Modeling stable boundary layers using the Explicit Algebraic Reynolds-stress model

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Explicit algebraic Reynolds-stress model (EARSM) for ABL

EARSM with buoyancy effects was formulated in *Lazeroms et al. (2013)* from the differential Reynolds-stress model

$$\frac{\mathrm{D}\,\overline{u_i u_j}}{\mathrm{D}t} = \mathcal{P}_{ij} + \mathcal{G}_{ij} + \Pi_{ij} - \varepsilon_{ij} + \mathcal{D}_{ij}, \qquad (1)$$

$$\frac{\mathrm{D}\,\overline{u_i \theta}}{\mathrm{D}t} = \mathcal{P}_{\theta i} + \mathcal{G}_{\theta i} + \Pi_{\theta i} - \varepsilon_{\theta i} + \mathcal{D}_{\theta i}. \qquad (2)$$

The EARSM has the following properties

- **explicit** algebraic relations for $\overline{u_i u_j}$, $\overline{u_i \theta} = f\left(\frac{\partial \mathbb{V}}{\partial z}, \frac{\partial \Theta}{\partial z}, \frac{\overline{u_i{}^2}}{2}, \frac{\overline{\theta^2}}{2}, \varepsilon\right)$,
- **physical** turbulence model (similar to Mellor-Yamada level 3 model),
- computationally requirement similar to "K-theory" models.

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Consistent boundary condition treatment in EARSM

- no predefining Monin-Obukhov similarity theory in boundary condition treatment, instead we want EARSM with proper boundary conditions to predict MOST scaling

For more details see article in Boundary-Layer Meteorology that is coming out.

Consistent boundary condition treatment in EARSM

If the **first points** of the model domain (z_1) **is placed close to the surface**, the buoyancy effects there are not significant so the standard log-law is valid

$$u_* = \frac{\kappa}{\ln \frac{z_1}{z_0}} \mathbb{V}(z_1), \quad (3)$$

$$\theta_* = - \frac{\overline{w\theta}(z_1)}{u_*}. \quad (4)$$

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Boundary conditions in EARSM:

$$U(z_0) = 0, \qquad V(z_0) = 0, \qquad \Theta(z_0) = \Theta_S(z_{0_h}) + \frac{Pr_t \theta_*}{\kappa} \ln \frac{z_0}{z_{0_h}},$$

$$K(z_0) = \frac{{u_*}^2}{\sqrt{f_m}}, \qquad K_\theta(z_0) = \frac{r Pr_t}{\sqrt{f_m}} {\theta_*}^2, \qquad \varepsilon(z_0) = \frac{{u_*}^3}{\kappa(z_1 - z_0)} \ln \frac{z_1}{z_0}$$

NOTE: The boundary condition for dissipation of TKE is not the standard relation $\varepsilon = \frac{{u_*}^3}{\kappa z}$.

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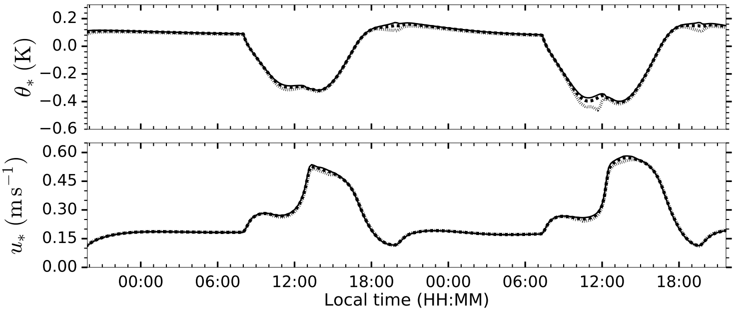
GABLS2 simulation, surface fluxes with consistent boundary conditions

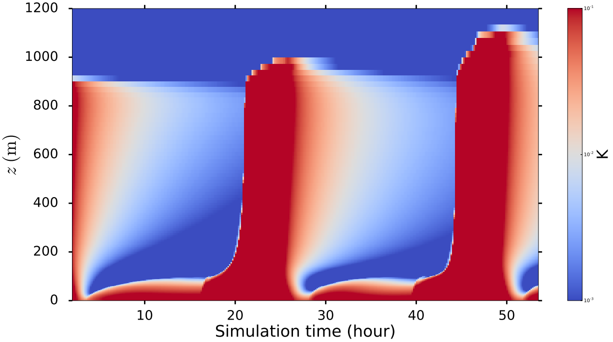


θ_* and u_* computed when $z_1 = 0.5$ m (——) , 1 m (- - - -), and 1.75 m(········).

