# Multiphasic equations for a NWP system

CNRM/GMME/Méso-NH

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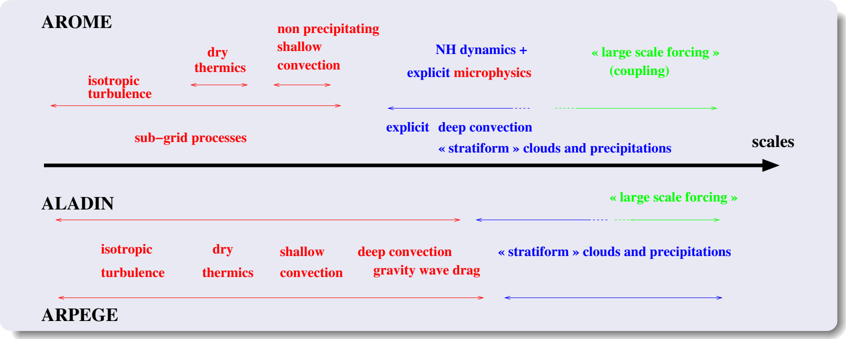
# Resolved versus parametrised processes



# Clouds in NWP models

## Large scale physics

For « large scale » model, we usually suppose(d) that a diagnostic representation of condensates is enough (no condensates in the air state forcasted by the model).

- We use diagnostic formulation from the mean water vapor content when needed (radiation, output)
- As the condensates (cloud and rain) are not pronostic, they are not directly known by the dynamics : no advection, no inertia, no weight of condensates

# Clouds in NWP models

## Cloud scale physics

In order to solve explicitly clouds formation and life cycle, we need a NH-dynamics, but also a finer description of microphysics processes.

- Explicit (pronostic) evolution equations for condensates (multiphasic system)
- Interaction between condensates and the dynamics (advection, weight of condensed phases)
- Realistic sedimentation (precipitation) with « finite » falling velocity (precipitating species are a componant of an air parcel)
- Detailled microphysics with a complete phase transition description (see J. P. Pinty's talk)

# Large scale physics / cloud scale physics in practice

## Water phase changes in a « large scale » physics

- equations for gaseous species only
- cond/evap with the pseudo-adiabatic hypothesis (no consensed phase in the atmosphere, precipitations are known only through their consequences on $T$ and $q_v$)

$$\frac{\partial \rho_v}{\partial t} = \frac{\partial P}{\partial z}$$

## Water phase changes in a cloud scale physics

- equations for gases and condensates
- parametrisation of detailled microphysical processes (but there may still be problem with unresolved clouds)

$$\frac{\partial \rho_v}{\partial t} = -\frac{\partial(\rho_c + \rho_r)}{\partial z} + \frac{\partial P}{\partial z}$$

# Large scale physics / cloud scale physics

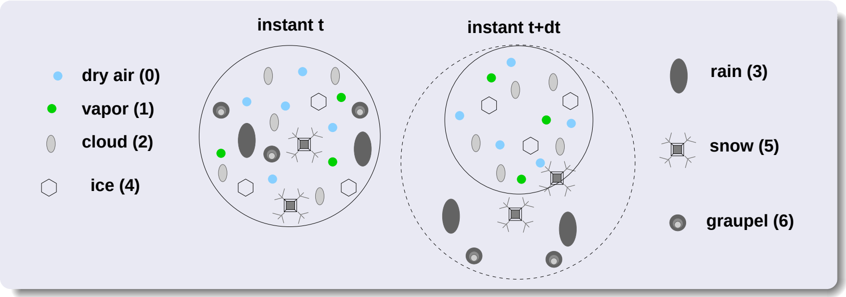
## Our worry

To have a consistant set of equations with hypotheses and approximations « under control » and valid for pseudo-adiabatic or multiphasic, ($\delta m = 0$ or $\delta m = 1$), hydrostatic or non-hydrostatic **NWP** model.

## Warnings

It is a « burning » subject with recent references only

# The multiphasic system of Arome



# Questions

## Variables in a multiphasic atmospheric parcel

- Which $\rho$ ?
- Which $p$ and $T$ ?
- Which wind ? What velocities should we manipulate ?
- Advection of what by what ?

