate version-4 of ARPEGE). The flux at the inversion level, i.e. at the top of the PBL, was replaced by "- RENTR" times the surface flux. For RENTR=0.20, and with a security to only allow an enhancement of the fluxes, the formulation is $(\overline{w'\theta'})_{inv} \equiv \ MAX[\ (\overline{w'\theta'})_{inv}; -0.2\ (\overline{w'\theta'})_{surf}\ ]$. It is the more simple formulation of Tenekes (1973).

In the more recent cycle of IFS, for instance in the cycle 33t1, or in the cycle 32t0_op1 corresponding to the Climate version-5 of ARPEGE, the scheme of Lock et al. (2000, Eq.(5) p-3191) is implemented, still at the end of VDFEXCU. The computations of the new exchange coefficients are made for each vertical levels, leading to the new vertical profiles $K_m(z)$ (for momentum) and $K_h(z)$ (for heat and moisture), both valid within the whole PBL. They are computed as the sum of two profiles, extending from the surface to the height $z_i$ for the surface driven part $K^{surf}(z)$, extending from $z_b$ to $z_{ml}$ for the top-driven part $K^{Sc}(z)$. The surface driven part and the top-driven part may overlap, or they may not.

The IFS formulas for $K_h(z)$ and $K_m(z)$ are given by

$$K_h^{surf}(z) = 0.85\,\kappa\,V_{surf}\ z\ \left(1-\frac{z}{z_i}\right)^2\ , \tag{72}$$

$$K_h^{Sc}(z) = 0.85\,\kappa\,V_{Sc}\ \frac{(z-z_b)^2}{z_{ml}}\ \sqrt{1-\frac{z-z_b}{z_{ml}}}\ , \tag{73}$$

$$K_m^{Sc}(z) = 0.75\,K_h^{Sc}(z)\ , \tag{74}$$

$$K_h(z) \equiv \ \mathrm{MAX}\,[\,K_h(z)\ ;\ K_h^{Sc}(z)+K_h^{surf}(z)\,]\ , \tag{75}$$

$$K_m(z) \equiv \ \mathrm{MAX}\,[\,K_m(z)\ ;\ K_m^{Sc}(z)+K_m^{surf}(z)\,]\ . \tag{76}$$

The Von Karman constant is $\kappa = 0.4$. There are three different scale heights $z_i$, $z_b$ and $z_{ml}$, with two velocity scales $V_{surf}$ and $V_{Sc}$ (see Lock et al., 2000). The Prandtl number is 0.75 for $K^{Sc}(z)$.

The code presently implemented in the climate version 5 of ARPEGE is different from the two IFS's schemes. It is based on the ideas of Grenier (2002), with a formulation more non-local and with more physical processes taken into account than in the older code of the IFS. The difference from the more recent code of the IFS is that the change in $K_h$ and $K_m$ will only possibly exist at the inversion level $z_{inv}$, the exchange coefficient will not be modified for the other levels.

The first step is to compute the inversion height $z_{inv}$. It could be defined as a jump in either $\partial\overline{\theta}/\partial z$ or $\theta$, or when the Ridchardson number $R_i$ becomes greater than a threshold value $(R_i)_{crit}$. It is rather computed in the code starting from the ground as the height of

the first level where an important decrease in $\overline{e}$ occurs, corresponding to the top of the PBL and the beginning of the stable layers located above the PBL.

The second step is to replace the exchange coefficients previously computed by (10) to (14) by another coefficient, computed at the inversion, denoted by $K_{inv}$ and verifying

$$\begin{aligned} (\overline{w'\theta_{vl}'})(z_{inv}) &= -\, w_{ent}\, \Delta_{inv}(\theta_{vl})\,, & (77)\\ &= -\, K_{inv}\, \frac{\Delta_{inv}(\theta_{vl})}{\Delta_{inv}(z)} \quad \Rightarrow \quad K_{inv} \;=\; w_{ent}\, \Delta_{inv}(z)\,, & (78) \end{aligned}$$

where $w_{ent}$ is called "entrainment velocity" (at the inversion) and where $\Delta_{inv}(\theta_{vl})$ is the buoyancy jump in $\theta_{vl}$ and across the layer surrounding this inversion $\Delta_{inv}(z)$.

Following the results of Grenier and Bretherton (2001) and some unpublished ideas of Grenier (Workshop EUROCS at Utrecht, in April 2002), the Top-PBL entrainment exchange coefficient $K_{inv}$ is defined at the inversion level by

$$K_{inv} \;=\; w_{ent}\, \Delta_{inv}(z) \;=\; A_{inv}\, \frac{<\overline{e}>^{3/2}}{L_{inv}\; N^2_{inv}}\,. \qquad (79)$$

Presently, there is no Prandtl number and all the exchange coefficients (10) to (14) are replaced by the same value (79).

Grenier (2002) has introduced the average value over the whole PBL of the turbulent kinetic energy, denoted by

$$<\overline{e}> \;=\; \frac{1}{z_{inv}} \int_0^{z_{inv}} \overline{e}(z)\, dz\,. \qquad (80)$$

The unknown parameters in (79) are the Richardson number at the inversion $N^2_{inv}$, the mixing length at the inversion $L_{inv}$ and an adimentional coefficient $A_{inv}$. These parameters are defined by

$$\begin{aligned} N^2_{inv} &= \beta \left[\frac{\Delta_{inv}(\theta_{vl})}{\Delta_{inv}(z)}\right]_{(z=z_{inv})}\,, & (81)\\ L_{inv} &= 0.085\; z_{inv}\,, & (82)\\ A_{inv} &= A_1 \left[1 + A_2\, \frac{L_{v/f}\; <\overline{q_c}>}{c_p\, \Delta_{inv}(\theta_{lt})}\right]\,. & (83) \end{aligned}$$

There is a security for the computation of $K_{inv}$ in (79) where, from (81), the division by $N^2_{inv}$ leads to a division by the term $\Delta_{inv}(\theta_{vl})$ which can be tiny or even equal to 0. A minimum value of 1.5 K or so is managed by setting AJBUMIN= 0.005 in the NAMELIST, with $\Delta_{inv}(\theta_{vl}) > \text{AJBUMIN} \star \theta$.

