

*Regional Cooperation for  
Limited Area Modeling in Central Europe*



## Dynamics in RC LACE

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thanks to Jozef Vivoda, Mario Hrastinski, Alexandra Craciun  
and other colleagues



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- ❑ NH dynamics as a departure from HPE [Jozef Vivoda]
- ❑ Dynamic definition of the time scheme [Jozef Vivoda, Alexandra Craciun]
- ❑ VFE new formulation for HPE [Jozef Vivoda]
- ❑ Grey zone of turbulence? [Mario Hrastinski]

## NH dynamics as a departure from HPE

## ALADIN/HIRLAM system dynamics

- ❑ uses a hybrid terrain following vertical coordinate  $\eta$  based on hydrostatic pressure  $\pi$
- ❑ uses hydrostatic primitive equation system (HPE) or fully compressible nonhydrostatic Euler equations (EE); recently implemented quasi elastic equation system (QE)
- ❑ prognostic variables  $\vec{v}, T, q_s = \ln(\pi_s)$ , in EE with  $w, \hat{q} = \ln(\frac{p}{\pi})$
- ❑ here adiabatic system with no moisture

Total time derivative

$$\dot{\psi} = \frac{\partial \psi}{\partial t} + \vec{v} \cdot \vec{\nabla} \psi + \dot{\eta} \frac{\partial \psi}{\partial \eta}$$

Prognostic equations for model variables may be written as

EE system

$$\begin{aligned}\dot{\vec{v}} &= -RT \frac{\vec{\nabla} p}{p} - \frac{\partial p}{\partial \pi} \vec{\nabla} \phi \\ \dot{w} &= \left( \frac{\partial p}{\partial \pi} - 1 \right) g \\ \dot{T} &= \frac{RT \dot{p}}{c_p p} \\ \dot{\hat{q}} &= - \left( \frac{\dot{\pi}}{\pi} + \frac{c_p}{c_v} D_3 \right) \\ \dot{q}_s &= - \frac{1}{\pi_s} \vec{\nabla} \cdot \int_0^1 \frac{\partial \pi}{\partial \eta} \vec{v} \, d\eta'\end{aligned}$$

which collapsed with the hydrostatic assumption to

HPE system

$$\begin{aligned}\dot{\vec{v}} &= -RT \frac{\vec{\nabla} \pi}{\pi} - \vec{\nabla} \phi \\ \dot{T} &= \frac{RT \dot{\pi}}{c_p \pi} \\ \dot{q}_s &= - \frac{1}{\pi_s} \vec{\nabla} \cdot \int_0^1 \frac{\partial \pi}{\partial \eta} \vec{v} \, d\eta'\end{aligned}$$

We introduce the measure of nonhydrostaticity  $\varepsilon_{NH}$

Horizontal momentum

$$\dot{\vec{v}} = -RT \frac{\vec{\nabla} \pi}{\pi} - \varepsilon_{NH} RT \vec{\nabla} \hat{q} - \vec{\nabla} \phi - \varepsilon_{NH} \left( \frac{\partial p}{\partial \pi} - 1 \right) \vec{\nabla} \phi$$

Vertical momentum

$$\dot{w} = \varepsilon_{NH} \left( \frac{\partial p}{\partial \pi} - 1 \right) g$$

Temperature

$$\dot{T} = (1 - \varepsilon_{NH}^2) \frac{RT}{c_p} \frac{\dot{\pi}}{\pi} - \varepsilon_{NH}^2 \frac{RT}{c_v} D_3$$

Pressure departure

$$\dot{\hat{q}} = -\varepsilon_{NH} \left( \frac{\dot{\pi}}{\pi} + \frac{c_p}{c_v} D_3 \right)$$

Surface pressure

$$\dot{q}_s = - \frac{1}{\pi_s} \vec{\nabla} \cdot \int_0^1 \frac{\partial \pi}{\partial \eta} \vec{v} d\eta'$$

