WHY DO WE NEED THE NH MODEL ?

Basic Set of Equations

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INTRODUCTION

The goal of this first "dynamical" lecture is to introduce the concept of nonhydrostatism, and the associated system of equations.

- Operational ARPEGE, IFS and Aladin are "hydrostatic" models
- AROME and Aladin-NH are "nonhydrostatic" (NH) models

- What is the hydrostatic approximation ?
- When to use NH models ?
- Why using more complicated NH models ?
- Anelastic NH or Fully Compressible NH ?
- How look the resulting NH systems ?
  - in height-based coordinates
  - in mass-based coordinates

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Outline

History

"Exact" Fluid Mechanics equations known for long times (Euler ?)
$\longrightarrow$ OK for solving equations analytically (idealised cases)

- For old numerical models, CPU and memory were poor
  $\longrightarrow$ try to find cheaper sub-systems to solve
- very old models (Δx = 500-100km) :
  $\longrightarrow$ barotropic systems/approximation (no vertical motions)
- old models (Δx = 100-10 km) :
  $\longrightarrow$ hydrostatic systems/approximation (small vertical accelerations)

- Now (Δx = 10-1 km) :
  $\longrightarrow$ nonhydrostatic systems (full vertical motions)

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Hydrostatic approximation

Equation of vertical motion in z coordinate :

$$\gamma \frac{dw}{dt} = g - \frac{RT}{p} \frac{\partial p}{\partial z}$$

Hydrostatic approximation :

postulate $(dw/dt) \ll g$

$\rightarrow$ i.e. replace the marker γ by zero above

$\rightarrow$ The quantity (dw/dt) no longer appears anywhere in the system

$\rightarrow$ The vertical velocity w becomes a diagnostic quantity

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Hydrostatic approximation

- w diagnostic does not mean w or (dw/dt) will remain zero.
- w is diagnosed from other fields ($\Rightarrow$ may change in time)
- even, no warranty that (dw/dt) will remain smaller than g !
  (this is similar to linear approximation in systems)
- There should be an abort test when (dw/dt) not smaller than g

Hydrostatic approximation

Euler Equations in unbounded "perfect" fluid :
$\longrightarrow$ 5 prognostic variables, e.g. (u, v, w, T, p).

A surprising consequence of hydrostatic approximation :

Hydrostatic Equations in unbounded "perfect" fluid :
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One constraint $\longrightarrow$ 2 pronostic variables less ! ! !

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When to use NH models ?

Truism : when (dw/dt) is likely to become not smaller than g.

Processes which generate largest vertical accelerations :

- flow over orography
- flow in convective areas

For these two kinds of flow, the NH limit will be reached first
In numerical models, this requires resolution about one kilometer (steep slopes, resolved clouds).

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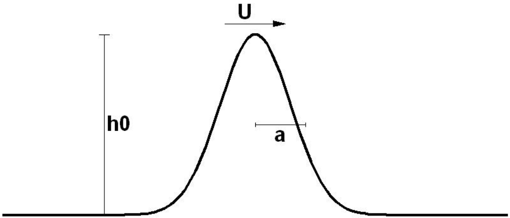
When to use NH models ?

Example : bell-shape mountain



$$h(x) = \exp[-(x/a)^2]$$

$$\dot{w} = U^2 h''(x)$$

$$|\dot{w}(0)| = \frac{2U^2 h_0}{a^2}$$

$$|\dot{w}(0)| = 0.1g \Rightarrow a = U\sqrt{\frac{2h_0}{0.1g}}$$

For $U = 30$ m/s, $h_0 =$1000 m $\longrightarrow$ $a \approx 1500$ m (i.e. kilometric scale)

Outline

Why using complicated NH models ?

For NWP models with orography, physics,... :
when Δx 10 km, no noticeable differences between H and NH,
but for Δx 2.5 km, noticeable differences begin to appear.

Bad side of the thing :
differences seem mainly restricted to "chaotic areas"

Why using complicated NH models ?

Skeptical persons could argue that :
"you just exchange one noise for another !"

However, one could argue that :
As soon as you have differences, NH is better.

Proposed work :
- Diagnose "how often" the flow is a really NH in forecasts
- Try to find cases where NH makes a "synoptic" difference

Outline

Anelastic NH or Fully Compressible NH ?

There are possible "intermediate" approximations between Hydrostatism and Euler equations : anelastic approximations

Assume there exist a reference state $\overline{\rho(z)}$ for which the local departure of density $\ll \overline{\rho(z)}$.

Controversed approximation (large domains, Rossby wave distorsion...)
In large scale modelling (NWP and Climate), people prefer to solve Euler Equations.