## How can we treat condensates ?

- As individuals wih their own evolution laws ?
- As continuous components of a mixture ?

# The local scale (our scale of the continuum)

## A continuum of droplets and drops

$$\rho_c = \frac{m_c}{V_{gaz}} \quad \rho_r = \frac{m_r}{V_{gaz}}$$

$$\rho = \sum_k \rho_k$$

## Local equilibrium hypothesis

$$T_{eq} = T_c = T_r = T_{gaz}$$

$$\vec{v}_{eq} = \vec{v}_c = \vec{v}_r = \vec{v}_{gaz}$$

# Barycentric parameter at the local scale

## Barycentric formulation

$$\rho\psi = \sum_l \rho_l \psi_l \quad \text{or} \quad \psi = \sum_l q_l \psi_l$$

$$\text{local departure} \quad : \quad \tilde{\psi}_l = \psi_l - \psi$$

## Consequences of equilibrium hypotheses

$$\vec{v} = \sum_l q_l \vec{v}_l = (\sum_l q_l)\vec{v}_{eq} = \vec{v}_{eq}$$

$$h = \sum_l q_l h_l == \left(\sum_l q_l {h_l}^{\circ}\right) + \left(\sum_l q_l {c_p}_l\right) T = h^{\circ} + c_p T$$

with $c_p = \sum_l q_l {c_p}_l$

# Averaging from the local scale to the model scale

## Volumic averaging

$$\overline{\psi} = \frac{1}{V} \int_V \psi dv$$

with $\psi' = \psi - \overline{\psi}$ and $\overline{\psi'} = 0$

## Mass-weighted averaging

$$\widehat{\psi} = \frac{1}{m} \int_m \psi dm$$

with $\psi'' = \psi - \widehat{\psi}$ and $\widehat{\psi''} = 0$

# Averaging in a multiphasic system

Averaging are weighted by the volume or the mass of the mixture

## Volumic averaging

$$\overline{\psi} = \frac{1}{V} \int_V \psi dv$$

$$\overline{\psi_l} = \frac{1}{V} \int_V \psi_l dv$$

# Averaging in a multiphasic system

## Mass-weighted averaging of a barycentric parameter

$$\widehat{\psi} = \frac{1}{m} \int_m \psi dm = \frac{1}{m} \int_V \rho \psi dv$$

## Mass-weighted averaging of a parameter for the species $l$

$$\widehat{\psi}_l = \frac{1}{m} \int_m \psi_l dm = \frac{1}{m} \int_V \rho \psi_l dv$$

with

$$\psi_l = \widehat{\psi}_l + \psi_l''$$

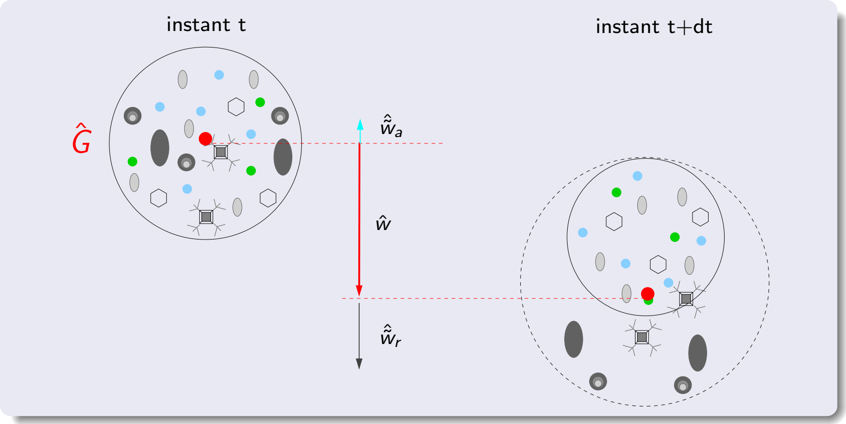
but

$$\psi_l = \widehat{\psi} + \psi_l'''$$

and

$$\widehat{\psi_l'''} = \widehat{\psi}_l - \widehat{\psi} = \widehat{\tilde{\psi}}_l \neq 0$$