**HYDROSTATIC SYSTEM** with  $\varepsilon_{NH} = 0$

**+ NH DEPARTURE**

with  $\varepsilon_{NH} = 1$

The system is closed with diagnostic relations

Hydrostatic pressure time change

$$\dot{\pi} = \vec{v} \cdot \vec{\nabla} \pi - \int_0^{\eta} \vec{\nabla} \cdot \frac{\partial \pi}{\partial \eta} \vec{v} \, d\eta'$$

Geopotential

$$\phi = \phi_s - \int_{\eta}^1 \left[ (1 - \varepsilon_{NH}) \frac{RT}{\pi} \frac{\partial \pi}{\partial \eta} + \varepsilon_{NH} \frac{RT}{p} \frac{\partial \pi}{\partial \eta} \right] d\eta'$$

Total divergence and modified vertical divergence

$$D_3 = D + d \quad \text{where} \quad d = -\frac{gp}{RT} \frac{\partial w}{\partial \pi} + \frac{p}{RT} \frac{\partial \vec{v}}{\partial \pi} \cdot \vec{\nabla} \phi$$

[Thanks to Pierre Bénard]

For the SI time scheme we define the isothermal, resting, horizontally homogeneous reference state  $X^*$  in the hydrostatic equilibrium given by 2 values  $T^*, \pi_s^*$ .

We use horizontal divergence  $D$  and modified vertical divergence  $d$  instead of  $\vec{v}$  and  $w$  for stability reasons.

We define linear vertical operators

Linear vertical operators

$$G^* X = \int_{\eta}^1 \frac{m^*}{\pi^*} X d\eta'$$

$$S^* X = \frac{1}{\pi^*} \int_0^{\eta} m^* X d\eta'$$

$$N^* X = \frac{1}{\pi_s^*} \int_0^1 m^* X d\eta$$

$$L^* X = \frac{\pi^*}{m^*} \frac{\partial}{\partial \eta} \left[ \varepsilon_{NH} \left( \frac{\pi^*}{m^*} \frac{\partial}{\partial \eta} + 1 \right) \right] X$$



When semi-Lagrangian advection is applied

$$\frac{dX}{dt} = \mathcal{M}(X)$$

We linearize around the reference state and get the linear terms  $\mathcal{L} \cdot X$  and the nonlinear terms  $\mathcal{N}(X)$ . Then we treat linear terms implicitly and nonlinear terms in an iterative centered implicit manner.

$$\frac{X^{+i} - X^0}{\delta t} = \frac{\mathcal{L} \cdot X^{+i} + \mathcal{L} \cdot X^0}{2} + \frac{\mathcal{N}(X^{+(i-1)}) + \mathcal{N}(X^0)}{2}$$

We get the Helmholtz equation

$$\left[ 1 - \frac{\delta t}{2} \mathcal{L} \right] X^{+i} = X^0$$

## Linearized Euler equations

### Horizontal momentum

$$\frac{\partial D}{\partial t} = -RG^* \Delta T - RT^* \Delta q_s - \Delta \phi_s$$

$$+ \varepsilon_{NH} RT^* (G^* - 1) \Delta \hat{q}$$

### Vertical momentum

$$\frac{\partial d}{\partial t} = -\frac{g^2}{RT_a^*} L^* \hat{q}$$

### Temperature

$$\frac{\partial T}{\partial t} = -\frac{RT^*}{c_p} S^* D +$$

$$+ \varepsilon_{NH}^2 \left[ \frac{RT^*}{c_p} S^* D - \frac{RT^*}{c_v} (D + d) \right]$$

### Pressure departure

$$\frac{\partial \hat{q}}{\partial t} = \varepsilon_{NH} S^* D - \varepsilon_{NH} \frac{c_p}{c_v} (D + d)$$

### Surface pressure

$$\frac{\partial q_s}{\partial t} = - N^* D$$

$$\varepsilon_{NH} = 0 \text{ HPE}$$

$$\varepsilon_{NH} = 1 \text{ Euler equations}$$

[Thanks to Fabrice Voitus]

