SPHERE TO SPHERE TRANSFORMS IN SPECTRAL SPACE IN THE CYCLE 43 OF ARPEGE/IFS: CONFIGURATION 911.

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Abstract:

This documentation describes configuration formerly numbered 911 which performs dilatation/contraction matrices: these matrices allow to do spectral transformation between a stretched spectral space and between its unstretched counterpart. Some algorithmic aspects and technical aspects are described. Some namelists are provided to use the configuration 911, which is now externalised in the project "utilities". Such transformations can be used in spherical geometry only, and for METEO-FRANCE purpose only (not at ECMWF).

Résumé:

Cette documentation décrit la configuration anciennement numérotée 911 qui fabrique des matrices de dilatation/rotation. Ces matrices permettent de faire des transformations spectrales entre un espace spectral étiré et sa contrepartie non étirée. On décrit quelques aspects algorithmiques et techniques. On fournit un exemple de namelist pour utiliser cette configuration, qui est maintenant externalisée dans la librairie "utilities". Cette transformation n'est utilisée qu'à METEO-FRANCE.

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1 Introduction.

This documentation describes the configuration formerly numbered 911.

In the stretched configuration of ARPEGE/IFS the most part of calculations is done on the stretched (computational) geometry. But there are some spectral calculations which have to be done on an unstretched sphere (for example filtering of some derivatives in FULL-POS). So one needs an efficient routine which does direct and inverse transformations between the spectral space of the computational sphere and its unstretched counterpart. Transformation includes a set of dilatation/contraction matrices which take a lot of place of memory, so these matrices are computed and stored separately. These matrices are written on a LFI file.

Dilatation/contraction matrices can also be computed in the set-up part of FULL-POS.

∗ Distributed memory:

- 911 works for one level of MPI distribution.
- Some parts of the code use OpenMp distribution.

∗ Modifications since cycle 42: none.

2 Dilatation and contraction matrices.

2.1 General considerations and stability.

One uses the algorithm described by Rochas (Rochas, 1990). c is the stretching coefficient. For each zonal wavenumber m algorithm computes the dilatation matrix B_m (containing the coefficients $\beta^m_{n,r}$) and the contraction matrix A_m (containing the coefficients $\alpha^m_{n,r}$). Two truncations are defined: a small truncation N_s corresponding to the spectral representation of the variable resolution computational sphere, and a large truncation N_c corresponding to the spectral representation of the constant resolution geographical sphere. The theoretical value of N_c would be infinite. In practical N_c is slightly greater than cN_s. In order to test the stability of the algorithm of matricial computation, one computes for each zonal wavenumber m (varying from 0 to N_s) the " L^∞ norm" $\mid B_m A_m - I \mid_\infty$ which is equal to the maximum of the absolute values of the coefficients of matrix $B_m A_m - I$, where I is the identity matrix.

$*$ Computation of the norm $\mid B_m A_m - I \mid_\infty$.