The part of the entrainment coefficient $A_{inv}$ which is control by $A_2$ is called the "evaporative enhancement of entrainment" in Grenier and Bretherton (2001). The jump in potential temperature $\Delta_{inv}(\theta_{lt})$ corresponds to another moist potential temperature, different from the Lilly one $\theta_{vl}$ given in (7). This conservative potential temperature $\theta_{lt}$ is a mix of the Betts variables, defined by

$$\theta_{lt} \;=\; \overline{\theta}_l\, (1 + 0.608\, \overline{q}_t)\,. \qquad (84)$$

TAB. 3 – Value of $A_1$ and $A_2$ for the Strato-Cumulus case FIRE-I

| | Sc | Sc | Sc | Sc | Cu |
|---|---|---|---|---|---|
| $A_1$ | 0.50 | 0.30 | 0.20 | 0.15 | 200 |
| $A_2$ | 0 | 20 | 60 | 80 | 0 |

There is a second security in the computation of $A_{inv}$ in (83) where, the division by $\Delta_{inv}(\theta_{lt})$ could lead to a division by a tiny value, possibly equal to 0. A minimum value AGREDTH from 1.5 K to 2.5 K is available in the NAMELIST, with $\Delta_{inv}(\theta_{lt}) >$ AGREDTH.

Similarly to the use of the PBL average value (80) in (79), suggested by Grenier (2002), E. Bazile has suggested in 2008 to replace in (83) the local value $q_c$ by the average value over the whole PBL (liquid plus solid), denoted by

$$< \overline{q_c} > \;=\; \frac{1}{z_{inv}} \int_0^{z_{inv}} \overline{q_c}(z)\, dz\,. \tag{85}$$

The choice for the numerical values of $A_1$ and $A_2$ has been made with an analysis of the best tuning for the FIRE-I SCM case. For the L31 hybrid levels currently used in the ARPEGE-GCM, it appears that good results are obtained with the following sets of values described in the "Sc" columns of the Table (3).

For the Strato-Cumulus regions, i.e. where the jump in potential temperature $\Delta_{inv}(\theta_{lt})$ is large (for instance equal to 8 K for FIRE-I), the previous choice of $A_1$ and $A_2$ is retained in the GCM.

Clearly, the more the "evaporative enhancement of entrainment" is active (i.e. the more $A_2$ is large), the less $A_1$ needs to be active. One of the weakness of the present parameterization is that for any other vertical resolution, it could be necessary to seek for and test a new set of values.

For the Cumulus regions, let us say for $\Delta_{inv}(\theta_{lt}) < 1.5K$, none of the values shown in the Table (3) gives good results in GCM mode (not enough fluxes at the Top of the PBL). It has been decided to retain this threshold of 1.5 K or so, in order to switch to the tuning $A_2 = 0$ and $A_1 = 200$ for the Cumulus regions, where the Top-PBL entrainment acts at the base of the cloud (whereas it acts at the top of the Strato-Cumulus clouds).

It is possible to rewrite $K_{inv}$ given by (79) in a form similar to (13) or (14), in order to define a kind of "stability function" (the term into brackets).

$$K_{inv} \;=\; \frac{A_{inv}}{2}\, L_{inv} < \overline{e} >^{1/2} \left[ \frac{2 < \overline{e} >}{(L_{inv})^2\, N_{inv}^2} \right]. \tag{86}$$

The term into brackets is a kind of bulk stability function $[\, (L_D)^2_{inv} \,/\, (L_{inv})^2 \,]$, similar to the functions $\phi_3$ and $\psi_3$ defined in (13) and (14), where $(L_D)^2_{inv} \equiv 2 < \overline{e} > /N^2_{inv}$ is the square of a kind of bulk Deardorff length, similar to (58).

### 1.3.11 A3-k : The modular set of subroutines

The very first prognostic TKE scheme used at Météo-France has been coded and used in the old meso-scale PERIDOT LAM. There was at that time two subroutines, called YCOEFKE and YEVOLET. The corresponding ARPEGE subroutines has been coded by P. Lacarrère, with the standard names ACCOEFKE and ACEVOLET.

- YCOEFKE ⇒ ACCOEFKE : (i) compute the surface value $\overline{e}_S$ by (60) ; (ii) compute the upward and downward buoyancy lengths $L_{up}$ and $L_{down}$ by (53) and (54), then the mixing length by (56) and the dissipation length by (57) ; (iii) compute the exchange coefficients $K_m$ and $K_T$ with the set of the constants given in the first column of the Table (1) ; use of a counter-gradient term ; no stability function like $\phi_3$ (or equivalently with $\phi_3 = 1$) ; use of the dry variables $\theta$ and $q_v$, instead of the Betts conservatives variables.
- YEVOLET ⇒ ACEVOLET : numerical computation of the TKE equation (1), with an implicit solver for the vertical diffusion (24) and with $\alpha_{imp} = 1.5$ and $\alpha_{exp} = -0.5$ given by (69). The dissipation term (5) is computed with the help of the same (1.5 ; $-0.5$) implicit scheme. The dynamical and the dry thermal production terms are computed both in this subroutine.

Later on, the code of ARPEGE has been largely modified. The motivation was a wish to get a more modular code, with in particular a separation of a subroutine where only the mixing and dissipation lengths are computed, in a way similar to what is done in Meso-NH and AROME, leading to the three subroutines ACBL89, ACTURB and ACEVOLET presently used in ARPEGE and ALADIN.