# Example



# State equation at local scale

## An equation for perfect gaz only

$$p = p_a + e = \rho_a R_a T + \rho_v R_v T$$

or

$$p = \rho R_h T$$

with $R_h = q_a R_a + q_v Rv$ and $q_a = \rho_a/\rho$, $q_v = \rho_v/\rho$

# State equation at the model scale

## Mean state equations for dry air and water vapor

$$\begin{aligned} \overline{p}_a &= \overline{\rho}\widehat{q}_a R_a \widehat{T} + R_a \overline{\rho q_a'' T''} \\ \overline{e} &= \overline{\rho}\widehat{q}_v R_v \widehat{T} + R_v \overline{\rho q_v'' T''} \end{aligned}$$

## Hypothesis

To conserve the shape of the gaz state law, we have to neglect non-linear terms

$$\overline{p} = \overline{p}_a + \overline{e} = \overline{\rho}(R_a \widehat{q}_a + R_v \widehat{q}_v)\widehat{T}$$

# Total mass conservation

## Budget of mass in any geometric volum $V$

$$\frac{\partial m}{\partial t} = \frac{\partial \left(\int_V \rho dv\right)}{\partial t} = -\int_S \sum_l (\rho_l \vec{u}_l . \vec{n}) dS + \int_V \sum_l \dot{\rho}_l dv$$

## No mass source at the local scale

$$\sum_l \dot{\rho}_l = 0$$

# Continuity equation

## Eulerian form of the local multiphasic continuity equation

$$\frac{\partial \rho}{\partial t} = -\mathrm{div}(\rho \vec{u})$$

## Eulerian form of the avegarge multiphasic continuity equation

$$\frac{\partial \overline{\rho}}{\partial t} = -\mathrm{div}(\overline{\rho \vec{u}}) = -\mathrm{div}(\overline{\rho}\widehat{\vec{u}})$$

# An alternative : dry air conservation (Bannon, 2002)

**Budget of dry air in a geometric volum $V$**

$$\frac{\partial m_a}{\partial t} = \frac{\partial \left( \int_V \rho_a dv \right)}{\partial t} = - \int_S \rho_a \vec{u}_a . \vec{n} dS + \int_V \dot{\rho}_a dv$$

**No dry air source**

$$\dot{\rho}_a = 0$$

**Eulerian form of the dry air continuity equation**

$$\frac{\partial \rho_a}{\partial t} = -\mathrm{div}(\rho_a \vec{u_a})$$

# Budget equation for any local variable $\psi$

## Budget for any geometric volume $V$

$$\frac{\partial \int_V \sum_l (\rho_l \psi_l)\, dv}{\partial t} = -\int_S \sum_l (\rho_l \psi_l \vec{u}_l . \vec{n})\, ds + \int_V \sum_l \dot{S}_l dv$$

## Local form of the budget equation

$$\frac{\partial (\rho\psi)}{\partial t} = -\mathrm{div}[\sum_l (\rho_l \psi_l \vec{u}_l)] + \sum_l \dot{S}_l$$

# Advection term

## Barycentric advection

$$\rho\frac{\partial(\psi)}{\partial t} + \rho\vec{u}.\vec{\mathrm{grad}}(\psi) = \underbrace{-\frac{\partial[\sum_l(\rho_l\psi_l\tilde{w}_l)]}{\partial z}}_{(+)} + \sum_l \dot{S}_l$$

with $\psi = \sum_l q_l\psi_l$ and $\sum_l(\rho_l\tilde{w}_l) = 0$

## Alternative : Dry air advection (Bannon, 2002)

$$\rho_a\frac{\partial(\psi)}{\partial t} + \rho_a\vec{u}_a.\vec{\mathrm{grad}}(\psi) = \underbrace{-\frac{\partial[\sum_l(\rho_l\psi_l\breve{w}_l)]}{\partial z}}_{(+)} + \sum_l \dot{S}_l$$

