Preparation of ALADIN-NH initial and lateral boundary conditions

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Introduction

- depending on driving model, ALADIN-NH initial and lateral boundary conditions are prepared by configurations (E)E927:

E927	ARPEGE $\longrightarrow$ ALADIN
EE927	ALADIN $\longrightarrow$ ALADIN

- basically these configurations perform change of geometry including necessary horizontal and vertical interpolations

- when driving model is hydrostatic, they must also invent NH prognostic fields based on true pressure p and vertical velocity $w \equiv \dot{z}$

How to prepare namelist for H $\rightarrow$ NH configuration (E)E927?

- take namelist for H $\rightarrow$ H configuration (E)E927 and make following changes:
  - specify that input FA file is hydrostatic:

```
&NAMCTO
  LNHDYN=.F.,
```

  - use NH vertical discretization:

```
&NAMDYN
  NDLNPR=1,
```

  - ask for NH fields in output FA file (`n` being the number of hydrostatic 3D dynamic fields):

```
&NAMFPC
  CFP3DF(n+1)='PRESS.DEPART',
  CFP3DF(n+2)='VERTIC.DIVER',
```

What is stored in output FA file?

- from practical reasons, FA file does not contain directly NH prognostic variables, but their unscaled values:

`S<lev>PRESS.DEPART`	$\tilde{p}_l \equiv p_l - \pi_l$
`S<lev>VERTIC.DIVER`	$-g(w_{\tilde{l}} - w_{\tilde{l}-1})$

- after being read from FA file, NH prognostic variables must be scaled (depending on choice of `NPDVAR`, `NVDVAR`)

- before being written to FA file, NH prognostic variables must be unscaled

- advantage of this approach is that FA header need not contain information about integration settings like `NPDVAR`, `NVDVAR`, `SIPR`, `SITR`

How are invented NH fields from hydrostatic initial/coupling state?

- NH initial/coupling state is assumed to be in hydrostatic equilibrium:

$$\frac{\partial p}{\partial z} = -\rho g \quad \Rightarrow \quad p = \pi$$

- vertical velocity w is set equal to hydrostatically diagnosed vertical velocity in the absence of diabatic terms:

$$w = (w_H)_{\text{adiab}}$$

Diagnostics of hydrostatic vertical velocity in z-system (1)

- hydrostatic approximation replaces prognostic equation for w by diagnostic relation between p and ρ:

$$\begin{aligned}\text{NH: } \frac{\mathrm{d}w}{\mathrm{d}t} &= -\frac{1}{\rho} \cdot \frac{\partial p}{\partial z} - g + \mathcal{W} \\ \text{H} \quad : \quad 0 &= \frac{\partial p}{\partial z} + \rho g\end{aligned}$$

- if H system is formulated in z-coordinate, w remains in vertical advection term and in 3D divergence:

$$\begin{aligned}\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} &= -\frac{1}{\rho}\nabla_z p + \boldsymbol{\mathcal{V}} \\ \frac{\mathrm{d}p}{\mathrm{d}t} &= -\varkappa p \left(\nabla_z \cdot \mathbf{v} + \frac{\partial w}{\partial z}\right) + \frac{p}{T} \cdot \frac{Q}{c_v} \\ \frac{\mathrm{d}T}{\mathrm{d}t} &= -(\varkappa - 1)T \left(\nabla_z \cdot \mathbf{v} + \frac{\partial w}{\partial z}\right) + \frac{Q}{c_v}\end{aligned}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} \equiv \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla_z + w\frac{\partial}{\partial z} \qquad \rho \equiv \frac{p}{RT} \qquad \varkappa \equiv \frac{c_p}{c_v}$$

Diagnostics of hydrostatic vertical velocity in z-system (2)

- diagnostic formula for w must be found in order to close the system
- it uses the fact that in H system evolution of p and ρ is not independent, but it must respect hydrostatic assumption:

$$\frac{\mathrm{d}p}{\mathrm{d}t} = -\varkappa p \left(\nabla_z \cdot \mathbf{v} + \frac{\partial w}{\partial z} \right) + \frac{p}{T} \cdot \frac{Q}{c_v}$$
$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\rho \left(\nabla_z \cdot \mathbf{v} + \frac{\partial w}{\partial z} \right)$$

$$\frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{\partial p}{\partial z} + \rho g \right) = 0 \quad \Rightarrow \quad \frac{\partial}{\partial z} \frac{\mathrm{d}p}{\mathrm{d}t} - \frac{\partial \mathbf{v}}{\partial z} \cdot \nabla_z p - \frac{\partial w}{\partial z} \cdot \frac{\partial p}{\partial z} + g \frac{\mathrm{d}\rho}{\mathrm{d}t} = 0$$

$$\Downarrow$$

$$\frac{\partial}{\partial z} \left(\varkappa p \frac{\partial w}{\partial z} \right) = \frac{\partial}{\partial z} \left(-\varkappa p \nabla_z \cdot \mathbf{v} + \frac{p}{T} \cdot \frac{Q}{c_v} \right) - \frac{\partial \mathbf{v}}{\partial z} \cdot \nabla_z p - \rho g \nabla_z \cdot \mathbf{v}$$

Diagnostics of hydrostatic vertical velocity in z-system (3)

- resulting diagnostic formula is Richardson equation

- since it is second order differential equation for w, two boundary conditions must be specified in order to determine unique solution

- at domain bottom given by relation $gz = \phi_S(x, y)$, free slip boundary condition is usually applied:

$$gw_S = \mathbf{v}_S \cdot \nabla_z \phi_S$$

- at domain top, elastic boundary condition $p_T =$ const (combined with assumption that no mass crosses the boundary) takes the form:

$$\left(\varkappa p \frac{\partial w}{\partial z}\right)_T = \left(-\varkappa p \nabla_z \cdot \mathbf{v} + \frac{p}{T} \cdot \frac{Q}{c_v}\right)_T$$

Diagnostics of hydrostatic vertical velocity in p-system (1)