- The most voluminous matrices are obtained for m=0.
- For small truncations, the most unstable algorithm (the highest value of $\mid B_0 A_0 - I \mid_\infty$) are obtained for m=0. This property is no longer valid for higher truncations ($N_\mathrm{s} > 1000$).
- Norm is lower when calculations are done with 128 bits. All the following results are given for
  - 128 bits calculations when computing the Legendre polynomials used in the matrices.
  - 64 bits calculations when computing the matrices.
- For $N_\mathrm{c} = cN_\mathrm{s}$, $\mid B_0 A_0 - I \mid_\infty$ is $\mathcal{O}(0.01)$ (so not negligible relative to 1) for the cases for which computation of such a norm has been done (for c=2, 3 or 4). The consequence is that a higher value of N_c is necessary to obtain more negligible values of $\mid B_0 A_0 - I \mid_\infty$, some calculations show that a satisfactory value is $N_\mathrm{c} \simeq 1.7cN_\mathrm{s}$ for $N_\mathrm{s} \simeq 20$ and $N_\mathrm{c} \simeq 1.15cN_\mathrm{s}$ for $N_\mathrm{c} \simeq 200$ to obtain a norm lower than 10^{-13}, if the stretching coefficient is between 1.5 and 4. Table (2.1) gives some examples of values of N_c which have to be used for different stretching coefficients between 1.5 and 4.0 and a list of admissible truncations. Values of N_c are taken preferably among admissible values (i.e. which allow use of fast Fourier transforms). See documentation (IDTS) about spectral transforms for more details about admissible truncations.
- $\log(\mid B_0 A_0 - I \mid_\infty)$ is nearly proportional to the quantity $N_\mathrm{c}/(c * N_\mathrm{s}) - 1$ when over 10^{-13}.
- For a given truncation N_s, the ratio $N_\mathrm{c}/(c * N_\mathrm{s})$ which gives a norm of 10^{-13} (or any other threshold) converges towards 1 when c converges towards 1.
- For a given stretching coefficient c, the ratio $N_\mathrm{c}/(c*N_\mathrm{s})$ which gives a norm of 10^{-13} (or any other threshold) converges towards 1 when the lower truncation N_s becomes infinite.
- Algorithm can be used even for high stretching coefficients ($c \simeq 10$) and for high truncations ($N_\mathrm{c} \simeq 1000$).

N_s	N_c $c=1.5$	N_c $c=2.0$	N_c $c=2.2$	N_c $c=2.4$	N_c $c=2.5$	N_c $c=3.0$	N_c $c=3.5$	N_c $c=4.0$
30	63	89	99	107	119	143	159	179
34	71	99	107	107	127	149	179	199
38	79	107	119	119	143	161	191	215
44	89	119	143	127	149	191	215	249
46	95	127	143	143	159	191	239	255
48	95	127	143	149	161	199	239	269
52	99	143	159	159	179	215	249	287
58	119	149	179	179	191	239	269	319
62	119	159	179	191	199	239	287	323
70	143	179	199	215	239	269	319	359
74	143	191	215	239	239	287	359	383
78	143	199	215	239	249	299	359	399
80	149	199	239	239	249	299	359	399
88	159	215	239	269	269	323	383	449
94	179	239	249	287	287	359	431	479
98	179	239	269	287	299	359	431	479
106	191	255	287	319	319	383	449	511
118	215	287	319	359	359	431	499	575
124	215	299	319	359	383	449	539	599
126	239	299	359	359	383	449	539	599
134	239	319	359	383	399	479	575	639
142	249	359	383	399	431	499	599	674
148	255	359	383	431	431	539	624	719
158	269	383	399	449	479	575	639	728
160	287	383	431	449	479	575	647	749
178	299	431	449	485	511	624	719	863
190	319	431	479	539	539	647	767	863
198	359	449	499	539	575	674	799	899
214	359	479	539	575	599	728	863	971
224	383	499	575	599	624	767	899	1023
238	399	539	599	639	674	799	959	1079
242	399	539	599	647	674	809	959	1079
248	431	575	624	674	719	863	971	1124
254	431	575	624	674	719	863	999	1151
268	449	599	674	719	749	899	1079	1199
286	479	639	719	767	799	959	1124	1279
298	485	674	719	799	863	999	1151	1349
318	539	719	767	863	899	1079	1249	1439
322	539	719	799	863	899	1079	1249	1439
358	599	799	863	959	971	1199	1439	1599
374	624	809	899	971	1023	1249	1439	1727
382	624	863	959	999	1079	1249	1457	1727
398	639	863	959	1079	1079	1295	1535	1727
404	647	899	959	1079	1124	1349	1535	1799
430	719	959	1023	1124	1199	1439	1727	1874
448	719	959	1079	1199	1214	1457	1727	1943
478	767	1023	1124	1249	1279	1535	1799	2159
484	799	1079	1151	1249	1295	1599	1874	2159
498	799	1079	1199	1279	1349	1599	1874	2159
510	809	1124	1199	1349	1439	1727	1919	2186
538	863	1151	1279	1439	1439	1727	2024	2303
574	959	1249	1349	1499	1535	1874	2159	2499
598	959	1279	1439	1535	1599	1919	2249	2559

Table 2.1: Optimal value of N_c to be taken for N_s between 30 and 599.