- **grid insensitive solution** (even when $u_* = \mathbb{V}(z_1)\kappa/\ln(z_1/z_0)$)
- model is able to capture dynamics of the "u_* - θ_* " coupling.

GABLS2 simulation, residual turbulence in the ABL

- EARSM **predicts residual turbulence** that remains in the higher part of the ABL during the stable period of the GABLS2 simulation.

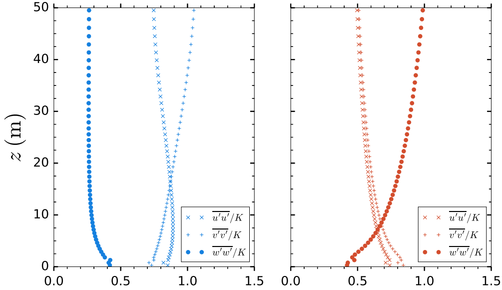


Contour plot of K-variation change with height in time during the GABLS2 simulation. Colormap is log-linear between the limits of 10^{-3} and 10^{-1}.

GABLS2 simulation, stable vs. unstable ABL turbulence

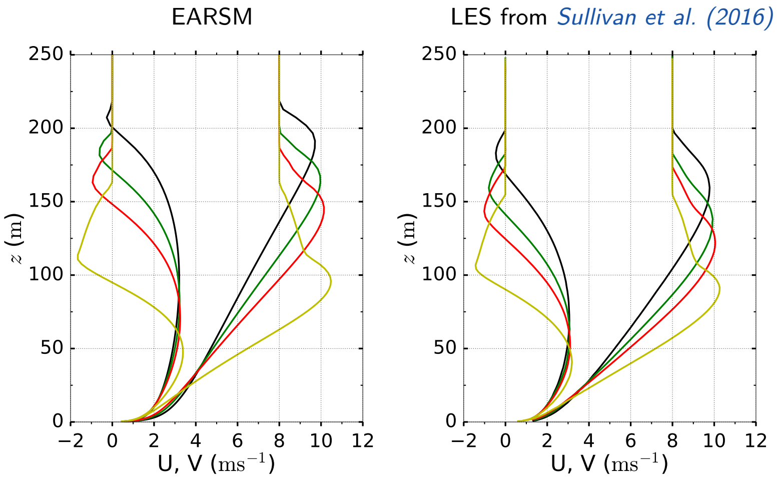
EARSM clearly distinguishes between stable and unstable ABL turbulence.

- During stable ABL EARSM predicts $\overline{uu}, \overline{vv} > \overline{ww}$ (damping of vertical motions).
- During unstable ABL EARSM predicts $\overline{ww} > \overline{uu}, \overline{vv}$ (convection).

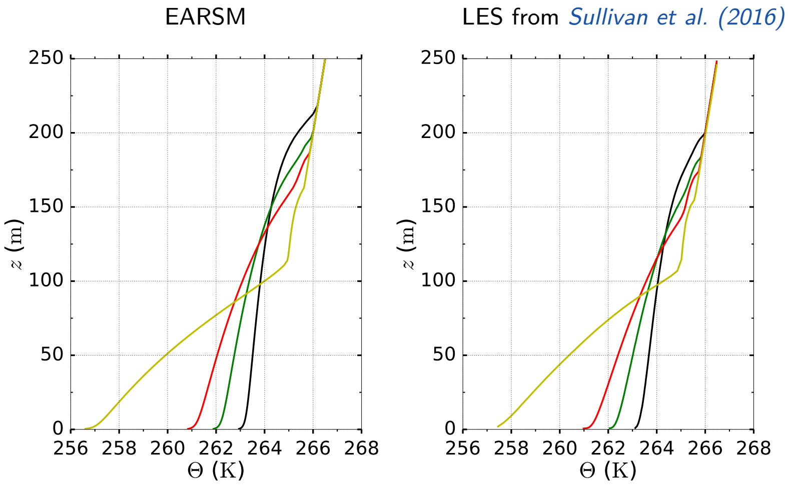


Stable and **unstable** ABL period from GABLS2.