- situation in p-system is different since continuity equation becomes diagnostic relation for pressure vertical velocity $\omega \equiv \dot{p}$
- H equations can be closed without diagnosing w (it is sufficient to diagnose 3D divergence)
- still w can be diagnosed by combining continuity equation with prognostic equation for p:

$$\frac{\partial \omega}{\partial p} = -\nabla_p \cdot \mathbf{v} \quad \Rightarrow \quad \omega = \omega_T - \int_{p_T}^{p} \nabla_p \cdot \mathbf{v} \, \mathrm{d}p'$$

$$\frac{\mathrm{d}p}{\mathrm{d}t} = -\varkappa p \left(\nabla_p \cdot \mathbf{v} + \rho \nabla_p \phi \cdot \frac{\partial \mathbf{v}}{\partial p} - \rho g \frac{\partial w}{\partial p} \right) + \frac{p}{T} \cdot \frac{Q}{c_v}$$

$$\Downarrow$$

$$-\varkappa p \rho g \frac{\partial w}{\partial p} = -\varkappa p \left(\nabla_p \cdot \mathbf{v} + \rho \nabla_p \phi \cdot \frac{\partial \mathbf{v}}{\partial p} \right) + \frac{p}{T} \cdot \frac{Q}{c_v} + \int_{p_T}^{p} \nabla_p \cdot \mathbf{v} \, \mathrm{d}p' - \omega_T$$

Diagnostics of hydrostatic vertical velocity in p-system (2)

- resulting diagnostic formula for w corresponds to vertically integrated Richardson equation

- it is only first order differential equation for w, but unique solution still requires 2 boundary conditions (one for w_S, another one for ω_T)

- free slip bottom boundary condition now have the form:

$$gw_S = \mathbf{v}_S \cdot \nabla_p \phi_S$$

- elastic top boundary condition $p_T = \text{const}$ (combined with assumption that no mass crosses the boundary) translates to:

$$\omega_T = 0$$

Interpretation of invented NH initial/coupling state

- so far NH initial/coupling state was constructed assuming hydrostatic equilibrium with hydrostatically diagnosed vertical velocity w:

$$p = \pi \qquad w = w_H$$

- using prognostic equation for NH pressure departure $\mathcal{P} \equiv (p - \pi)/\pi$ it can be shown that these requirements are equivalent to:

$$\mathcal{P}|_{t=t_0} = 0 \qquad \left.\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}t}\right|_{t=t_0} = 0$$

- it means that initial/coupling NH fields are constructed in such way that they do not bring any acoustic energy into the system

Why does it work?

- integration starting from invented NH initial state quickly adapts to local forcings ⇒ realistic NH state is reached after short time

- this process can be illustrated using non-linear 2D potential flow:

  - uniform background flow with horizontal wind speed $15\,\mathrm{ms}^{-1}$ over bell shaped mountain with half width $a = 100\,\mathrm{m}$ and height $h = 100\,m$

  - isothermal background stratification with temperature 239 K

  - background surface pressure 101 325 Pa

  - periodic domain with 64 gridpoints (8 points in coupling zone), horizontal resolution $\Delta x = 20\,\mathrm{m}$

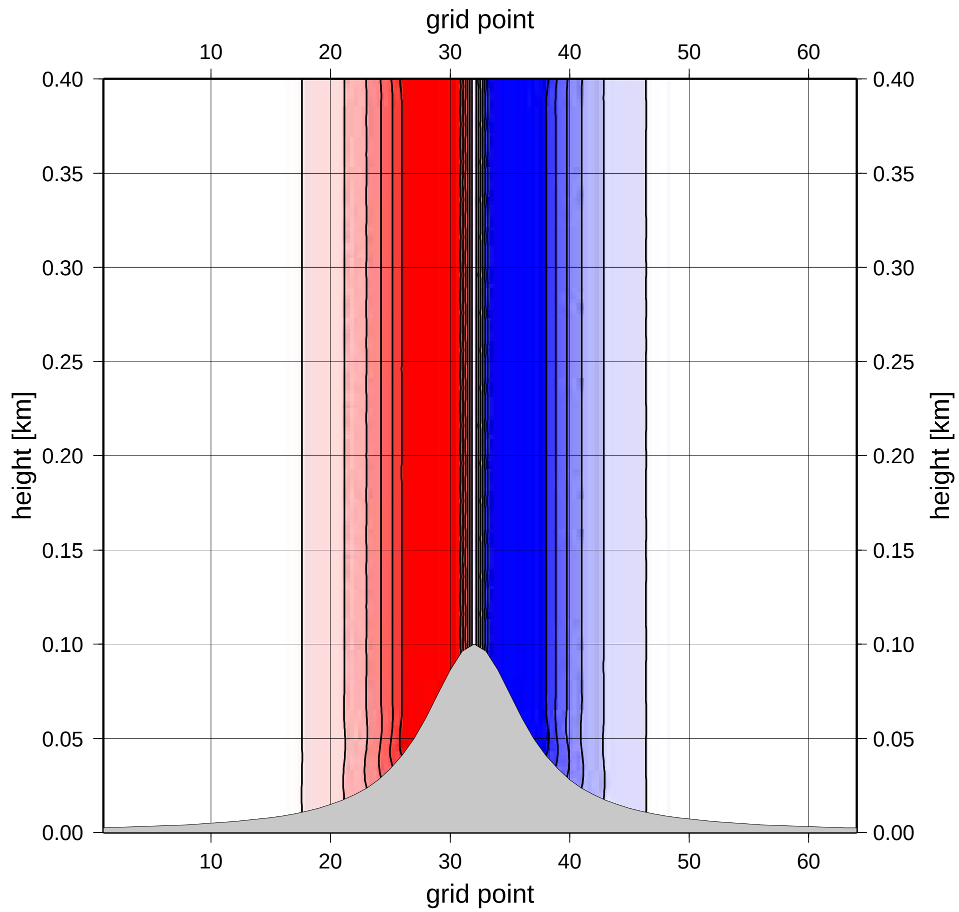
  - 39 levels vertical levels with $\Delta z \approx 20\,\mathrm{m}$, sponge layer between 450 and 750 m

  - time constant LBC (coupling files identical to init file)