N_s	N_c $c = 1.5$	N_c $c = 2.0$	N_c $c = 2.2$	N_c $c = 2.4$	N_c $c = 2.5$	N_c $c = 3.0$	N_c $c = 3.5$	N_c $c = 4.0$
624	999	1349	1457	1599	1727	1999	2399	2699
638	1023	1349	1499	1619	1727	2024	2399	2879
646	1023	1439	1499	1727	1727	2159	2399	2879
674	1079	1439	1599	1727	1799	2159	2499	2879
718	1151	1535	1727	1874	1919	2303	2699	3071
728	1151	1535	1727	1874	1919	2303	2699	3071
748	1199	1599	1727	1919	1999	2399	2879	3199
766	1199	1619	1799	1943	2024	2429	2879	3239
798	1249	1727	1874	2024	2159	2559	2999	3374
808	1279	1727	1874	2047	2159	2559	2999	3455
862	1349	1874	1999	2186	2303	2879	3199	3644
898	1439	1874	2159	2303	2399	2879	3374	3839
958	1499	1999	2249	2399	2499	3071	3599	4049
970	1535	2024	2249	2429	2559	3071	3599	4095
998	1599	2159	2303	2499	2699	3124	3644	4319
1022	1599	2159	2399	2559	2699	3199	3749	4319
1078	1727	2249	2499	2699	2879	3374	3999	4499
1124	1799	2399	2591	2879	2999	3599	4095	4799
1150	1799	2399	2699	2879	2999	3599	4319	4799
1198	1874	2499	2879	2999	3124	3749	4373	4999
1214	1919	2559	2879	3071	3199	3839	4499	5119
1248	1943	2591	2879	3124	3239	3887	4607	5183
1278	1999	2699	2915	3199	3374	3999	4799	5399
1294	1999	2699	2999	3239	3374	4049	4799	5399
1348	2159	2879	3171	3374	3599	4319	4999	5624
1438	2249	2999	3374	3599	3749	4499	5399	5999
1456	2249	3071	3374	3644	3839	4607	5399	6074
1498	2399	3124	3455	3749	3887	4799	5624	6249
1534	2399	3199	3599	3839	3999	4799	5624	6399
1598	2499	3374	3644	3999	4319	4999	5831	6749
1618	2499	3374	3749	3999	4319	5119	5999	6749
1726	2699	3599	3999	4319	4499	5399	6249	7199
1798	2879	3749	4095	4499	4799	5624	6479	7499
1874	2879	3887	4319	4799	4859	5831	6749	7775
1918	2999	3999	4373	4799	4999	5999	6911	7999
1942	2999	3999	4499	4799	4999	5999	7199	7999
1998	3071	4319	4607	4999	5183	6249	7199	8639
2024	3124	4319	4607	4999	5399	6249	7289	8639
2046	3199	4319	4799	5119	5399	6399	7499	8639
2158	3374	4499	4999	5399	5624	6749	7775	8999
2186	3374	4499	4999	5399	5624	6749	7999	8999
2248	3455	4607	5119	5624	5759	6911	8099	9374
2302	3599	4799	5183	5759	5999	7199	8639	9599
2398	3749	4999	5399	5999	6143	7499	8639	9999
2428	3749	4999	5624	5999	6249	7499	8747	9999
2498	3839	5119	5624	6143	6399	7679	8999	10239
2558	3999	5399	5759	6399	6560	7999	9215	10799
2590	3999	5399	5831	6399	6749	7999	9374	10799
2698	4319	5624	6074	6749	6911	8639	9719	11249
2878	4499	5999	6479	7199	7499	8999	10367	11999
2914	4419	5999	6560	7199	7499	8999	10799	11999
2998	4607	6143	6749	7499	7679	9215	10799	12287

Table 2.1b: Optimal value of N_c to be taken for N_s between 624 and 2999.