- ACBL89 : compute the upward and downward buoyancy lengths $L_{up}$ and $L_{down}$, then the mixing length $L_m$ and the dissipation length $L_\epsilon$ by (55) and (59) ; compute the stability functions $\phi_3 = \psi_3$ given by (15) and (16), with the set of the constants given in the last column of the Table (1), corresponding to CCH02. It is assumed that the grid-cell mean liquid and solid cloud water contents are available as input of ACBL89 and ACTURB.
- ACTURB : compute $\overline{e}_S$ by (60) ; compute the exchange coefficients $K_m$ and $K_T$ with the set of the (CCH02) constants given in the last column of the Table (1) ; no longer use of a counter-gradient term ; compute the moist thermal production term (27) with the help of (47) and (50), where the fluxes of the conservative variables are given by (28) and (29), with $a$ and $D_2$ given by (31) and (42), with $\lambda_3$ given by the Table (2) ; compute the grid-cell mean cloud cover $\overline{N}_s$, the grid-cell mean cloud water content $\overline{q_{cs}}$ and the subgrid normalized second order flux $F_2(Q_1, A_S)$ with the use of (48) to (50) ; compute the Top-PBL height and the exchange coefficient at the Top-PBL inversion.
- ACEVOLET : numerical computation of the TKE equation (1), with an implicit solver for the vertical diffusion (24), with $\alpha_{imp} = 1.5$ and $\alpha_{exp} = -0.5$ given by (69).

The dissipation term (5) is computed with the help of the same $(1.5\,;\,-0.5)$ implicit scheme. The moist thermal production term is an input of the subroutine (computed in ACTURB). The dynamical production term is computed in this subroutine.

## 1.4 A4 : Results and limitations

### 1.4.1 A4-a : Validations with SCM cases

The realism of the dry and moist versions of the TKE prognostic scheme has been evaluated with several SCM cases.

- Until the beginning od the years 2000, only two cases were available : the shallow convection case BOMEX and the deep convection case TOGA-COARE.
- The EUROCS project (2000-03) has bring another set of cases and promoted the use of joint SCM and GCM validations and inter-comparisons in order to improve the physics of the GCM. Since then, the physics of ARPEGE are evaluated with the use of the FIRE-I Stratocumulus case, the ARM-Cumulus case, the Guichard (2002) deep convection case, among others... A new set of parameterization, close to the one corresponding to the Climate version-5 of ARPEGE, has been tested in 2002 within the frame of the international intercomparison managed by Lenderink et al. (2004). Indeed, the standard Version 3 of the ARPEGE GCM has been compared with a new physics containing the prognostic CBR00 TKE scheme, the Lopez (2002) bulk scheme for the precipitations and the Bechtold (2001) shallow convection scheme.
- Since 2002, the same Lopez (2002) and CBR00 schemes has been associated in most of the following Climate validations with the shallow and Deep convections scheme of Gueremy (2005). The Lopez (2002) bulk scheme for the precipitation has been improved and put into the operational suite of the NWP version of ARPEGE. The CBR00 prognostic scheme has been tested in NWP versions of ARPEGE since 2006-07, with a fine cross validation of the Meso-NH and AROME (a priori) similar schemes, a validation made in 2008 on the GABLS-I dry case, involving altogether the GMME, GMAP and GMGEC teams of the CNRM.
- The old SCM cases are still available :
  - BOMEX ;
  - TOGA-COARE ;
  - ARM-Cumulus ;

  But some new cases has been built :
  - GABLS-I and GABLS-II (dry continental polar cases) / collaboration with CNRM-/GMAP ;
  - RICO-I and RICO-II (precipitating shallow convection over sea), workshop GEWEX/GCSS of New-York (2006) and NetFAM-COST of Toulouse (2007) / collaborations with CNRM/GMME and IPSL/LMD ;
  - The deep convection case of F. Guichard / collaborations with CNRM/GMME ;

- The numerous cases of AYOTTE (dry continental PBL cases, with several kinds of surface friction velocities and different jump in $\theta$ at the top-PBL) / collaborations with IPSL/LMD and ENM/UFR.

The WEB page of the EUROCS is
http ://www.cnrm.meteo.fr/gcss/EUROCS/EUROCS.html
The link with the SCM RICO cases are
http ://www.gewex.org/gcss.html and http ://www.knmi.nl/samenw/rico/
see also http ://www.eol.ucar.edu/projects/rico/

### 1.4.2 A4-b : Validation with 2D/Cross-Sections (transects)

In addition to the 1D-SCM and 3D-GCM cross validations, a new kind of intercomparisons has been set up and promoted during the EUROCS program. It has been suggested to run the model in GCM (or NWP) mode, for some particular months, with prescribed SSTs, and to extract the output data on a set of selected points and for several pressure levels, all forming a cross-section over the North-Estern Pacific, from the Equator to the west coast of the USA.

The Climate version-3 of ARPEGE has been evaluated in 2001-02 for the standard physics (see the intercomparison in Siebesma et al., 2004). The Pacific Cross-section Intercomparison (or PCI) has then been renewed in the frame of the GEWEX organisation, with Joao Teixera as coordinator, with more and more US models included.

The two diagnostic (standard) and prognostic (new) physics of the ARPEGE-GCM has been validated for the GEWEX/PCI, with evaluations presented at the Pan GCSS at Athens (2005), in New-York (2006) and in Toulouse (NetFam-COST in 2007). Until 2007, the Lopez and CBR00 schemes were associated with the Gueremy's shallow and deep convections scheme.

In 2008, it has been decided to test the Lopez and CBR00 schemes with the Bechtold (2001) shallow convection and the Bougeault (1985) deep convection, jointly in the GCM-V5 and the NWP versions of ARPEGE. Results obtained with the PCI has been presented at Toulouse in 2008 (Pan-GCSS, 2008), with a special interest paid to the positive impact of the Top-PBL entrainment.

The links with the PCI WEB sites of GEWEX are
http ://www.igidl.ul.pt/cgul/projects/gpci.htm
http ://gcss-dime.giss.nasa.gov/gpci/modsim_gpci_models.html

Just like the EUROCS and GEWEX PCI approaches, it has been decided in the frame of the AMMA program to make an inter-comparison of the models for a South-North Cross-section extending from 40 N to 20 S, with data averaged from 10 W et 10 E.

Some of the WEB sites of the AMMA-MIP (AMMA-cross) of the CNRM, MEDIAS and IPSL/LMD are
http ://amma-mip.lmd.jussieu.fr/Welcome.html
http ://www.cnrm.meteo.fr/amma-moana/transect/

http ://www.cnrm.meteo.fr/gcss/API/API.html
http ://web.lmd.jussieu.fr/∼hourdin/AMMA/
http ://web.lmd.jussieu.fr/∼musat/AMMA-MIP/inter/
http ://amma.mediasfrance.org/
http ://amma.mediasfrance.org/france/index

### 1.4.3 A4-c : Validation in 3D mode (GCM and PNT)

The main objective of the Climate team in GMGEC is the validation of the Climate version of ARPEGE in 3D-GCM mode. During the EUROCS program, and since then, the numerous 1D-SCM and Cross-sections validations has been a powerful tool to understand, modify and improve the 3D physics of the model.