  - adiabatic integration using SL2TL NESC scheme with $\Delta t = 1\,\mathrm{s}$

NL potential flow – initial state

hydrostatic vertical velocity w_H

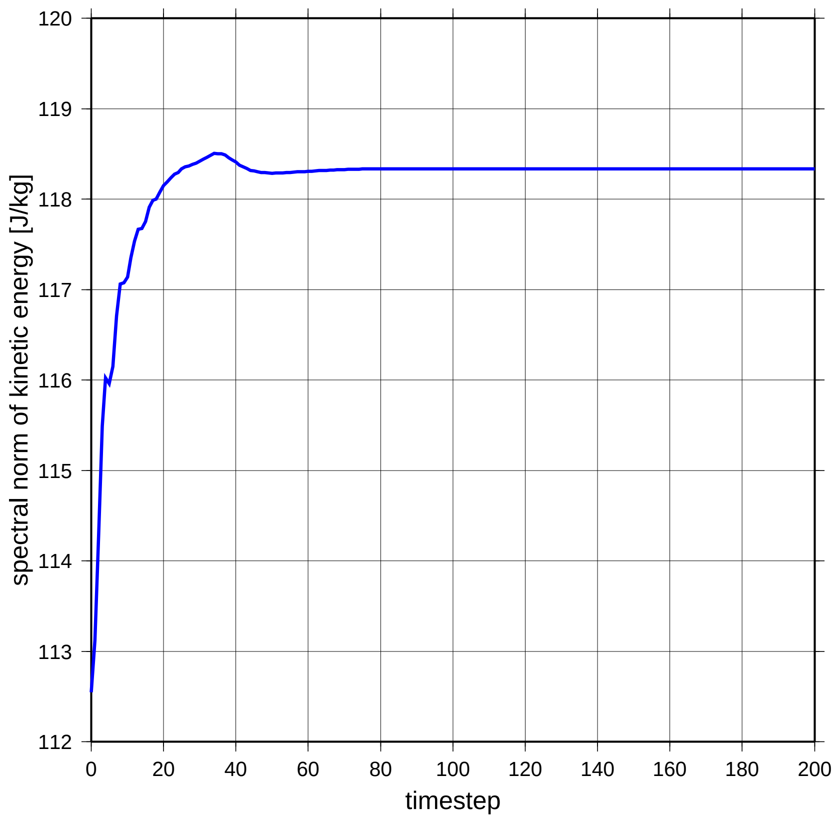


$$\Delta t = 1.0\,\mathrm{s},\ N_{\mathrm{step}} = +0000$$

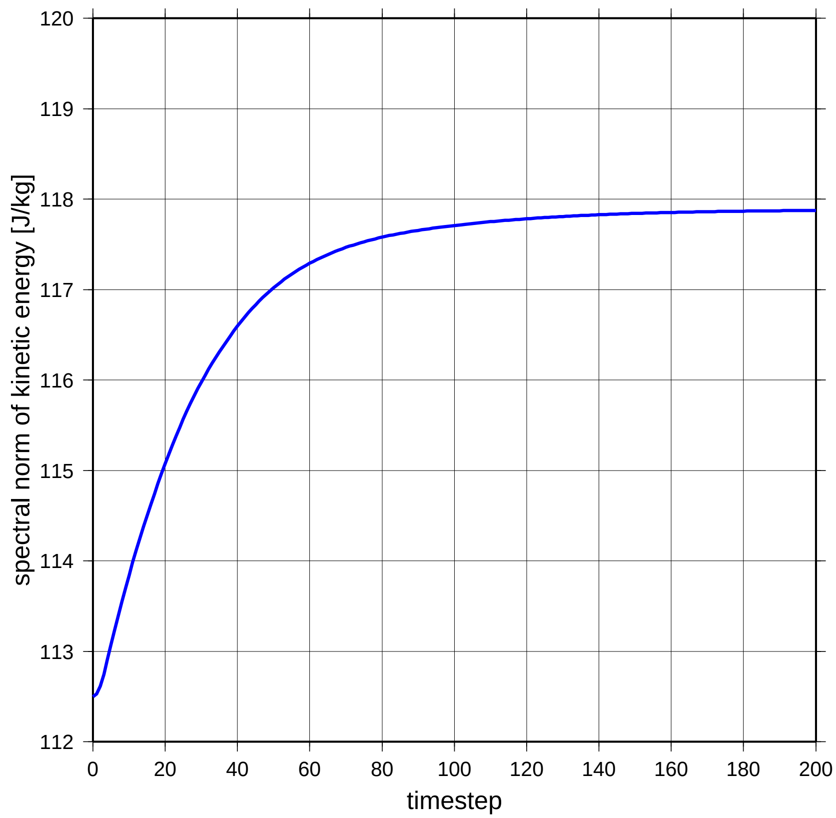
NL potential flow – H versus NH evolution