2.2 General algorithm to compute the upper truncation N_c: subroutine SUNCMAX.

∗ **Calculation of N_c:** The default value of N_c is computed as follows:

- A threshold S_0 is given for $| B_0 A_0 - I |_\infty$ (this threshold is set to 10^{-13} in the code).
- One uses the property: $\log(| B_0 A_0 - I |_\infty)$ is nearly proportional to the quantity $N_c/(c * N_s) - 1$ for a given threshold S_0 when c and N_s are given:
- The ratio:

$$a_2 = \frac{\log(| B_0 A_0 - I |_\infty)}{N_c/(c * N_s) - 1}$$

  can be fitted by a curve:

$$a_2 = \alpha(c) {N_s}^r + \beta(c)$$

- Exponent r is quasi-independent from c and a value of 2/3 is satisfactory; this value has been taken in the code.
- α only depends on the stretching coefficient c and has been fitted by the following function:

$$\alpha = \frac{-1.08}{(c-1)^{0.86}} - 6.54$$

- β only depends on the stretching coefficient c and has been fitted by the following function:

$$\beta = \frac{0.65}{(c-1)^{0.86}} + 5.91$$

- One computes α, then β, then a_2, then the integer immediately above:

$$(c * N_s) \left(1 + \frac{a_2}{S_0}\right)$$

  This integer is not necessary an admissible truncation; so one takes for N_c the admissible truncation immediately above this integer.
- For $S_0 = 10^{-13}$ the previous algorithm generally gives the same values as in table 2.1, with some isolated exceptions. So one does a correction to obtain in all cases the values of table 2.1 and a monotonic solution: N_c has to be a monotonic function of S_0, c and N_s.

∗ **Subroutine SUNCMAX:** **SUNCMAX** is called by **MASTER911** to compute an acceptable default value for the upper truncation N_c. **SUNCMAX** calls **SUADMI** and **SUNCET13**.

- **SUADMI** says if a truncation is admissible or not, for stretched geometry and returns the adjacent admissible truncations.
- **SUNCET13** returns the values of N_c given by table 2.1 for a subset of admissible truncations and stretching coefficients.

2.3 Matrices size.

Two files of identical size are made, the first one containing dilatation matrices, the second one containing contraction matrices. Each file contains a number of values equal to:

$$\frac{(N_s+1)(N_s+2)}{2}(N_c+1) - \frac{(N_s)(N_s+1)(N_s+2)}{6}$$

This number has to be multiplied by 8 to obtain the number of octets. Local name of file is **MATDILA** for dilatation, **MATCONT** for contraction.

2.4 Description of subroutine SUDIL.

Most of the code is now in the library “utilities” (directory “rdc”). Some “ifsaux”, “algor” and “trans” routines may be required.

```
MASTER911 ->
 * SUNCMAX -> SUADMI and SUNCET13
 * SUDIL ->
   - SETUP_TRANS0 and SETUP_TRANS
   - TRANS_INQ
   - DILAT_MAPPING -> MXMAOP
   - DILAT_CALC
   - DILAT_DEVIATION
   - DILATB -> MXMAOP and LFIECR
   - DILAT_CONTROL
   - the processor communication routines MPL_RECV, MPL_SEND, MPL_BARRIER.
   - Some LFI.. routines.
```

Action of each routine:

- **SUDIL**: head routine.
- **SETUP_TRANS0** and **SETUP_TRANS**: calculation of the useful Legendre polynomials. One computes the Legendre polynomials corresponding to the truncation N_s on the variable resolution sphere latitudes which are associated to the Gaussian latitudes of the geographical sphere. One computes the Legendre polynomials corresponding to the truncation N_c only for the zonal wavenumbers m below or equal than N_s, on the Gaussian latitudes of the sphere of truncation N_c.
- **TRANS_INQ** gets the Gaussian weights and the useful Legendre polynomials.
- **LFIOUV** opens LFI files which will contain matrices.
- **LFIFER** closes LFI files.
- **LFIECR** writes on LFI files.
- **DILAT_MAPPING**: pre-calculations.
- **DILAT_CALC**: calculation of dilatation and contraction matrices.
- **DILAT_DEVIATION**: calculation of product contraction times dilatation, computes deviation from identity matrix.
- **DILATB**: write matrices on LFI files; in some cases prints matrices coefficients.
- **DILAT_CONTROL**: prints deviation from identity matrix.