After discretization, the elimination of all variables except  $D$  yields

the reduced system

$$[1 - \delta t^2 B^* \Delta] D^+ = 0$$

with (using  $\varkappa = \frac{c_v}{c_p}$ )

vertical structure matrix

$$\begin{aligned}
 B^* &= B_{HY}^* + B_{NH}^* \\
 B_{HY}^* &= RT^* (1 - \varkappa) G^* S^* + RT^* N^* \\
 B_{NH}^* &= \frac{RT^*}{\varkappa} (I - \varkappa G^*) \varepsilon_{NH} \left( I - \delta t^2 \frac{g^2}{\varkappa RT_a^*} L^* \varepsilon_{NH} \right)^{-1} (I - \varkappa S^*)
 \end{aligned}$$

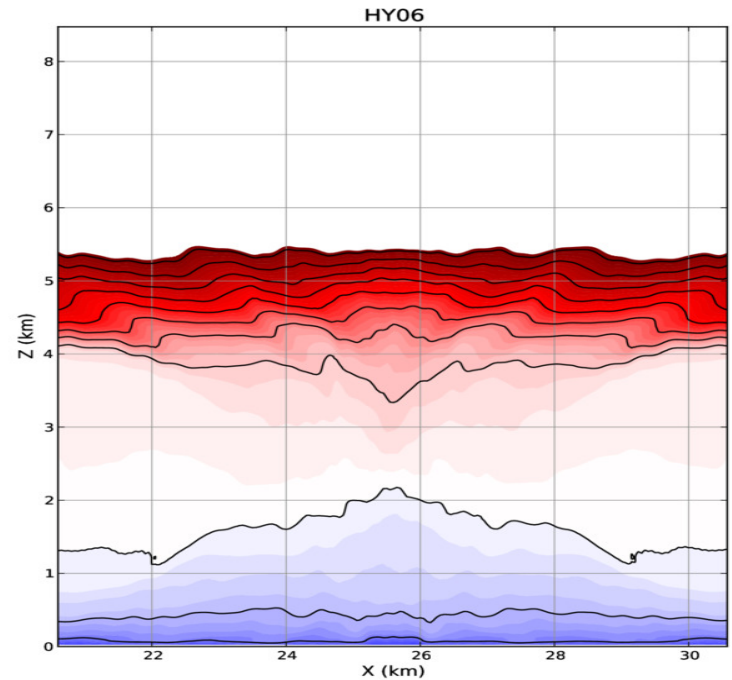
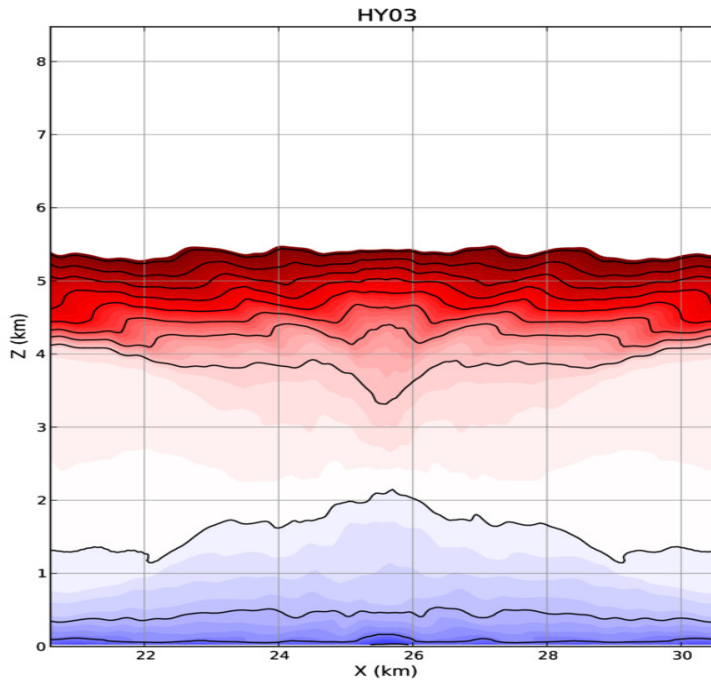
## Conclusions:

- the whole procedure for the semi-implicit time scheme may be solved similarly for HPE and EE, only with different vertical structure matrix (Fabrice Voitus).
- Moreover, the vertical structure matrix may be formulated as the sum of a hydrostatic part and a nonhydrostatic departure.
- Values  $0 < \varepsilon_{NH} < 1$  do not have a physical meaning in the full model. But we may use them in linear model for the semi-implicit time scheme and investigate the stability of the proposed solution.
- We may envisage values  $0 < \varepsilon_{NH} < 1$  in dependence on the vertical coordinate  $\eta$ , allowing for smooth transition from fully elastic nonhydrostatic Euler equations to hydrostatic primitive equations near the model top where we care about stability more than accuracy (at least in LAM).