Outline

Basic set of equations ("Cartesian" z-coordinate, dry,...)

$$
\begin{aligned}
\frac{d\mathbf{V}}{dt} &= \frac{1}{\rho}\boldsymbol{\nabla}_z p \\
\gamma\frac{dw}{dt} &= -g - \frac{1}{\rho}\frac{\partial p}{\partial z} \\
\frac{dT}{dt} &= -\frac{RT}{C_v}D_3 + \frac{Q}{C_v} \\
\frac{d\rho}{dt} &= -\rho D_3
\end{aligned}
$$

where :

$$
\begin{aligned}
\frac{d}{dt} &= \frac{\partial}{\partial t} + \mathbf{V}\cdot\boldsymbol{\nabla}_z + w\frac{\partial}{\partial z} \\
D_3 &= \boldsymbol{\nabla}_z\cdot\mathbf{V} + \frac{\partial w}{\partial z} \\
p &= \rho RT
\end{aligned}
$$

Transformation to general vertical coordinate s

Following e.g. Laprise, 1992, MWR, 197–207 :

If "s" is any vertical coordinate, the above continuity equation writes :

$$\left[\frac{\partial}{\partial t}\left(\rho\frac{\partial z}{\partial s}\right)\right]_s + \boldsymbol{\nabla}_s \cdot \left(\rho \mathbf{V}\frac{\partial z}{\partial s}\right) + \frac{\partial}{\partial s}\left(\rho\dot{s}\frac{\partial z}{\partial s}\right) = 0$$

Demonstration on request, or at :
http ://www.cnrm.meteo.fr/gmapdoc/modeles/Dynamique/massc.ps

Transformation to general vertical coordinate "π"

$$\left[\frac{\partial}{\partial t}\left(\rho\frac{\partial z}{\partial s}\right)\right]_s + \nabla_s \cdot \left(\rho \mathbf{V}\frac{\partial z}{\partial s}\right) + \frac{\partial}{\partial s}\left(\rho\dot{s}\frac{\partial z}{\partial s}\right) = 0$$

Choose vertical coordinate "π" such as the first term vanishes :

$$\rho\frac{\partial z}{\partial s} = Cst \qquad \longrightarrow \qquad \frac{\partial \pi}{\partial z} = -\rho g$$

Transformation rules :

$$\begin{aligned} \frac{\partial}{\partial z} &= -\frac{gp}{RT}\frac{\partial}{\partial \pi} \\ \nabla_z &= \nabla_\pi + \frac{gp}{RT}(\nabla_\pi z)\frac{\partial}{\partial \pi} \end{aligned}$$

Basic system in "π" coordinate

$$
\begin{aligned}
\frac{d\mathbf{V}}{dt} &= -\frac{RT}{p}\nabla_\pi p + \frac{\partial p}{\partial \pi}\nabla_\pi \phi \\
\gamma \frac{dw}{dt} &= g\left(\frac{\partial p}{\partial \pi} - 1\right) \\
\frac{dT}{dt} &= -\frac{RT}{C_v}D_3 + \frac{Q}{C_v} \\
\frac{dp}{dt} &= -p\frac{C_p}{C_v}D_3 + \frac{Qp}{C_v T}
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\frac{d}{dt} &= \frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla_\pi + \dot{\pi}\frac{\partial}{\partial \pi} \\
D_3 &= \nabla_\pi \cdot \mathbf{V} + \frac{p}{RT}(\nabla_\pi \phi)\cdot\left(\frac{\partial \mathbf{V}}{\partial \pi}\right) - \frac{gp}{RT}\frac{\partial w}{\partial \pi}
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where :

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\begin{aligned}
\dot{\pi} &= -\int_{\pi_0}^{\pi} \boldsymbol{\nabla}_{\pi} \cdot \mathbf{V} \, d\pi' \\
\phi &= -\int_{\pi_0}^{\pi} \frac{RT}{p} \, d\pi'
\end{aligned}
$$

Comments about "π" coordinate

π is in fact a mass-based coordinate.

The above system is tractable only for domains bounded by constant and uniform π at top and bottom (π_T and π_s).

$\Longrightarrow$ not well-suited for NWP.

$\Longrightarrow$ reformulate in terrain-following coordinate

Pure π-based terrain-following $\sigma = \pi/\pi_s$ not well-suited for analysis

Define a hybrid π-based terrain-following coordinate η.

Transformation to "η" coordinate

A hybrid π-based terrain-following coordinate can be defined implicitly :

$$\pi = A(\eta) + B(\eta)\pi_s$$

where A and B are two specified functions.

levels where $A = 0 \rightarrow \eta =$ pure (stretched) terrain following coordinate
levels where $B = 0 \rightarrow \eta =$ pure (stretched) π coordinate

Transformation to "η" coordinate

vertical metric of the coordinate :

$$m = \frac{\partial \pi}{\partial \eta} = \frac{dA}{d\eta} + \pi_s \frac{dB}{d\eta}$$

Transformation rules :

$$\frac{\partial}{\partial \pi} = \frac{1}{m} \frac{\partial}{\partial \eta}$$

$$\nabla_{\pi} = \nabla_{\eta} - (\nabla_{\eta} \pi) \frac{1}{m} \frac{\partial}{\partial \pi}$$

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\end{aligned}$$

$$\phi = \int_\eta^{\eta_0} \frac{mRT}{p}d\eta$$

Basic system in "η" coordinate

In the spectral part, alternative NH variables are used (for stability reasons)

$$\hat{q} = \ln(p/\pi)$$

$$d_4 = D3 - \nabla_\eta \cdot \mathbf{V}$$

Number of prognostic variables

- Archetypal system (X, Y, Z) variables
- 1D vertical system EE in $z \longrightarrow$ EE in $\pi \longrightarrow$ HE in π
- 1D vertical system EE in $z \longrightarrow$ EE in $\pi \longrightarrow$ HE in π
- 1D HE in z with 3 var. Progn. $\longrightarrow$ is it possible ?