with $\psi = \sum_l r_l\psi_l$ but $\sum_l(\rho_l\breve{w}_l) \neq 0$

# Turbulent / organised diffusive terms

## Budget equation for any modele scale variable

$$
\begin{aligned}
\frac{\partial(\overline{\rho}\widehat{\psi})}{\partial t} = & -\mathrm{div}(\overline{\rho}\widehat{\widehat{\psi}\vec{u}}) - \mathrm{div}(\overline{\rho\psi''\vec{u}''}) \\
& -\frac{\partial[\sum_l \overline{\rho}\widehat{q}_l\widehat{\psi}_l\widehat{\tilde{w}}_l]}{\partial z} - \frac{\partial[\sum_l \widehat{q}_l\overline{\rho\psi_l''\tilde{w}_l''}]}{\partial z} \\
& -\frac{\partial[\sum_l \widehat{\psi}_l\overline{\rho q_l''\tilde{w}_l''}]}{\partial z} - \frac{\partial[\sum_l \widehat{\tilde{w}}_l\overline{\rho q_l''\psi_l''}]}{\partial z} - \frac{\partial[\sum_l \overline{\rho q_l''\psi_l''\tilde{w}_l''}]}{\partial z} \\
& + \sum_l \overline{\dot{S}}_l
\end{aligned}
$$

## Simplified form

$$
\overline{\rho}\frac{\partial(\widehat{\psi})}{\partial t} + \underbrace{\overline{\rho}\widehat{\vec{u}}.\vec{\mathrm{grad}}(\widehat{\psi})}_{-\ advection} = -\underbrace{\mathrm{div}(\overline{\rho\psi''\vec{u}''})}_{turbulent} - \underbrace{\frac{\partial[\sum_l \overline{\rho}\widehat{q}_l\widehat{\psi}_l\widehat{\tilde{w}}_l]}{\partial z}}_{organised} + \sum_l \overline{\dot{S}}_l
$$

# Mass budget for species $l$

**Mass of $l$ variations in any volume $V$**

$$\frac{\partial m_l}{\partial t} = -\int_S \rho_l \vec{u}_l.\vec{n} dS + \int_V \dot{\rho}_l dv$$

with $m_l = \overline{\rho_l} V = \int_V \rho_l dv = \int_V \rho q_l dv$ and $\dot{\rho}_l$ the source/sink

# Model scale equation for $q_l$

## Eulerian form

$$\frac{\partial \overline{\rho}\widehat{q_l}}{\partial t} = -\mathrm{div}\left(\overline{\rho}\widehat{q_l}\widehat{\vec{u}}\right) - \frac{\partial\left(\overline{\rho}\widehat{q_l}\widehat{\tilde{w}_l}\right)}{\partial z} - \frac{\partial\left(\overline{\rho q_l'' w_l'''}\right)}{\partial z} + \overline{\dot{\rho}_l}$$

## Lagrangian form

$$\overline{\rho}\frac{\widehat{D}\widehat{q_l}}{Dt} = \overline{\rho}\left(\frac{\partial \widehat{q_l}}{\partial t} + \widehat{\vec{u}}.\vec{\mathrm{grad}}(\widehat{q_l})\right) = -\frac{\partial\left(\overline{\rho}\widehat{q_l}\widehat{\tilde{w}_l}\right)}{\partial z} - \frac{\partial\left(\overline{\rho q_l'' w_l'''}\right)}{\partial z} + \overline{\dot{\rho}_l}$$

# Barycentric momentum budget

**Variations of one componant of the barycentric momentum for any volume $V$**

$$\frac{\partial\left(\int_V\left(\sum_l \rho_l u_l^\alpha\right)dv\right)}{\partial t} = -\int_S\left(\sum_l \rho_l u_l^\alpha \vec{u}_l\right).\vec{n}ds$$

$$+\int_V\left(\sum_l \rho_l g^\alpha\right)dv$$

$$+\int_V\left(\sum_l \rho_l[2\vec{\Omega}\wedge\vec{u}_l]^\alpha\right)dv$$

$$+\int_S[\vec{\vec{\tau}}_s.\vec{n}]^\alpha ds$$

# The stess tensor

## Hypothesis on the stress tensor

We suppose that $\overrightarrow{\overrightarrow{\tau}}_s$ may be written as the sum of two tensors :

$$\overrightarrow{\overrightarrow{\tau}}_s = -p \overrightarrow{\overrightarrow{\delta}} + \overrightarrow{\overrightarrow{\sigma}}$$

with $p$ the total pressure of the gas and $\overrightarrow{\overrightarrow{\sigma}}$ the viscous stress tensor (« molecular diffusion »).