## Straka density current test: central part

HYD clasically

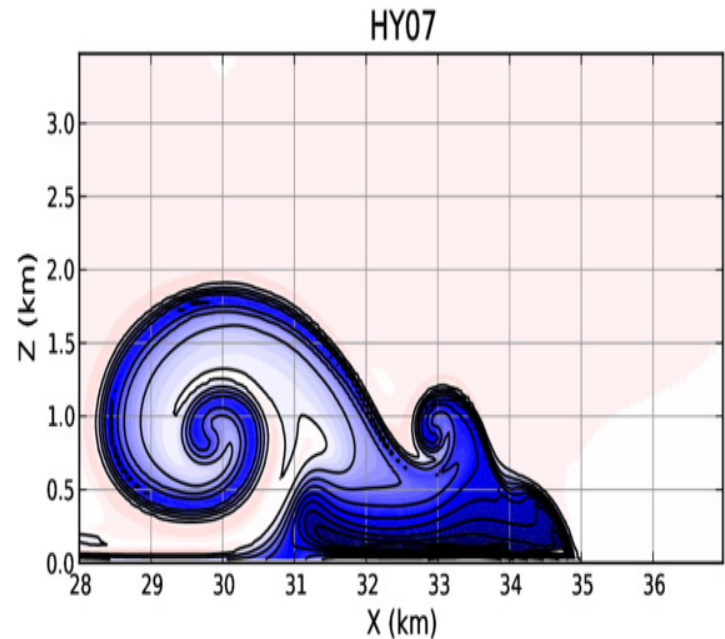
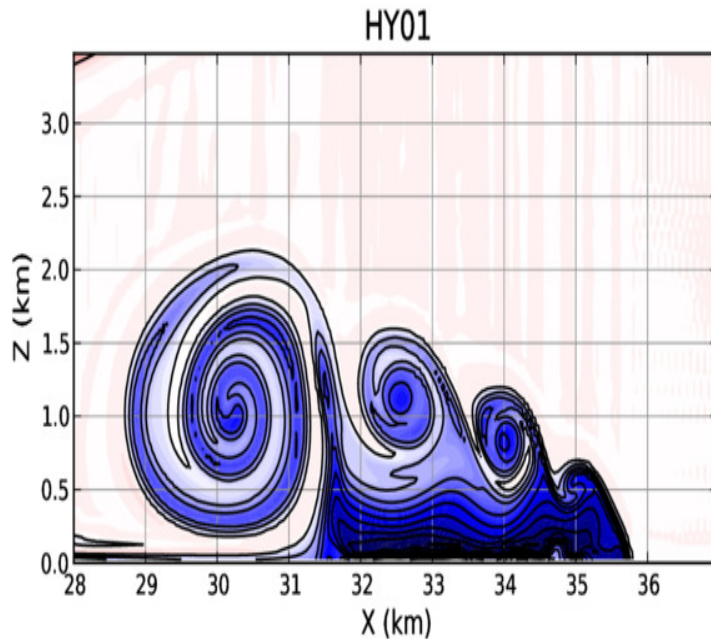
HYD with  $\varepsilon_{NH} = 0$   
from hybrid system