Most of the new parameterizations of the Climate Version-5 were already available in the test versions of the Climate Version-4.6 of ARPEGE, including the prognostic CBR00 turbulent scheme and the prognostic Lopez precipitation scheme, with the Gueremy's (shallow+deep) convections used up to 2007.

This set of new parameterizations has been tested and compared to the standard versions, for several 1D-SCM cases (AYOTTE, GABLS, FIRE-I, BOMEX, ARM-Cu, RICO, Guichard-Deep, TOGA-COARE) and for several Cross-sections intercomparisons (North-Eastern Pacific, African Monsoon). Some high resolutions runs has also been realized with a 10 km ALADIN LAM simulations during the preparation of the AMMA campaign (series of 5 days runs).

There is no problem of stability, both with the time step and the horizontal or vertical resolutions. The usual time step of 1800 s can be used for the T63 / 64x128 Gauss grid / L31. The time step of 450 s for the 10 km ALADIN LAM is correct, too.

More recently, starting from the year 2008, it has been decided to built a set of common parameterizations for the Climate and the NWP versions of ARPEGE, with almost the same versions for the prognostic CBR00 turbulence scheme and the Lopez prognostic precipitation scheme. The differences from the previous simulations concern the Gueremy's (shallow+deep) convections which has been replaced by the new set of Bechtold (shallow) and modified Bougeault/Gerard (deep) ones, including also several debugs and several improvements made by the NWP team in the CBR00 and Lopez schemes. The bougeault/Gerard (deep) convection has been limited by E. Bazile with the constraint to have clouds with a minimum depth (3000 m or so).

It has been possible to prove with several NWP runs that there is almost no fibrillation with the CBR00 TKE prognostic equation (except at the top of the Hymalaya), whereas these fibrillations are large with the Louis scheme (study of Y. Bouteloup).

There are obvious advantages and improvements to use the new physics, improvements observed in all the 1D-SCM, the Cross-Sections and the 3D-GCM runs :

- the marine Srato-Cumulus are more realistic in the new physics, both in NWP and in GCM mode ;
- as expected, there is an important and positive impact of the Top-PBL entrainment scheme which explains and controls most of the improvement of these marine Srato-

Cumulus.

There are also new or other problems that are not seen with the standard physics :

- the low levels wind are too strong, mainly over the African monsoon and the "Somali Jet" regions ;
- the energy budget is not closed for the atmosphere (from $\pm 5$ up to $\pm 10\ W\ m^2$) with the Gueremy's (shallow+deep) convections ;
- the temperature are too high in JJA over the African Heat-low, the Arabic desert and the central Europe.

All these problems are under investigation.

## 1.5 A5 : Next modifications and possible improvements

- Test of the mixing lengths defined in Teixera et Cheinet (2004) or in Teixera et al. (2004), proportional to $\tau \sqrt{\overline{e}}$, where the time scale is typically of $\tau = 600$ s for the instable cases and with $\tau = \min(600, 0.76\ /N)$ s for the stable conditions. Note that for the CBR00 scheme, and from (58), $L_m$ is close to the Deardorff's length which writes for the stable cases $L_m \approx (1.41/N)\ \sqrt{\overline{e}}$, i.e. a formulation similar to what is defined in Cheinet (2004) or Teixera et al. (2004). The main difference is for the unstable cases, where the TKE computed with the CBR00 moist prognostic TKE equation must lead to the BL89 lengths $L_{up}$ and $L_{down}$, also the mixing and dissipation length $L_m$ and $L_\epsilon$ given by (55), which could be different from what is suggested by Cheinet (2004) or in Teixera et al. (2004).
- Test of the mixing length of Lenderink and Holtslag (2004). The mixing length is computed for the instable case starting from some non-local mixing length similar to the BL89 ones defined by (53) and (54). Some simple analytical formulations are given, depending on the Ridchardson number $F(R_i)$ ; There is no upward and downward integrals, i.e. with a much cheaper numerical cost when the number of vertical levels increase. Then an integral length $L_{int}$ is defined. For the stable cases, the result is presented in the same form as in CBR00, Cheinet (2004) or Teixera et al. (2004), i.e. proportional to $\tau \sqrt{\overline{e}}$ and with $\tau = 0.2\ /N$.
- Test of the surface driven and Top-PBL entrainment profiles from Lock et al. (2000, Eq.(5) p-3191), as presently used in the IFS code and implemented at the end of VDFEXCU.
- Test of the Roeckner (1995) and Roeckner et al. (1996) modifications made at the MPI for the prognostic TKE equation. The aim is to get a more implicit numerical scheme for getting a joint impact of three of the terms of (1), the terms proportional to $\sqrt{\overline{e}}$, i.e. the dynamical production (25), the thermal production (28) and (29), the dissipation (5). For these three terms, the usual equation (1) for $\overline{e}$ is transformed into a prognostic equation for $\sqrt{\overline{e}}$ and it is solved numerically with an implicit scheme, leading to a simple quadratic equation. The advantage of this method is that, for longer time step, the time evolution of $\overline{e}(t)$ could follow more accurately the diurnal

cycle.