spectral norms of kinetic energy

NH model



H model

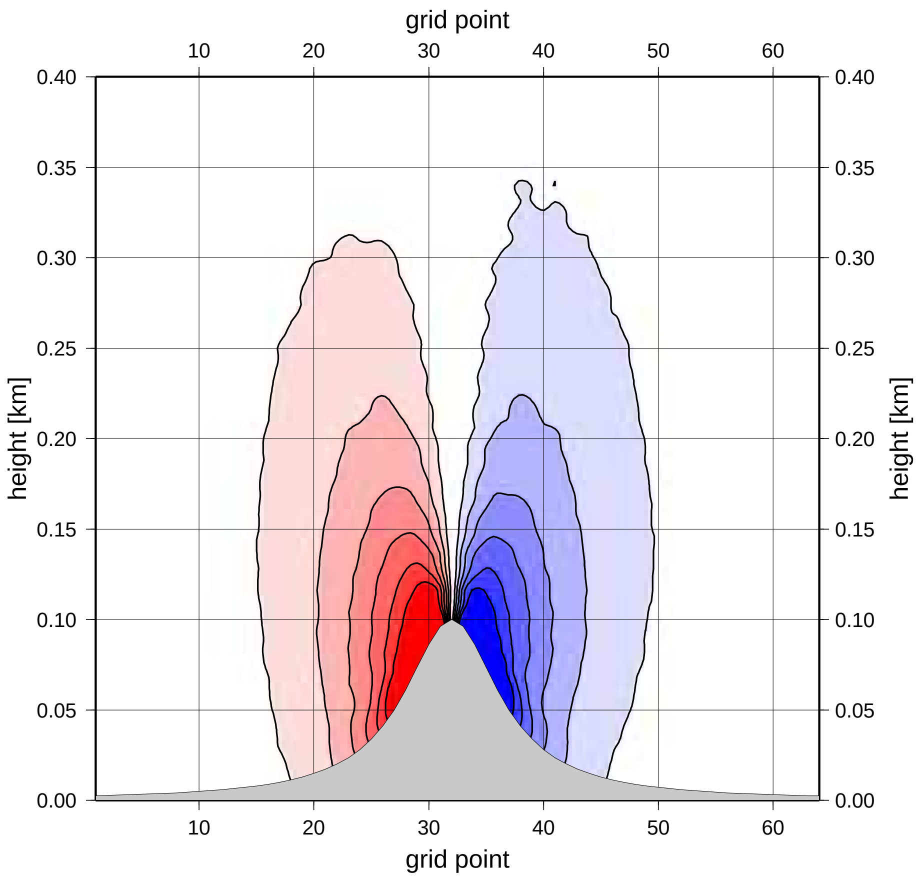


$$\Delta t = 1.0\,\mathrm{s}$$

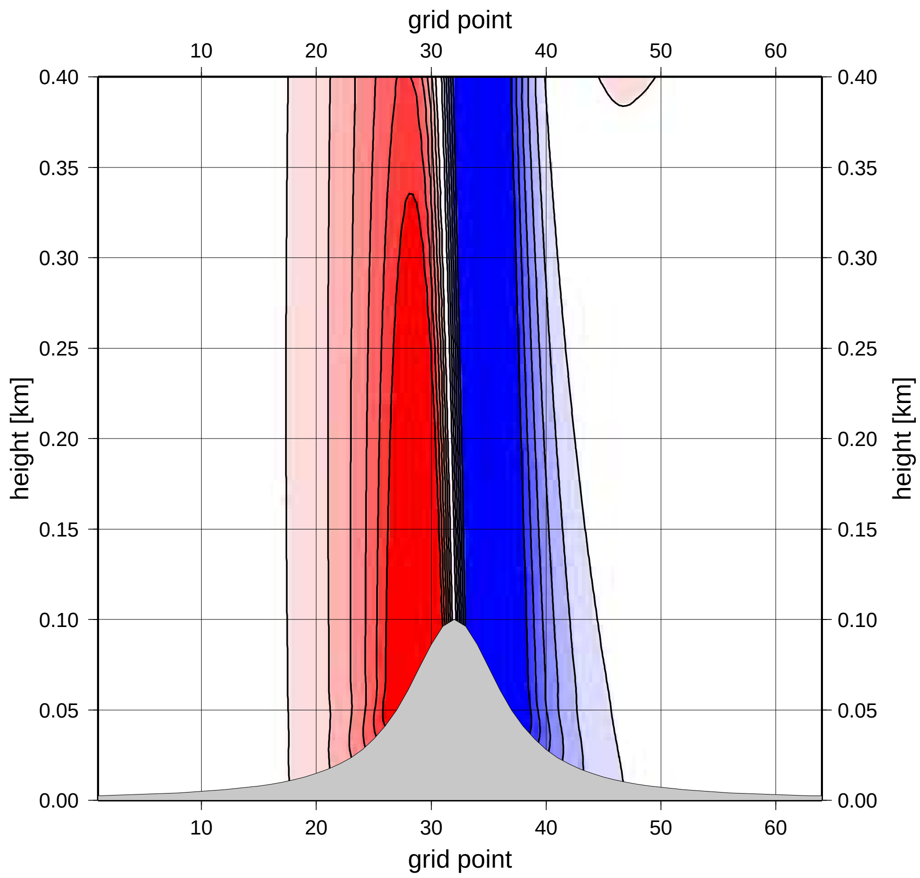
NL potential flow – H versus NH results

vertical velocity w

NH model, w_{NH}



H model, w_H

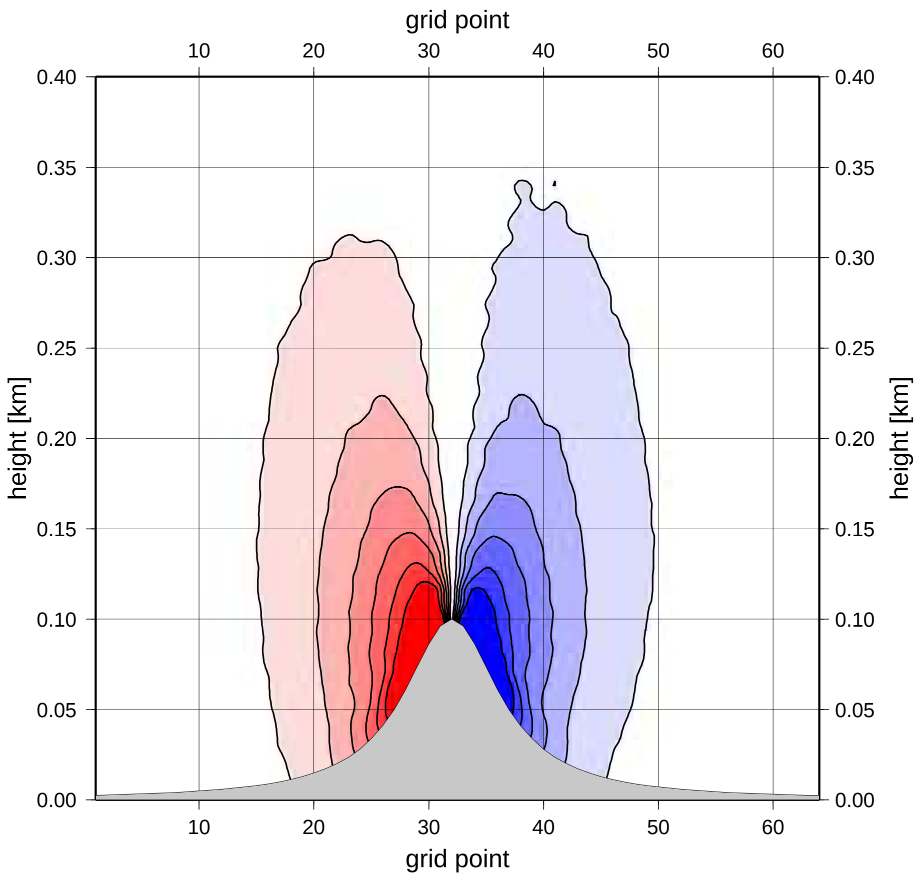


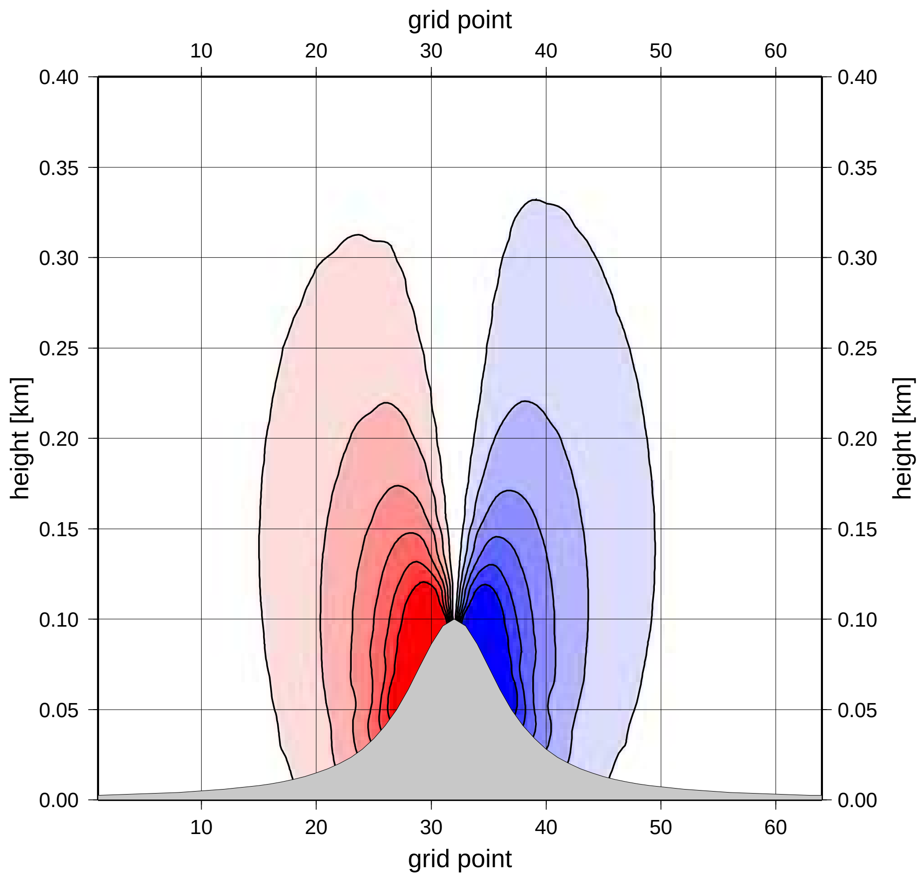