## Straka density current test: right hand part

NH clasically

hybrid system with constant  $\varepsilon_{NH} = 0.5$



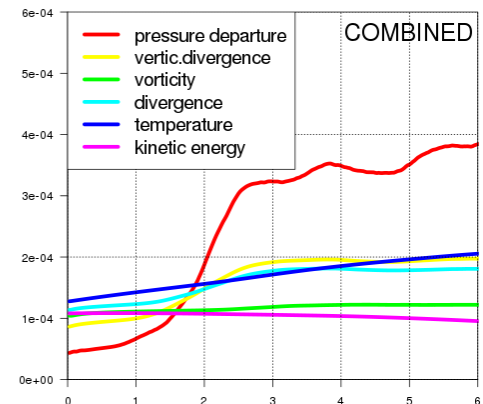
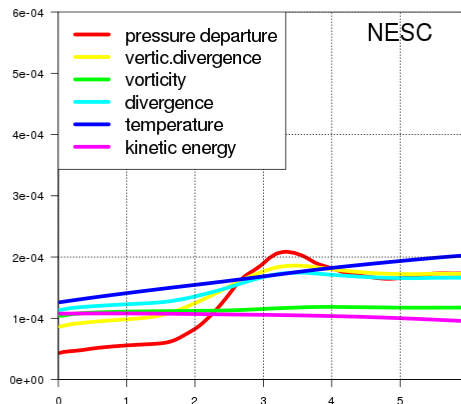
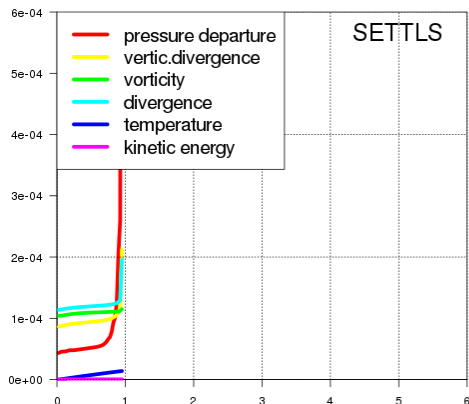
## Dynamic definition of the time scheme

- ❑ we try to find a balance between less accurate/more stable non extrapolating scheme NESC and second order accurate/less stable scheme SETTLS for each grid point
- ❑ a combined scheme allowing to use a linear combination of these two approaches for each grid point was proposed and implemented
- ❑ we investigate which condition could serve as a good weight for this linear combination
- ❑ since the stability issues appear usually at the top of the domain, we tried as well to apply NESC scheme in upper layers and SETTLS scheme in lower layers and the combined scheme in between.



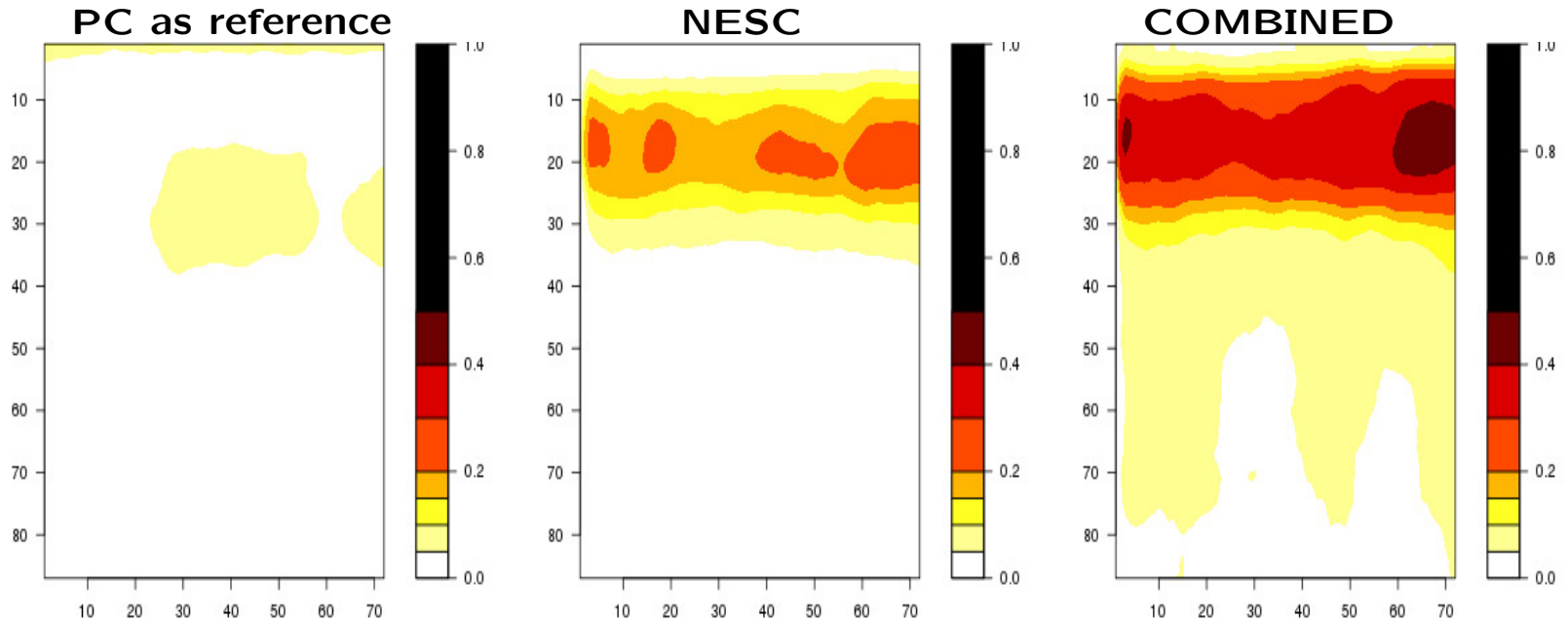
Tests on 30.10.2017 with severe wind over the Czech operational domain, 2.3km x 87 vertical levels.