- Test on the accuracy of a modification of the prognostic TKE equation when the same three terms of (1) proportional to $\sqrt{\overline{e}}$ are parameterized via a pure relaxation term $(\tilde{e} - \overline{e}) / \tau$, as coded in ALARO and following Redelsperger, Mahé and Carlotti (2001). The aim of Redelsperger, Mahé and Carlotti (2001) was not to improve the TKE equation (1) throughout the whole atmosphere, nor for the whole PBL, but possibly close to the surface. The advantage of the method is that the Louis and Geleyn formulations (1979, 1982, 1986) can be somehow transformed into a pseudo-prognostic TKE equation.
- The TKE variable $\overline{e}$ is presently defined on the half levels of the model, where the exchange coefficients are to be computed. This configuration lead to an accurate evaluation of the gradients, but it do not allow the activation of the horizontal nor the vertical advections schemes. Several advices were obtained (Ph. Bougeault, HIRLAM, MPI, ...) that (i) it is indeed important to put forward the accuracy of the scheme ; (ii) the advections are not so important until the resolution of 10 to 5 km ; (iii) it is detrimental to make half to full levels averages before the advections and return, leading to very bad results indeed, as verified in SCM cases by E. Bazile.
- The problem of too strong low-levels winds over some area of the tropical belt is one of the main drawback of the new physics, observed both in NWP and in Climate runs. The switch from the pure RS81 and CBR00 to the CCH02 tunings, as indicated in the section (1.3.2), has only partly solved the problem. Some other actions must be tested.
- It could be interesting to test the ideas similar to those promoted in Chaboureau and Bechtold (2002), i.e. to take into account the mass fluxes coming from the shallow convection and to add a "convective part" to the "turbulent part" of the standard deviation $\sigma_s$ given by (34), leading to a "total part" $(\sigma_s)^2_{tot} = (\sigma_s)^2_{turb} + (\sigma_s)^2_{conv}$. These ideas has already tested in Meso-NH and AROME, they are under considerations in NWP tests (S. Malardel, E. Bazile).
- Would it be possible and/or interesting to used the Meso-NH TURB-1D version of the turbulent code, possibly with a call via a specific interface ?

It is important to note that it is indeed possible to modify at will, via the NAMELIST variables, any changes in one or more of the constants of the CBR00 or CCH02 schemes. The long experience obtained altogether in Meso-scale, NWP and Climate modes has learned to us that arbitrary or too partial changes can be irrelevant.

As an example, the modifications described in the section (1.3.2) and leading to the use of the CCH02 tunings have needed two simultaneous actions : a decrease in $C_{pv}$ to increase the mixing of the wind AND a decrease in $C_\epsilon$ to enhance the dissipation. If only one of the two modification is made, some of the internal equilibrium are no longer verified and the simulations can become unrealistic. Other examples would be the change in the formulations of the mixing lengths, or the change in the way to solve numerically the TKE equation (1).

In such cases of partial or important changes, it would be probably necessary to redo all the validations for all available SCM, Cross-Sections and NWP+GCM configurations... with also possible side effects with the complex interactions between the turbulent scheme and the shallow and deep convections, the precipitations, the radiation, ...

# 2 Partie B : Interactions

## 2.1 B1 : Interactions with other parameterizations

***a) The subgrid variance of moisture.***

The computations of the subgrid variance of moisture (48) to (50) could (or should ?) be put aside of the turbulent scheme.

In the present formulation, it interacts on the definition of the thermal production (47) via the product $F_2\,\lambda_3$, also on the possible reprojection of the fluxes of mixing, see (87) to (89) in the following.

In a more general point of vue, a modification of the choices for the PDF $G(t)$ in (48) to (50) would modify the thermal production (47), but also $\sigma_s$ in (38) via the changes in the variables and their vertical gradients, and so the TKE values $\overline{e}(t)$ themselves, which would modify in turns $L_{up}$ and $L_{down}$ ... and in fact all the simulations.

***b) The condensation, evaporation and precipitation schemes.***

There are important links between the turbulent scheme and the bulk scheme of Lopez (2002) for the condensations and the precipitations.

The prognostic turbulent scheme for $\overline{e}(t)$ needs as input the liquid and solid grid-cell mean values for the cloud water. These grid-cell mean values initiate the computations of the variables (7) to (9), with indirect impacts on $Lup$ and $L_{down}$, then on all the next computations, including in particular the gradients (28) and (29) and the associated fluxes.

Reverse impacts exist from the turbulent scheme toward the bulk scheme of Lopez (2002). The condensed water before activation is diagnosed from (49). It is equal to $\overline{q_c}_s = 2\,\sigma_s\,F_1(Q_1, A_S)$ and it thus depends on $Q_1$ and $\sigma_s$, which depends via (38) on $\phi_3$, $L_m$ and the gradients of the Betts variables. All these quantities are computed in the core of the CBR00 turbulent scheme.

This condensed water before activation $\overline{q_c}_s$ is used in the bulk scheme of Lopez (2002) in ACQMESM to compute the source or sink terms for the equation of $\overline{q_c}$, by using the departure with the prognostic value and in terms of a relaxation terms like $(\overline{q_c}_s - \overline{q_c})/dt$, where $dt$ is the time step used in the physics.

***c) The convection schemes.***

In the code of the Climate version-5 of ARPEGE, the convection and the turbulent schemes give two sets of independent tendencies for the basic variables (temperature, wind, humidity). Therefore the convection and the turbulent schemes can act in a constructive or a destructive way when they overlap, including the dry)layer bellow the cloud base, where

most of the convection scheme generate a non-zero mass-flux, with non-zero convective tendencies.

An example of the interaction from the turbulence toward the convection has been implemented from Meso-NH and AROME ideas to the Gueremy's scheme. The convective mass-flux close to the surface depends on the surface turbulent kinetic energy $\overline{e}_S$, given by (60).

The reverse can be true for the possible interaction from convection toward the turbulent scheme, for instance if the mass-flux was used to define a convective standard deviation $(\sigma_s)^2_{conv}$, as presently tested in AROME and the NWP version of ARPEGE (see the end of the section (1.5) above).

***d) The radiation scheme.***

The "large scale" variables $\overline{N}_s$ and $\overline{q_{cs}}$ are computed in (48) and (49), in the turbulent CBR00 scheme. They correspond to the grid-cell average values for the Cloud Cover and the Condensed Could Water. They are added to the same informations coming from the Shallow and the Deep convections schemes, with some suitable assumptions for the overlapping of the cloud layers on the vertical (Maximum / Random / Maximum-Random).

## 2.2 B2 : Interactions with the data flow

The three subroutines ACBL89, ACTURB and ACEVOLET act in ARPEGE in a parallel way, using as input the same data at the initial time step "t" as the other parts of the physics (convection, radiation, precipitation, drag, ...), in order to compute the value for the next time step at "t+dt", i.e. the new TKE value $\overline{e}(t+dt)$ from $\overline{e}(t)$ by solving the TKE equation (1)).