$\Delta t = 1.0\,\mathrm{s},\ N_{\mathrm{step}} = +0200$

NL potential flow – NH results, w_H versus w_{NH}

vertical velocity w in NH model

w_{NH} w_H



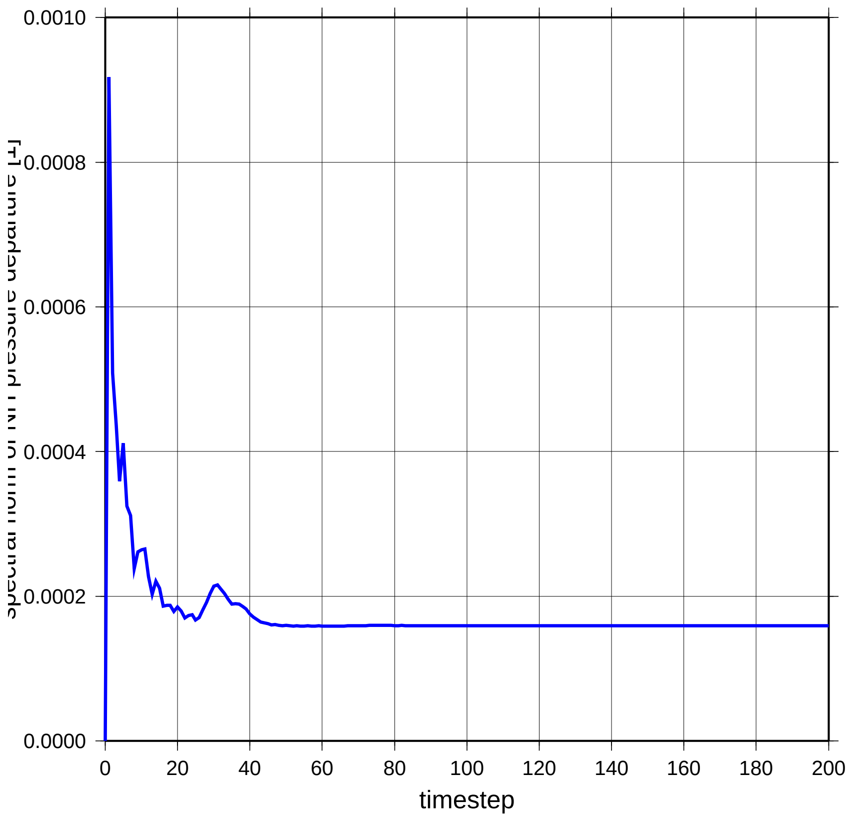


$\Delta t = 1.0\,\text{s},\ N_{\text{step}} = +0200$

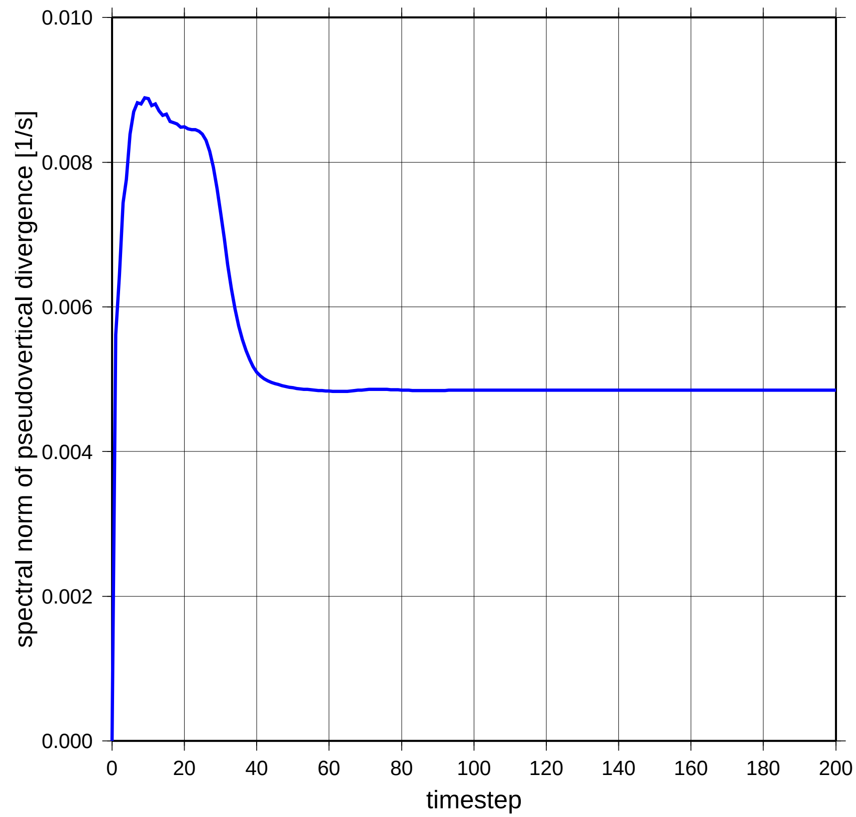
NL potential flow – adaptation of NH prognostic variables