2.5 Important namelist parameters.

$*$ **Main namelist parameters:** Configuration 911 has now its own namelist containing only one element: NAM911.

- **INSMAX** (N_c), **INSMIN** (N_s) and **INDGLG** (number of latitudes corresponding to a linear grid associated with truncation N_c).
- **ZSTRET** (stretching coefficient c).
- **LLPROVIDED_NSMAX**: says if **INSMAX** and **INDGLG** must be provided in the namelist (otherwise they are automatically computed).
- **IPRINTLEV**: level of printings.
- **LLIO_PNM**: T/F: PNM are stored/not stored in LFI files.

The number of MPI processors (**INPROC**) is not in NAM911 but taken from the script environment.

$*$ **Additional namelist parameters:** Other namelist variables are provided, for example to control memory, execution time, level of printings.

```
! IMPL_OUTPUT     : printings level in MPL communications (put 2 for more printings)
! IMBX_SIZE       : user-provided mailbox size
! IMP_TYPE        : 1=blocked   (MPI_SEND/RECV)
!                 : 2=buffered  (MPI_BSEND/MPI_BRECV)
!                 : 3=immediate (MPI_ISEND/MPI_IRECV)
! INSLIM          : intermediate truncation for optimisations, assumed to be >= INSMIN.
!                   if INSMIN <= INSLIM < INSMAX, DILA/CONT articles are split
```

```
!                     into WNUM and WNUC.
! LLOPT_NSLIM     : active only if INSMIN <= INSLIM < INSMAX; in this case
!                   articles named WNUC are not written.
! ITEST           : For testing in DILATB (between 0 and 2).
! ITESTIO         : For testing : 0 = No I/Os ; 1 = with I/Os
! IULOUT          : Logical unit.
```

∗ **Remark about granularity factor:** the granularity factor (local variable **IFACTD** in **MASTER911**) is computed inside the code as an internal intermediate variable. Optimal factor is proportional to the size of matrices; The default value is the integer closest to

$$\left[\frac{(N_\mathrm{s}+1)(N_\mathrm{s}+2)}{2}(N_\mathrm{c}+1) - \frac{(N_\mathrm{s})(N_\mathrm{s}+1)(N_\mathrm{s}+2)}{6}\right]/6232525$$

bounded by 1 and 120. For stretching coefficients lower than 4, **IFACTD** is equal to 1 if $N_\mathrm{s} < 66$. **IFACTD** is equal to 120 if $N_\mathrm{s} >= 485$.

∗ **Default values:** see in main routine **MASTER911**.

2.6 Example of namelist for configuration 911 of ARPEGE.

The example provided below is the namelist allowing to compute the dilatation/contraction matrices for the resolution TL1278c2.4 (linear grid).

```
&NAM911
  IPRINTLEV=0,
  IMPL_OUTPUT=0,
  IMBX_SIZE=2048000000,
  INSMIN=1278,
  ZSTRET=2.4,
  INSMAX=3199,
  INDGLG=3200,
  LLPROVIDED_NSMAX=.TRUE.,
/
```

3 References.

∗ **Internal notes:**

- (TDECTEC) 2015: IFS technical documentation (CY41R1). Part VI: technical and computational procedures. Available at “https://software.ecmwf.int/wiki/display/IFS/Official+IFS+Documentation”.
- Rochas, 1990: Dilatation dans l’espace spectral (internal note in French, included in the “note ARPEGE numéro 19”).
- (IDBAS) Yessad, K., 2016: Basics about ARPEGE/IFS, ALADIN and AROME in the cycle 43 of ARPEGE/IFS (internal note).
- (IDDM) Yessad, K., 2016: Distributed memory features in the cycle 43 of ARPEGE/IFS (internal note).
- (IDTS) Yessad, K., 2016: Spectral transforms in the cycle 43 of ARPEGE/IFS (internal note).