- Simple SETTLS is in this case unstable.
- Stabilized through the application of NESC scheme in points where the “stability criterion” was broken, or with a combined scheme.
- Stability criterion is based on the time evolution of the non-linear residual in the pressure departure equation.



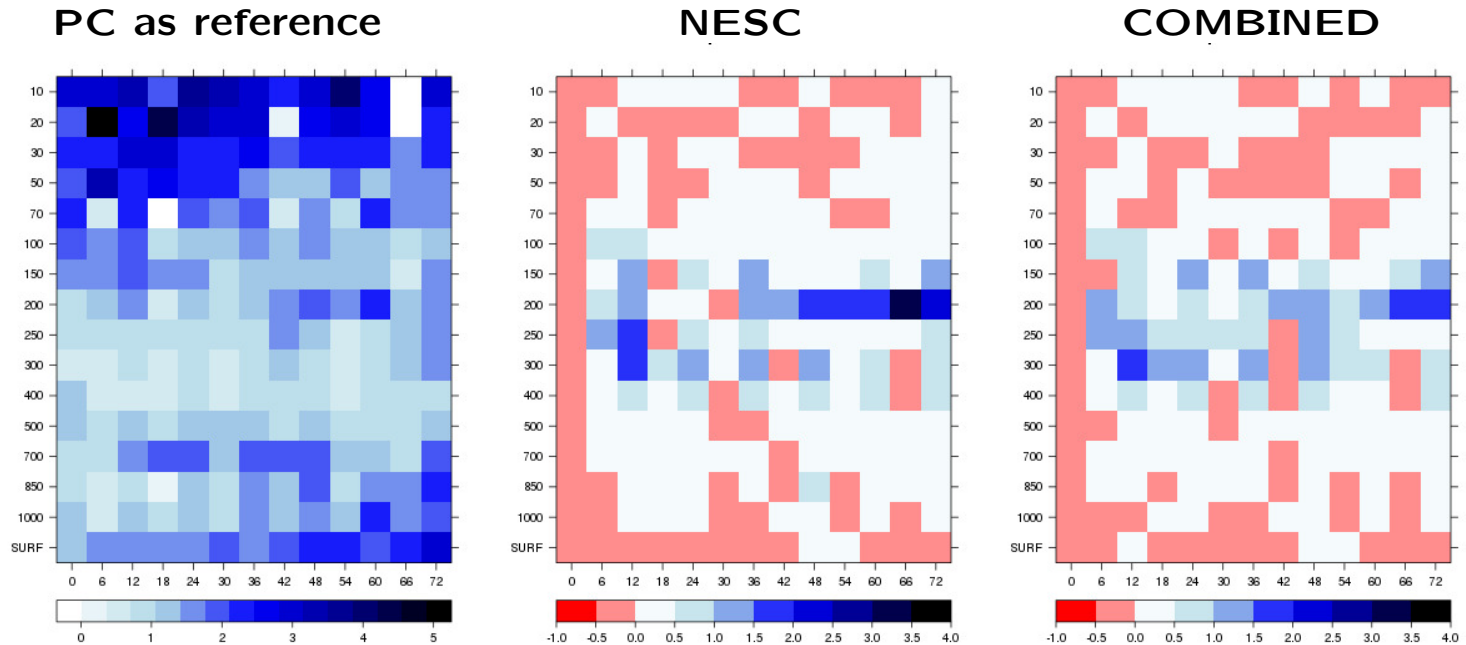
Time evolution of spectral norms of model prognostic variables.