evolution of spectral norms

NH pressure departure $\widehat{q}$



vertical divergence d



$$\Delta t = 1.0\,\mathrm{s}$$

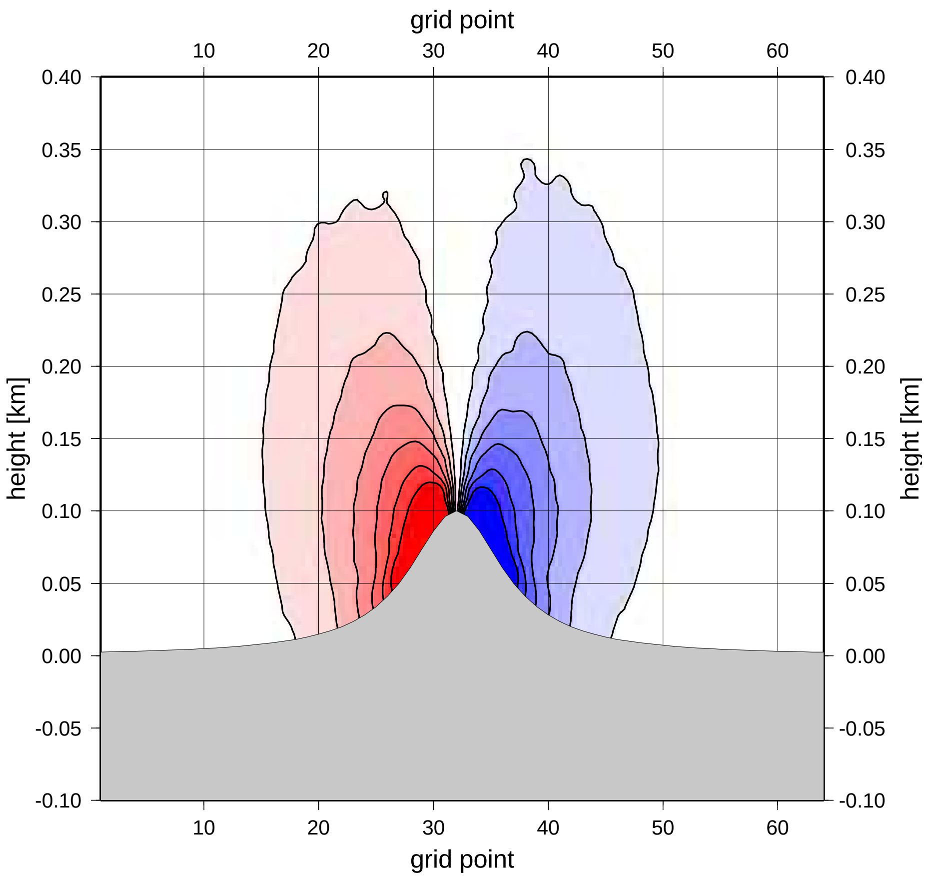
How do we know that flow is non-linear?

- in linear regimes system responds to forcing linearly
- if the forcing changes sign, response should also change the sign
- in orographic flows forcing is the slope of terrain (gradient of orography)
- to check for non-linearity it is enough to replace volcano by crater and look at response
- comparison should be done in $x\eta$-coordinate plane, since in xz-coordinate plane the two computational domains are different

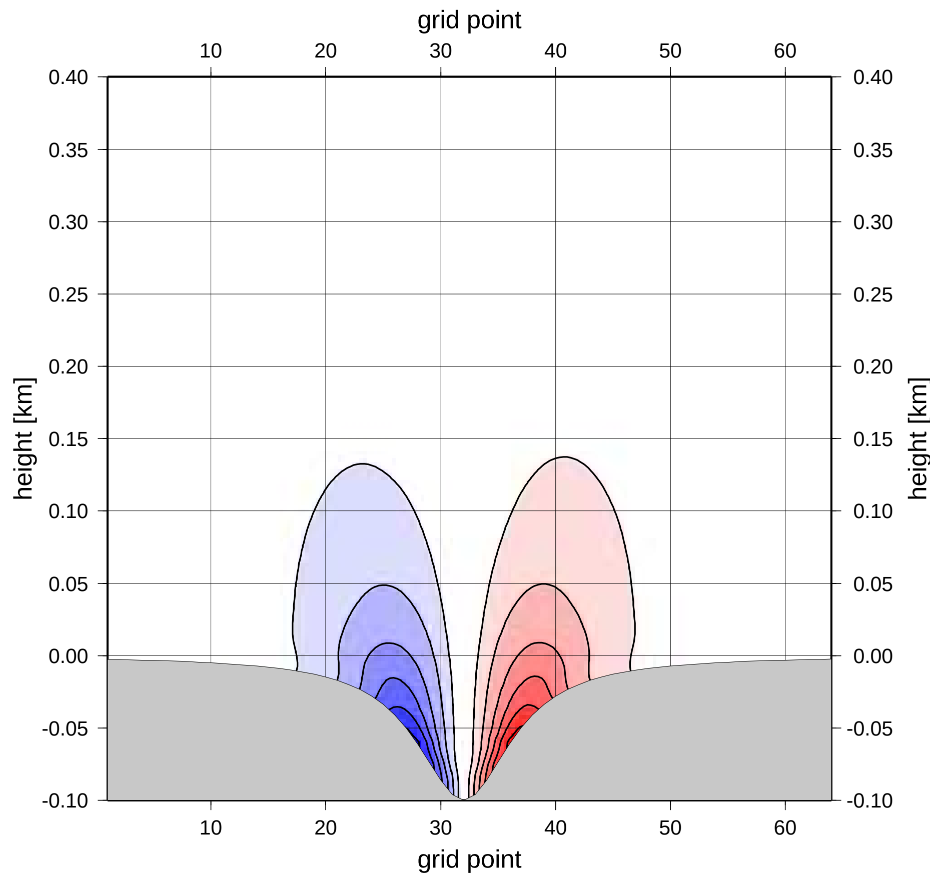
Proof of flow non-linearity (1)