There is no pseudo-prognostic or provisional values coming from the previous time step, due to the **Hypothesis 1** made in the section (1.3.1), with an approximation for the computation of $\phi_3$ that is not made in Meso-NH or AROME.

The moist Betts conservative variables (8) and (9) and the moist potential temperatures (7) and (84), are used all along the computations made in the subroutines ACBL89, ACTURB and ACEVOLET of the turbulent scheme. Since the prognostic variables of ARPEGE are the temperature, the water vapor and the condensed water, it is necessary to compute at each time step these moist Betts conservative variables and moist potential temperatures. These computations are made with the cloud water content as input, coming from the prognostic scheme of Lopez.

Once the turbulent exchange coefficients are computed in the code, the vertical mixing is computed by an inversion of a tri-diagonal matrix (made in ACDIFUS), with the moist Betts conservative variables used as input. Note, however, that $\overline{\theta}_l$ is replaced in ARPEGE by the moist static energy $S_l = c_p\,T + \phi - L_v\,q_l - L_f\,q_i$ . As a consequence, the output turbulent tendencies should be applied to the same moist Betts conservative variables i.e. to $S_l = c_p\,T + \phi - L_v\,q_l - L_f\,q_i$ and $q_t = q_v + q_l + q_i$.

An important hypothesis is made in the code of the Climate version-5 of ARPEGE. It is assumed that the output turbulent tendencies can be directly applied to the "dry"

prognostic variables $S = c_p\,T + \phi$ and $q_v$, without the possible "projection" of the fluxes suggested in the section 3.2(b).

The explanation and motivations for this hypothesis is that, for given conservative values for $S_l(t)$ and $q_t(t)$, i.e. if they are constant over a given layer, there is no warranty that the "projected" tendencies acting independently on $S$, $q_v$, $q_l$ and $q_i$ would lead to conservative values for $S_l(t+dt)$ and $q_t(t+dt)$.

The advantage of applying the moist tendencies to the dry variables is that, for any given conservative values for $S_l(t)$ and $q_t(t)$, the moist turbulent fluxes should be zero, the dry variables $S$, $q_v$, $q_l$ and $q_i$ would be unchanged and thus $S_l(t+dt)$ and $q_t(t+dt)$ would remain conservative, as expected.

The drawback of the method is that the liquid and solid cloud water are never mixed. Also, in case of an existing vertical gradient in $S_l(t)$ and/or $q_t(t)$, the moist turbulent fluxes are not equal to zero, and it should not be the same as the dry turbulent fluxes to be applied to $S$, $q_v$, $q_l$ and $q_i$.

Another interaction is that both $\overline{N}_s$ and $\overline{q_{cs}}$ depends on the turbulent scheme, via (48) and (49) which depends on $Q_1$, depending from (33) on $\sigma_s$, depending from (38) on $L_m$ and $\phi_3$, which are computed in the core of the turbulent scheme.

### 2.3 B3 : Interactions with the dynamics

There are only few direct links between the turbulent scheme and the dynamics of the model. The only one is the choice to keep the TKE on the half levels of the model and to have the horizontal and vertical advections switch off.

The underlying hypothesis it a belief that it is more important to have an accurate numerical scheme on the vertical (especially for the Top-PBL entrainment, see above) and because we are mainly concerned with the large scale models with grid-mesh coarser than 10 km (ALADIN LAM, and NWP + GCM versions of ARPEGE).

It seems that even for grid-mesh coarser than 3 km the impact of the advection is tiny (less than 10 %, according to test realized at GMME).

## 3 Partie C : Algoritmics - Informatic

### 3.1 C1 : Algorithmic choices - Strong Constraints

The prognostic version of the CBR00 TKE equation as used in ARPEGE is similar to the "TURB-1D" version of the Meso-NH CBR00 code. Only the vertical gradients of $u$, $v$, $T$, $q_v$, $q_l$ or $q_i$ are taken into account.

The horizontal gradients of these dynamical and thermal variables are not taken into account, mainly because the Gauss grid in ARPEGE is separated into different uneven packages, without the constraint that a given vertical column must be close to its neighbours, with the different packages possibly distributed onto different processors. This configuration could not allow the TURB-3D option and the associated computation of the horizontal

gradients.

## 3.2 C2 : Algorithmic choices - Weak Constraints

***a) The mixing lengths ?***

Since the computation of the mixing and dissipation lengths are put into a separate subroutine (ACBL89), it is easy possible to test other formulations, according to the ideas presented in the section (1.5), for instance.

An important work of optimization has been done for the BL89 subroutine (ACBL89), with a division by more than a factor 4 for the CPU cost on vectorized machine. The consequence, however, is the drawback of a less readable code, with numerous embedded SIGN, MAX, MIN functions. the same optimization have been implemented into the equivalent subroutine of Meso-NH and AROME (also called BL89).

The same buoyancy term $\beta\,[\,\overline{\theta}_{vl}(z') - \overline{\theta}_{vl}(z)\,]$ appears in the two formulations of the buoyancy lengths (53) and (54) for $L_{up}$ and $L_{down}$. The modification proposed by Sanchez or Cuxart has already been tested in Meso-NH and in older SCM and GCM versions of the Climate version of ARPEGE. It is proposed to think in terms of an environment which evolve with $z'$ at the same time as the parcel moves upward or downward, leading to $\beta\,[\,(\overline{\theta}_{vl})_{part}(z') - (\overline{\theta}_{vl})_{env}(z')\,]$. This kind of modifications seems interesting, at least in research mode, and on a theoretical point of vue. But up to now, the test in SCM or 3D mode has not given interesting results.

***b) Vertical mixing in terms of the conservatives variables ?***

For sake of possible numerical instabilities for longer time steps, the vertical mixing is made in ARPEGE with an implicit scheme (in ACDIFUS), with a strong coupling between the upper-air variables $S = c_p T + \phi$ and $q_v$ and the change in time of the surface variables $T_{surf}$ and $q_{surf}$ .

The implicit schemes corresponds to the inversion of a tri-diagonal matrix. The the upper-air variables $S$ and $q_v$ are the "dry static energy" and the "specific content of the water vapor", respectively.