# Barycentric momentum budget

**Variations of one componant of the barycentric momentum for any volume $V$**

$$\int_V \left[ \frac{\partial (\rho u^\alpha)}{\partial t} \right] dv = - \int_V \mathrm{div} \left( \sum_l \rho_l u_l^\alpha \vec{u}_l \right) dv + \int_V (\rho g^\alpha)\, dv + \int_V \left( \rho [2\vec{\Omega} \wedge \vec{u}]^\alpha \right) dv - \int_V [\vec{\mathrm{grad}}(p)]^\alpha dv + \int_V \mathrm{div}[\vec{\vec{\sigma}}_s^\alpha] dv$$

# Barycentric zonal momentum equation at model scale

## Eulerian budget

$$\frac{\partial(\overline{\rho}\widehat{u})}{\partial t} = -\mathrm{div}\left(\overline{\rho}\widehat{u}\widehat{\vec{u}}\right) - \mathrm{div}\left(\overline{\rho u''\vec{u}''}\right) + 2\overline{\rho}\Omega\sin(\varphi)\widehat{v} - \frac{\partial\overline{p}}{\partial x} + \mathrm{div}(\overline{\sigma_u})$$

## « Lagrangian » form

$$\begin{aligned}\overline{\rho}\frac{\widehat{D}\widehat{u}}{Dt} &= \overline{\rho}[\frac{\partial\widehat{u}}{\partial t} + \widehat{\vec{u}}.\vec{\mathrm{grad}}(\widehat{u})] \\ &= -\mathrm{div}\left(\overline{\rho\vec{u}''u''}\right) + 2\overline{\rho}\Omega\sin(\varphi)\widehat{v} - \frac{\partial\overline{p}}{\partial x} + \mathrm{div}(\overline{\sigma_u})\end{aligned}$$

# Barycentric NH vertical momentum equation at model scale

## « Lagrangian » form

$$
\begin{aligned}
\overline{\rho}\frac{\partial\left(\widehat{w}\right)}{\partial t} + \overline{\rho}\widehat{\vec{u}}.\vec{\mathrm{grad}}(\widehat{w}) \quad = \quad & -\mathrm{div}\left(\overline{\rho w'' \vec{u}''}\right) \\
& -\frac{\partial\left(\sum_l \overline{\rho_l \tilde{w}_l^2}\right)}{\partial z} \\
& -\overline{\rho}g \\
& -\frac{\partial\overline{p}}{\partial z} + \mathrm{div}(\overline{\sigma_w})
\end{aligned}
$$

# Barycentric NH vertical momentum equation at model scale

The second term on the right hand side of the last equation may be decomposed in :

$$
\begin{aligned}
-\frac{\partial\left(\sum_l \overline{\rho_l \tilde{w}_l^2}\right)}{\partial z} &= -\frac{\partial\left(\sum_l \overline{\rho q_l \tilde{w}_l^2}\right)}{\partial z} \\
&= -\frac{\partial\left(\sum_l \overline{\rho}\widehat{q}_l \widehat{\tilde{w}^2}_l\right)}{\partial z} \\
&\quad -\frac{\partial\left(\sum_l \overline{\rho} q_l'' \widehat{\tilde{w}^2}_l\right)}{\partial z} - 2\frac{\partial\left(\sum_l \overline{\rho}\widehat{q}_l \widehat{\tilde{w}}_l \tilde{w}_l''\right)}{\partial z} \\
&\quad -2\frac{\partial\left(\sum_l \overline{\rho} q_l'' \widehat{\tilde{w}}_l \tilde{w}_l''\right)}{\partial z} - \frac{\partial\left(\sum_l \overline{\rho} q_l'' (\tilde{w}_l'')^2\right)}{\partial z}
\end{aligned}
$$