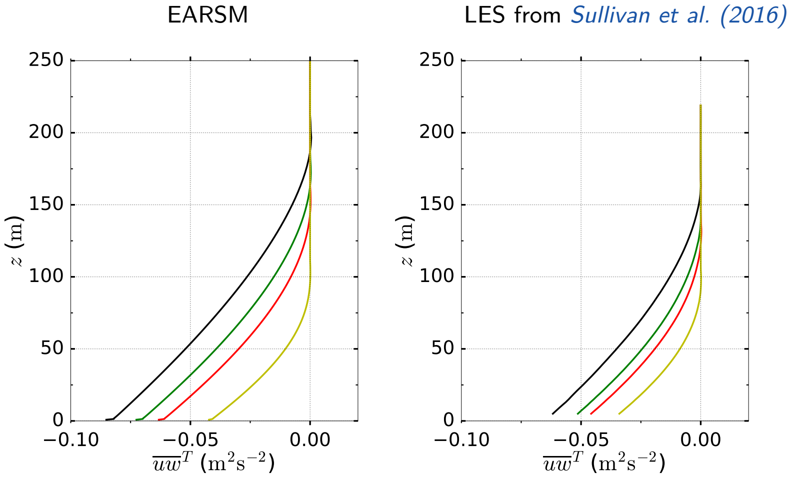
GABLS1-alike simulation with different surface cooling rates (0.25, 0.375, 0.5, 1.0) K/hr, horizontal wind speed



GABLS1-alike simulation with different surface cooling rates (0.25, 0.375, 0.5, 1.0) K/hr, potential temperature



GABLS1-alike simulation with different surface cooling rates (0.25, 0.375, 0.5, 1.0) K/hr



NOTE: Vertical turbulent momentum flux $\overline{uw}^T$ is rotated in the direction of the mean flow.

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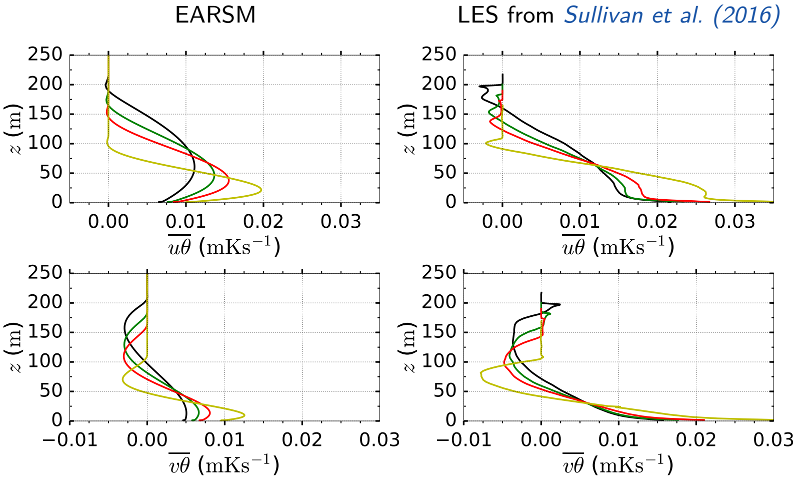
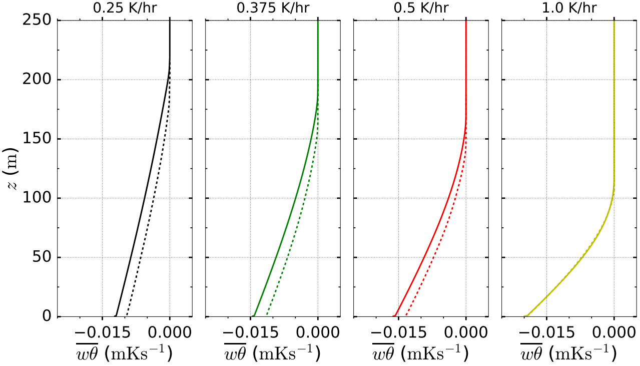
GABLS1-alike simulation with different surface cooling rates (0.25, 0.375, 0.5, 1.0) K/hr, horizontal heat fluxes



Simple models cannot predict these fluxes ⇒ $\overline{u\theta} = \overline{v\theta} = 0$

Future work: Predefined cooling rate vs. fixed surface cooling flux in GABLS1

GABLS1 is defined with the prescribed surface temperature forcing $\Rightarrow$ $T_s(t)$ is the same for LES and EARSM. However, heat flux at the surface is not necessarily the same. This can lead to disagreement between the LES and EARSM results.



For the case of 1 K/hr cooling rate the surface flux in EARSM and LES are the same. Leading to much better agreement between the EARSM and LES in the ABL.

Take-away notes

- **EARSM predicts many turbulence processes in the ABL (residual turbulence, stable/unstable turbulence anisotropy, transitioning boundary layers, etc.)**;
- **the model is explicit**, therefore the computational cost is not much larger than for some other explicit models;
- predicts **full turbulent momentum flux tensor and heat flux vector**;
- **consistent boundary condition treatment improves model behavior and produce grid-independent results;**