vertical velocity w, xz-plane

$h = 100\,\mathrm{m}$



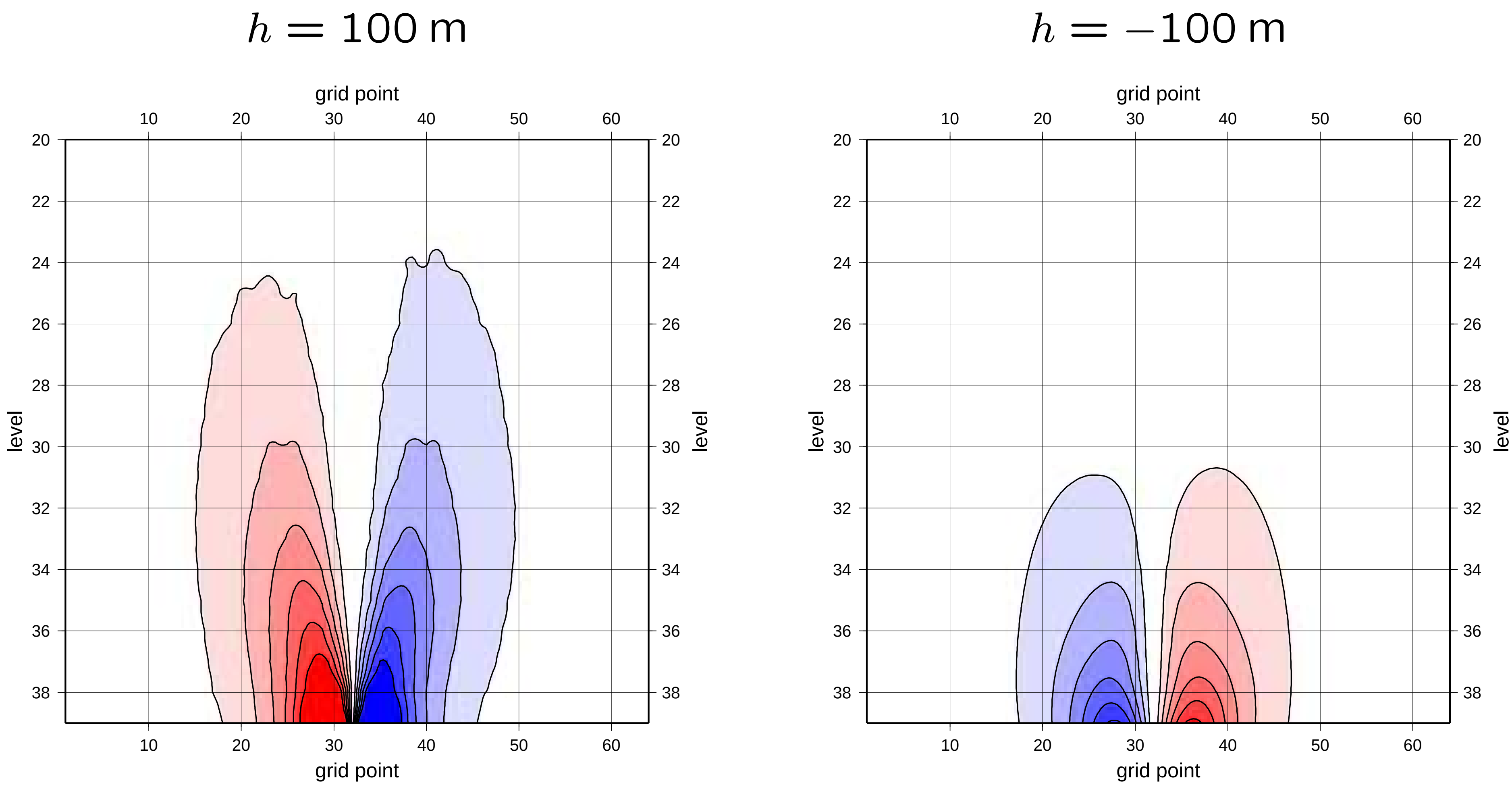
$h = -100\,\mathrm{m}$



$\Delta t = 1.0\,\mathrm{s},\ N_{\mathrm{step}} = +0200$

Proof of flow non-linearity (2)