# Barycentric NH vertical momentum budget

We keep only the first term of this development. The average NH equation for the vertical velocity is then finally :

$$\begin{aligned}
\overline{\rho}\frac{\partial\left(\widehat{w}\right)}{\partial t} + \overline{\rho}\widehat{\vec{u}}.\vec{\mathrm{grad}}(\widehat{w}) \quad = \quad & -\mathrm{div}\left(\overline{\rho w''\vec{u}''}\right) \\
& -\frac{\partial\left(\sum_l \overline{\rho}\widehat{q}_l\widehat{\tilde{w}^2}_l\right)}{\partial z} \\
& -\overline{\rho}g \\
& -\frac{\partial\overline{p}}{\partial z} + \mathrm{div}(\overline{\sigma_w})
\end{aligned}$$

# Total energy budget

## Method

The classical method to deduce the thermodynamic equation is to substract the equation for the kinetic energy from the equation for the total energy (kinetic energy + internal energy).

## In practice

- do this operation at the local scale
- apply the average operator on the resulting thermodynamic equation
- if we neglect some non linear terms, the exact form of the mean thermodynamics equation may depend on the form of the local equation to which the averaging is applied.

# Hydrostatic thermodynamic equations at model scale

**Neglecting all the term with a perturbation of $c_p$ or a perturbation of $\tilde{w}_k$ and the presso-correlations**

$$
\begin{aligned}
\frac{\partial \overline{\rho}\widehat{c_p}\widehat{T}}{\partial t} + \mathrm{div}\left(\overline{\rho}\widehat{c_p}\widehat{T}\widehat{\vec{u}}\right) = {} & \frac{\partial \overline{p}}{\partial t} + \widehat{\vec{u}}.\vec{\mathrm{grad}}\overline{p} \\
& + \mathrm{div}(\widehat{c_p}\overline{\rho T''\vec{u}''}) \\
& - \frac{\partial \left[\sum_k \left(\overline{\rho}{c_p}_k\widehat{q_k}\widehat{T}\widehat{\tilde{w}_k}\right)\right]}{\partial z} \\
& + \overline{\epsilon} + \mathrm{div}(\overline{\vec{J}_Q}) \\
& + L_v(T=0)\overline{\dot{\rho}_l} + L_i(T=0)\overline{\dot{\rho}_i}
\end{aligned}
$$

# Hydrostatic temperature equation at model scale

$$
\begin{aligned}
\overline{\rho}\widehat{c_p}\frac{\widehat{D}\widehat{T}}{Dt} &= \frac{\widehat{D}\overline{p}}{Dt} \\
&+\mathrm{div}(\widehat{c_p}\overline{\rho T''\vec{u}''}) \\
&-\sum_k\left(\overline{\rho}_k\widehat{\tilde{w}_k}\frac{\partial {c_p}_k\widehat{T}}{\partial z}\right) \\
&+\overline{\epsilon}+\mathrm{div}(\overline{\vec{J}_Q}) \\
&+L_v(T)\overline{\dot{\rho}_l}+L_i(T)\overline{\dot{\rho}_i}
\end{aligned}
$$

# NH total energy budget at local scale

$$\rho\left[\frac{\partial\left(e_i+\tilde{e}_c\right)}{\partial t}+\vec{u}.\vec{\mathrm{grad}}(e_i+\tilde{e}_c)\right] = -\frac{\partial\left(\sum_k(\rho_k ei_k+p_k)\tilde{w}_k\right)}{\partial z} -\rho\tilde{e}_c\frac{\partial(w)}{\partial z} -p\mathrm{div}(\vec{u})+\epsilon+\mathrm{div}(\vec{J}_Q)$$

# Practical applications for Arome

## Tendency and diagnostic (budget) interface

Because of the « finite » falling velocity of the precipitation, the current budget equations used in the Arpege/Aladin physics to compute the physical tendencies from fluxes and in the diagnostics (DDH) is not valid. A more generalised set of equations/interface as to be coded (B. Catry et al) and a nice set of fluxes have to be recovered from the microphysic processes (T. Kovavic et al).

## Barycentric departure fluxes

The « new » terms (« barycentric departure fluxes ») in the equations should be coded and analysed (computational cost compared to order of magnitude).