The code presently used in the Climate version-5 of ARPEGE has been written in order to deal as far as possible with the Betts conservative variables (8) and (9), via (27), (47), (51) and (52).

It is possible to further extend the use of $\theta_l$ and $q_t$. They are computed at the beginning of ACDIFUS, in terms of the "moist static energy" $S_l = c_p\,T + \phi - L_v\,q_l - L_f\,q_i$ and the total water specific content $q_t = q_v + q_l + q_i$. The tri-diagonal matrix is then inverted and it results the two fluxes of mixing : $\overline{w'S'_l}$ and $\overline{w'q'_t}$.

As indicated in the section (2.2), an attempt can be made to project these two conservative fluxes $\overline{w'S'_l}$ and $\overline{w'q'_t}$ onto three "equivalent" fluxes which could be applied to the "non-conservative" variables.

In the time being, there was in Meso-NH such an attempt, leading to the three following fluxes

$$\overline{w'S'} \;=\; \Big[\,(\,a\,F_2\,\lambda_3\,)\;L_{v/f}\,\Big]\;\overline{w'q'_t} \;+\; \Big[\,1\;-\;(\,a\,F_2\,\lambda_3\,)\;\alpha'_1\,L_{v/f}\,\Big]\;\overline{w'S'_l}\,, \qquad (87)$$

$$\overline{w'q_v{}'} = [\,1 - (\,a\,F_2\,\lambda_3\,)\,]\,\overline{w'q_t'} + [\,(\,a\,F_2\,\lambda_3\,)\,\alpha_1'\,]\,\overline{w'S_l'}\,, \quad (88)$$

$$\overline{w'q_c{}'} = [\,a\,F_2\,\lambda_3\,]\,\overline{w'q_t'} - [\,(\,a\,F_2\,\lambda_3\,)\,\alpha_1'\,]\,\overline{w'S_l'}\,. \quad (89)$$

The term $a$ is still given by (31), but the change of $\theta_l$ and $\theta$ into $S_l$ and $S$ implies the change of $\alpha_1$ given by (32) into $\alpha_1'$ given by

$$\alpha_1' = \frac{1}{c_p}\left(\frac{\partial\, q_{sat}}{\partial\, T}\right)_{(T=T_l)}\,. \quad (90)$$

The value of $F_2\,\lambda_3$ are chosen according to the same statistic scheme described in (50) and at the end of the section (1.3.5).

The use of the conservative variables in ACDIFUS as input for the solver of the vertical mixing has led to improvements in 1D-SCM and 3D-GCM simulations made with ARPEGE.

On the contrary, the projection of the turbulent fluxes for the conservative variables, made in output by using (87) to (89), has detrimental impacts for 1D-SCM and 3D-GCM. This part of the code can be switch on / off by a logical in the NAMELIST (presently set to .FALSE.).

It could be also possible to test other ideas, such as the one promoted by J.F. Gueremy which avoid the projection of the turbulent fluxes made in output, but rather a control of the exchange coefficients in input.

***c) Possible limitations for $Q_1$ ?***

Some securities and limitations are presently used in the standard diagnostic TKE scheme of Ricard and Royer (1993). In order to avoid irrealistic negative humidities or mixing lengths, the absolute values for the normalized saturation deficit $Q_1$ are controlled by minimum and maximum values, with $(Q_1)_{min} < |Q_1| < (Q_1)_{max}$.

These "securities" has been retained up to now in the Climate version 5 of ARPEGE, with possible (too ?) important impacts on $\sigma_s$ which verifies, according to (33), the relation $\sigma_s = a\,[\,q_t - q_{sat}(T_l)\,]/(2\,Q_1)$.

These "securities" can be switch on / off by a logical in the NAMELIST (presently set to .TRUE.).

***d) The super-adiabatic layers : Bug in the turbulent scheme ?***

Some strange patterns has been observed in the upper tropical troposhere since the very first tests made by C. Bossuet, with the dry version of the code written by P. Lacarrère.

Indeed, very large values for $\overline{e}$ are simulated in the upper tropical troposhere by the dry version of the prognostic TKE scheme, just above the top of the tropical convection, where possible super-adiabatic instabilities can exist, due to the radiative cooling occuring just above these cloud top.

It appears that large values for $\overline{e}$ are also simulated by the diagnostic Mellor and Yamada scheme used in the standard physics of the Climate version of ARPEGE (Ricard and Royer, 1993), and also by other diagnostic TKE scheme (like the one of J.F. Gueremy). The same large values for $\overline{e}$ were also observed for the more recent moist version of the CBR00 prognostic TKE scheme.

These large values for $\overline{e}$ had some important impact via the feedback leading to : large values for the mixing lengths, from (38) an increase in $\sigma_s$ and, from (33), values of $Q_1$ which generate a too large amount of the high levels cloud cover, with also too large values for the cloud ice content.

In the Climate version 4 of ARPEGE it was decided to test a corrective method based on a dry adjustment scheme, coming from an old version used in the old model SISYPHE. This code has been validated and tuned at GMGEC/EAC and at ENM/UFR, for several 1D-SCM and 3D-GCM simulations.

However, in the new Climate version 5 of ARPEGE, this scheme has not been implemented up to now. Even if the output of the TKE variable has not been analyzed as precisely as for the version 4, it seems that the feedback leading to too much high level clouds no longer exists, or is better controlled.

The reasons that may explain this change could be : (i) the important debug made in 2007 in the prognostic CBR00 code by Y. Bouteloup, E. Bazile et S. Malardel (CNRM/GMAP et CNRM/GMME) ; (ii) the RRTM LW code that has replaced the FMR15 one, with a better accuracy and less biases of the radiative cooling computations.

The 2007 debug of the turbulent scheme, with a problem solved in ACEVOLET together with other debugs or changes in ACBL89 and ACTURB, correspond to a decrease in the impact of the dynamical shear production term.

## 3.3 C3 : Discretisations - spatial or temporal

The temporal scheme presently used in the dynamics of ARPEGE is a Semi-Lagrangian and two-time levels semi-implicit scheme. The scheme used in the physics are, for the most part of them, implicit schemes.