But huge portion of grid points was treated by the NESC scheme.



**Red color** indicates more than 20% in the given layer.

And the objective scores (RMSE, STDEV , BIAS) for basic variables with the combined scheme are close to the case with pure NESG scheme.



Time evolution of RMSE compared to PC.

Similarly, the application of NESC only in upper levels is not stable enough and the stable case with NESC being applied in the big part of the domain is not accurate enough.

## Plans:

- ❑ Consider other stability criteria.
- ❑ Consider similarly PC scheme with corrector step activated only if a stability criteria is broken in a substantial part of the domain.

VFE new formulation for HPE

- ❑ Hybrid mass based vertical coordinate definition with hydrostatic pressure  $\pi$  in both HPE and EE systems (Laprise, 1992).
- ❑ Vertical discretization is based on finite difference method (VFD), or finite elements method (VFE).
- ❑ General order B-splines are available as the basis for the finite element method in the vertical.
- ❑ Only integral operator needed in HPE, while integral and derivative operators are needed in EE system.
- ❑ For both operators,  $\eta$  is only implicitly defined for VFD, while the explicit definition of  $\eta$  is needed in VFE.

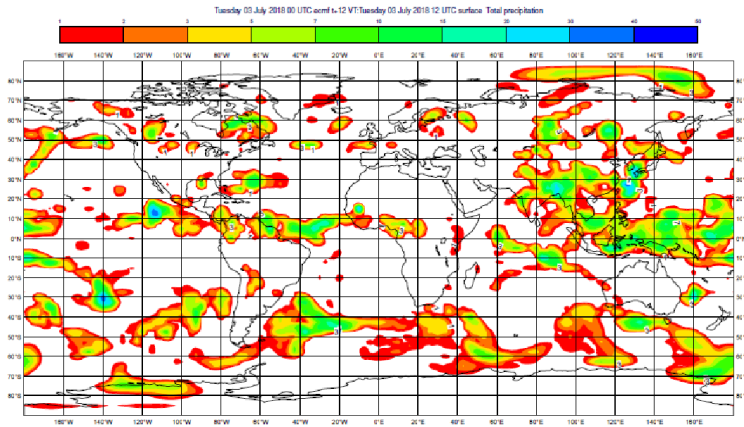
## Novelties in VFE formulation:

- ❑ Several new explicit definitions of  $\eta$  were proposed. More stable for higher density of levels close to domain top and bottom boundaries.
- ❑ VFE was tested with elimination of all variables but D in SI time scheme (as proposed by Fabrice Voitus). Results are in good agreement with previous iterative solution.
- ❑ Vertical velocity may be defined either on half levels, or on full levels. In that case there is no vertical staggering at all, but it does not seem to pose a problem.

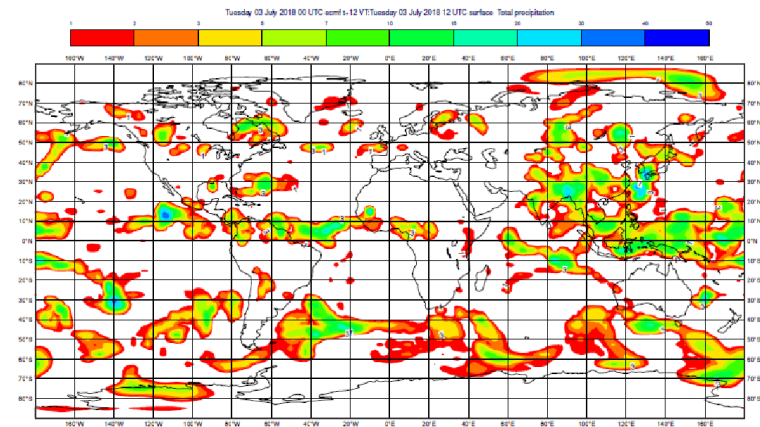
## Real experiment with NH IFS using VFE