vertical velocity w, $x\eta$-plane

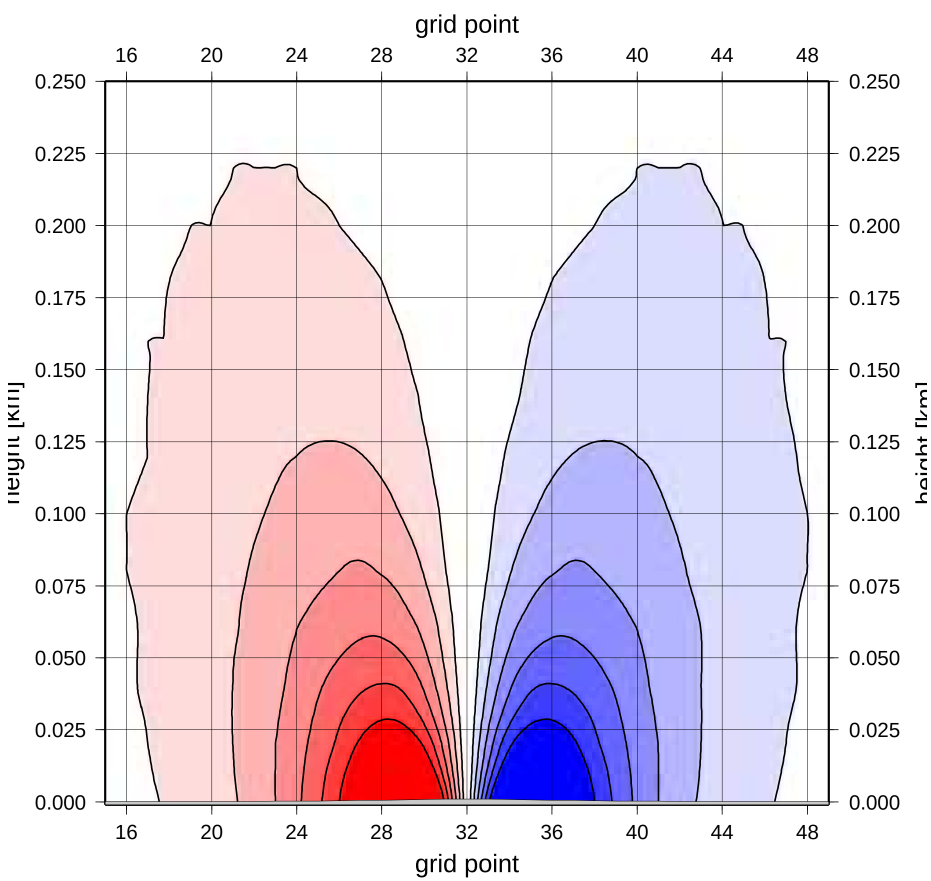


$\Delta t = 1.0\,\mathrm{s},\ N_{\mathrm{step}} = +0200$

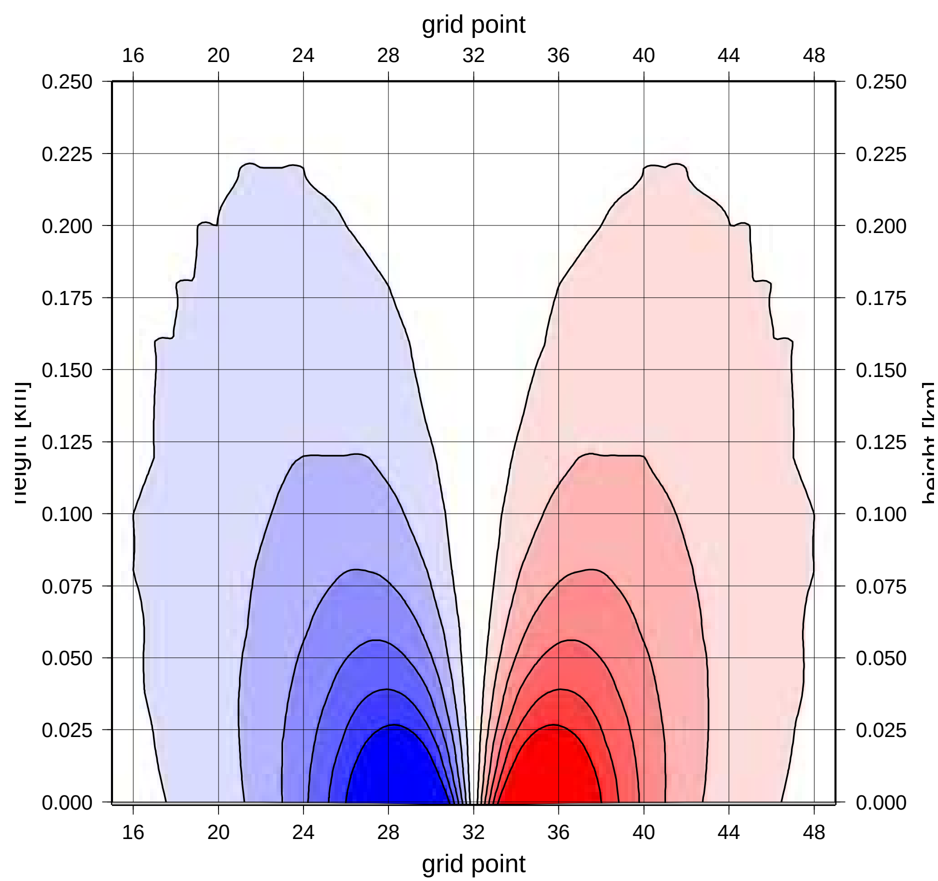
Example of linear potential flow

vertical velocity w, xz-plane

$h = 1\,\mathrm{m}$



$h = -1\,\mathrm{m}$



$\Delta t = 1.0\,\mathrm{s},\ N_{\mathrm{step}} = +0200$

Adaptation of NH prognostic variables – conclusions

- invented NH initial state quickly adapts to local forcings, providing realistic NH fields after short evolution time

- side effect of digital filter initialization is adaptation of NH initial fields

- in NH model, difference between prognostically computed vertical velocity w_{NH} and hydrostatically diagnosed vertical velocity w_H is usually small

- this is true also in NH regimes, where hydrostatic model is not able to provide correct vertical velocity field

Why is hydrostatic vertical velocity diagnosed without diabatic term Q?

- prognostic equations for p and T have the form:

$$\frac{\mathrm{d}p}{\mathrm{d}t} = -\varkappa p D_3 + \frac{p}{T} \cdot \frac{Q}{c_v} \qquad \frac{\mathrm{d}T}{\mathrm{d}t} = -(\varkappa - 1) T D_3 + \frac{Q}{c_v}$$

- however, in current ALADIN code they are modified in order to parameterize hydrostatic adjustment:

$$\frac{\mathrm{d}p}{\mathrm{d}t} = -\varkappa p D_3 + 0 \qquad \frac{\mathrm{d}T}{\mathrm{d}t} = -(\varkappa - 1) T D_3 + \frac{Q}{c_p}$$

- direct consequence is that Q term disappears from diagnostic formula for hydrostatic vertical velocity w

- this is advantageous, since hydrostatic vertical velocity w can be diagnosed without call to model physics (no need to evaluate diabatic heating rate Q)

Quick look at hydrostatic adjustment (1)

- assume that heat amount $\Delta\mathcal{Q}$ is quickly added to the air parcel with mass m, initial temperature T and pressure p

- if fast enough, heating will happen at constant volume, increasing parcel's temperature and pressure by amounts:

$$\Delta T = \frac{\Delta\mathcal{Q}}{mc_v} \qquad \Delta p = \frac{p}{T} \cdot \frac{\Delta\mathcal{Q}}{mc_v}$$

- due to pressure increase, parcel will not be in mechanical equilibrium with its surroundings $\Rightarrow$ it will expand adiabatically until the new equilibrium is reached

- work done during expansion at the expense of internal energy will be radiated away by acoustic waves

Quick look at hydrostatic adjustment (2)

- final state of parcel will be the same as if the heat $\Delta\mathcal{Q}$ was added at constant pressure (no generation of acoustic waves):

$$\Delta T = \frac{\Delta\mathcal{Q}}{mc_p} \qquad \Delta p = 0$$

- parameterization of hydrostatic adjustment is important in H model, since it prevents false dynamical response to pressure changes induced by diabatic heating (system cannot respond correctly, since hydrostatic approximation filters out acoustic waves)

- currently it is used also in ALADIN-NH where it might be beneficial (acoustic waves are present in NH model, but they are slowed down by semi-implicit scheme)

Quick look at hydrostatic adjustment (3)

- hydrostatic adjustment parameterization is incompatible with mass continuity equation, because:

  heated layer expands and displaces layers above, but we pretend there is no vertical velocity connected to this displacement $\Rightarrow$ density of heated layer decreases while there is no divergence of mass flux

- in order to reach consistency, corresponding source term must be introduced in continuity equation:

$$\frac{1}{\rho} \cdot \frac{\mathrm{d}\rho}{\mathrm{d}t} = -D_3 - \frac{Q}{c_p T}$$

- it means that with parameterization of hydrostatic adjustment diabatic heating acts as mass sink

- so far this problem was ignored in ALADIN $\Rightarrow$ it is probably not very harmful

Summary

- initial and lateral boundary conditions for ALADIN-NH are prepared by configurations (E)E927, there are only few namelist changes with respect to H case

- in case of H driving model, NH specific fields must be invented

- they are constructed in such way that initial/coupling NH state is in hydrostatic equilibrium and free from acoustic disturbances

- this approach works thanks to the fact that NH fields quickly adapt to local forcings (orography, diabatism, . . .)

- in order to be consistent with hydrostatic adjustment parameterization, hydrostatic vertical velocity w is diagnosed with omitted diabatic heating term