According to (69), the temporal scheme used to compute the change of $\overline{e}(t)$ into $\overline{e}(t+dt)$ is :
- the special implicit scheme - computed at the end of the time-step $(t+dt)$ - for the vertical mixing and the dissipation terms, with the coefficients 1.5 and $-0.5$.
- an explicit and decentered scheme - computed at the beginning of the time-step $(t)$ - for the dynamical shear and the thermal production terms.

The wind components, the temperature, the specific content for the water species are all available at the full-levels. The TKE values $\overline{e}(t)$ and $\overline{e}(t+dt)$ are computed at the half-levels of the model, on the same levels where the BL89 mixing length, the vertical velocity and the exchange coefficients are available.

It is possible to go to the full-levels, via half-levels to full-levels averages, then to make horizontal and vertical advections, then to come back to the half-levels via full-levels to half-levels averages, just by setting LECTFL=.TRUE. (see ACTKE). These actions are, however of a very "diffusive" nature and they lead to detrimental impacts, see the section (1.5.

## 3.4 C4 : The architecture - list of subroutines

The monitor of the ARPEGE physics is APLPAR. The prognostic TKE scheme is called by the main subroutine ACTKE, with the TKE variable "PTKE" as input array and the flux of the tendencies "PFTKE" as output array.

The list of subroutines called if LVDIF.AND.LECT=.TRUE. is

**APLPAR** : monitor of the ARPEGE physics
- > **ACHMT** : general surface layer computations, PCD, PCH,...
- > **ACTKE** : monitor of the TKE computations ;
  possible half<->full<->half averages (if LECTFL)
  - > **ACBL89** : compute $\phi_3$ and the BL89 mixing and dissipation lengths,
    with ZLMECT= $g\,L_m$ and ZUSLE= $1/(ALD * g * L_\epsilon)$
  - > **ACTURB** : compute the exchange coefficients PKUROV and PKTROV,
    with PKUROV= $\rho\,g\,K_m/\Delta(\phi)$ and PKTROV= $\rho\,g\,K_T/\Delta(\phi)$ ;
    compute the Brunt Väisälä frequency PNBVNO= $\underline{N^2}/(\rho\,g)$ ;
    also the thermal production PPRODTH= $(g/T) * \overline{w'(\theta)'_{vl}}$ ;
    also PL3F2= $\lambda_3\,F_2$ ; compute the stratiform (large-scale)
    values PNEBS (cloud-cover) and PQCS (specific water content) ;
    the surface layer values PGKCLS= $g\,K_{surf}$ and PECTCLS= $e_S$ ;
  - > **ACEVOLET** : compute the tendencies of PTKE in the flux form PFTKE, where
    $\partial(\text{PTKE})/\partial t \equiv -g\,\partial(\text{PFTKE})/\partial p$

## 3.5 C5 : The architecture - list of NAMELIST options

The main NAMELIST variables used in the CBR00 version of ARPEGE are

**&NAMPHY**

**LECT**= .TRUE. : the main switch for the CBR00 scheme
**LPBLE**= .TRUE. : use of the Top-PBL vertical entrainment
**LNEBECT**= .TRUE. : use of the Bougeault-Bechtod values ($F_0$F, $F_1$, $F_2$)
**LDIFCONS**= .TRUE. : use the Betts conservative variables (beginning of ACDIFUS)
**LECTFL**= .FALSE. : no half<->full<->half averages in ACTKE

**&NAMPHY0**

**AKN**= 0.126 : CCH02 value for $(K_m/L_m/\sqrt{\overline{e}})$ – see the Table (1)
**ALPHAT**= 1.13 : CCH02 value for $(K_T/K_m/\phi_3)$ – see the Table (1)
**ALPHAE**= 2.7 : CCH02 value for $(K_e/K_m)$ – see the Table (1)
**ALD**= 1.18 : CCH02 value for $(1/C_\epsilon\ )$ – see the Table (1)
**ACBRPHIM**= 2.2 : a maximum value for $(\phi_3)$ – see the Table (1)
**ECTMIN**= 1.0E−6 : a minimum value for $(\overline{e})$
**ALMAVE**= 10. : an asymptotic and minimum value for $L_m$

**ARSB2**= 0.833 : the coefficient $1/\sqrt{C_{\epsilon_\theta}}$ in (39)
**AGRE1**= 0.20 : first Sc value $A_1$ – Top-PBL entrainment ; see the Table (3)
**AGRE2**= 60. : second Sc value $A_2$ – Top-PBL entrainment ; see the Table (3)
**AGREF**= 200. : Cu value for $A_{inv} = A_1$ – Top-PBL entrainment ; see the Table (3)
**AGREDTH**= −1.5 : a threshold value to define Sc/Cu regimes
Top-PBL entrainment ; see the section (1.3.10)
**AJBUMIN**= 0.005 : a threshold value to avoid division by zero
Top-PBL entrainment ; see the section (1.3.10)

The GFL array associated with the prognostic TKE variable is defined as a GFL array, with appropriate lines added in the NAMELISTs (NAMFA and NAMGFL). Note that, presently, the advection status is switched off and the TKE variable is set to REFVALI at the first time step.

**&NAMFA**
**YFATKE%NBITS**=12, ; TKE variable

**&NAMGFL**
**YTKE_NL%LGP**=.TRUE., ; Grid-Point (or spectral) type ?
**YTKE_NL%LGPINGP**=.TRUE., ; Grid-Point field input as Grid-Point ?
**YTKE_NL%LT1**=.TRUE., ; Field in t+dt GFL ?
**YTKE_NL%LPHY**=.FALSE., ; Field in physics GFL ?
**YTKE_NL%LREQOUT**=.TRUE., ; Field required in output (or not) ?
**YTKE_NL%LADV**=.FALSE., ; Advections required (or not) ?
**YTKE_NL%LQM**=.TRUE., ; Quasi-Monotone interpolations required (or not) ?
**YTKE_NL%NREQIN**=-1, ; 1 if required in input ; 0 if not ; -1 if set to REFVALI
**YTKE_NL%REFVALI**=0.000001, ; TKE set to 1. E-6 at the first time step

# 4 Partie D : Références

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