- total precipitation after 12 hours of integration

VFD



VFE with cubic B-splines and new definition of  $\eta$





Grey zone of turbulence

- ❑ In ALARO configuration the turbulence scheme TOUCANS (Third Order Moments Unified Condensation Accounting and N-Dependent Solver) is applied, based on a unified treatment of stability functions, applicable in both stable and unstable regimes.
- ❑ It includes the prognostic equation for turbulent kinetic energy TKE and total turbulent energy TTE.

Are we entering the grey zone of turbulence?

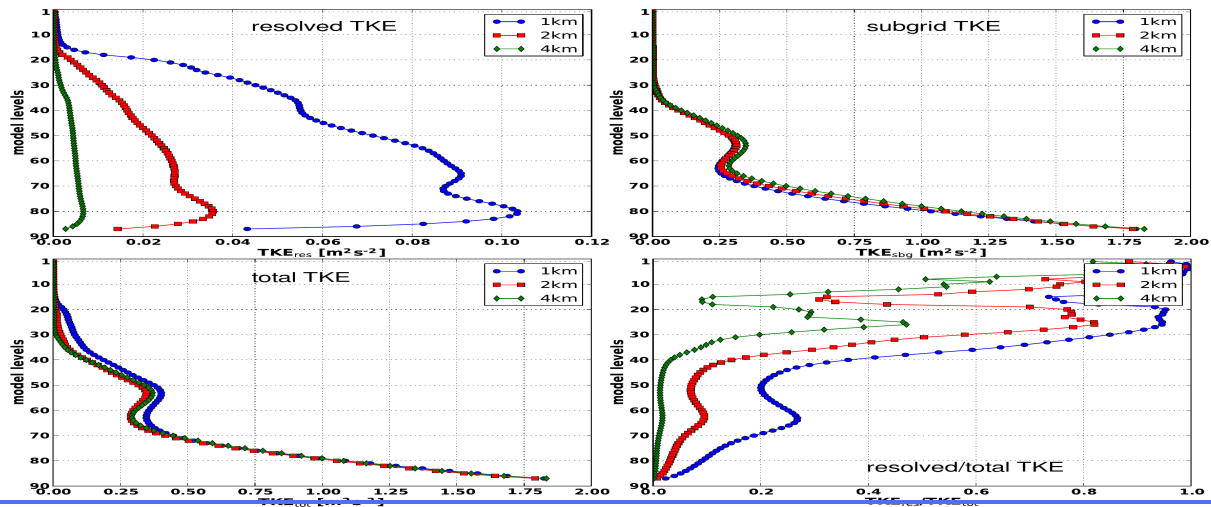
Depends on the definition of the grey zone, which is not unique, but with horizontal resolution below 1km yes.

Do we have to adjust the turbulence scheme in the grey zone of turbulence?

We estimated the resolved and the subgrid part of TKE as the first step.

## Calculation of the resolved TKE

- wind field components can be decomposed to slowly varying averages and fast varying turbulence perturbations (only valid if there is a spectral gap between these two scales)
- the averages are computed as a running average in time, for each grid point at all model levels
- then perturbations are calculated by subtraction of averages from the total values and resolved TKE is calculated



SLHD (Semi-Lagrangian Horizontal Diffusion, Váňa, 2008) is the nonlinear horizontal diffusion with three basic parts:

- ❑ grid-point diffusion dependent on the flow deformation
- ❑ reduced spectral diffusion acting on the upper domain
- ❑ supporting spectral diffusion

How the turbulence scheme interacts with SLHD?

Can we control SLHD to act as the horizontal part of the turbulence parametrization?

Thank you for your attention!