Vertical Finite Elements
some test

Álvaro Subías
Agencia Estatal de Meteorología
asubiasd@aemet.es

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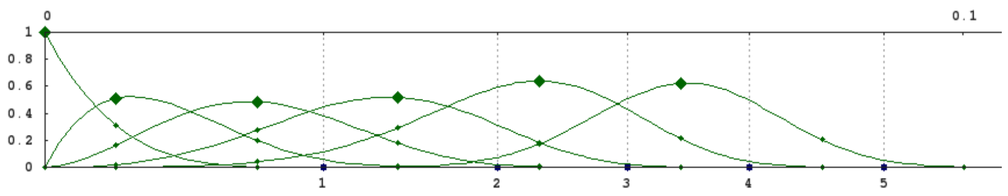
Overview

the interest in vfe approach is to develop a strictly full-level model to avoid several interpolations in the semi-lagrangian scheme arising from the use of half and full levels, we construct several operators making use of exact relations between integrals and derivatives of B-splines N_{ik}, we also construct new operators associated to c1 constraint using new basis ξ, σ. B-splines are piecewise polynomial functions of local support defined by an iterative way by knots t_i and the leaps between them $\Delta_{ik} := t_{i+k} - t_i$. The basis functions are such that their maxima should be as close to levels as possible (as pointed out by J. Vivoda), the definition of knots fulfilling that condition is not a well posed problem so an iterative process is set to minimize an error functional built as the L_p-norm of the difference of levels and maxima. The following results are computed in a code based on `harmonie − 40h1.1.beta.5`

$$\frac{\partial}{\partial t} N_{ik} = (k-1)\left[\frac{N_{i,k-1}}{\Delta_{i,k-1}} - \frac{N_{i+1,k-1}}{\Delta_{i+1,k-1}}\right]$$
$$\int_0^t N_{ik} = \frac{\Delta_{ik}}{k} \sum_{s=i}^{\infty} N_{s,k+1}$$



knots adjusted to the condition of maxima of splines at levels (green rhombus) after 50 iterations minimizing $L_{0.01}$-norm

C1 operators

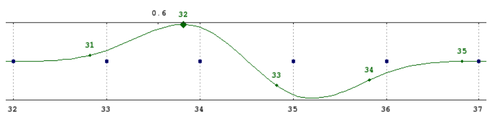
from the factorization of C1-constraint $(\mathcal{G}^*-1)(\mathcal{S}^*-1) = (1-\mathcal{N}^*)$ we develop discretized vertical integral operators which are called inside `sigam.f90, sitnu.f90` plus equivalent integral operators to be compatible in the non-linear part of the model. The basis functions are not exactly B-splines having negative values. We make use of coordinate $t := \frac{\pi^*}{\pi^*_s}$

vertical integral operators

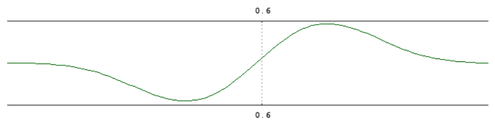
$$\mathcal{G}^* f := \int_t^1 f \frac{dt}{t}$$
$$\mathcal{S}^* f := \frac{1}{t}\int_0^t f\, dt$$
$$\mathcal{N}^* f := \int_0^1 f\, dt$$

basis functions

$$[\mathcal{G}^* - 1]\ \sigma_{ik} = \xi_{ik} - \delta_{iI}$$
$$[\mathcal{S}^* - 1]\ \xi_{ik} = \sigma_{ik}$$
$$[1 - \mathcal{N}^*]\ \xi_{ik} = \xi_{ik} - \delta_{iI}$$



$$\xi_{ik} := (\partial^* + 1) N_{ik}$$



$$\sigma_{ik} := -\partial^* N_{ik}$$

Integrals and derivatives

the option with `LGWADV` needs an invertibility relation between vertical divergence and vertical velocity. For the experiments launched this option has not been chosen, but invertible operators `RDERS`$(\mathrm{L}, \mathrm{L}+1)$ and `RINTE`$(\mathrm{L}+1, \mathrm{L})$ can be built with the switch `LDEC` $=$ `.FALSE.` when the following diagram is not decoupled (the equivalence relation $\sim$ identifies functions which differ by a constant)

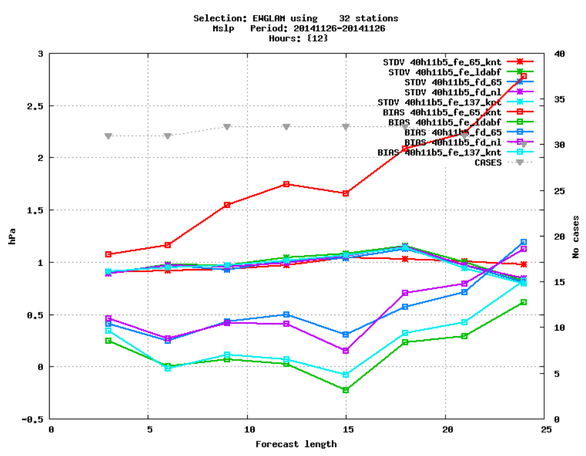
$$\begin{array}{ccc} S_k & \underset{\partial}{\overset{\int_0}{\rightleftarrows}} & S_{k+1}/\sim \\ \mathcal{P}_k \downarrow \uparrow \mathcal{Q}_k & & \mathcal{P}_{k+1} \downarrow \uparrow \mathcal{Q}_{k+1} \\ \mathcal{F}_{(0:L)} & \underset{\widehat{\partial}}{\overset{\widehat{\int}_0}{\rightleftarrows}} & \mathcal{F}_{(0:L+1)}/\sim \end{array}$$

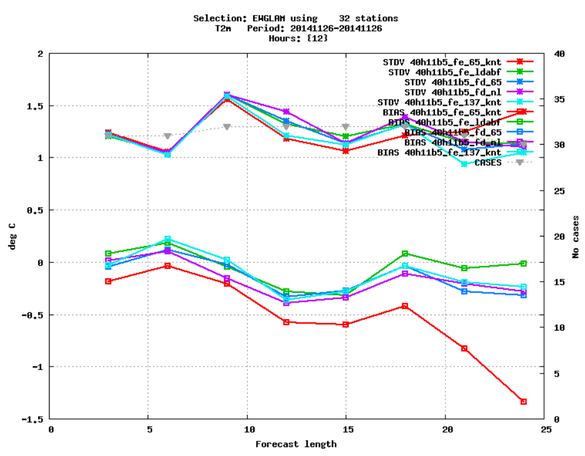
being $\mathcal{P}_k$, $\mathcal{Q}_k$ projection operators between grid point space and spline space. The calls to vertical integral and derivative operators are those that are at `harmonie − 40h1.1.beta.5` but with different setup of the operators

Single day integration

exp	lderib	lvfe_gw	lvfe_z_term	ldab	ldec	lknt	vlev	lvertfe
40h11b5_fe_65_knt				x		x	65	x
40h11b5_fe_ldabf		x	x		x	x	nl	x
40h11b5_fd_65							65	
40h11b5_fd_nl							nl	
40h11b5_fe_137_knt				x		x	137	x

list of experiments comparing finite elements and finite differences with different choices of vertical sets of levels: the *65* levels in harmonie_domain.pm, the *137* levels of ecmwf, and a set of 65 new levels computed with `http://www.cnrm-game-meteo.fr/gmapdoc/meshtml/AandB0_v1.html` and smoothed. The mslp bias depends very much on the choice of vertical levels, in particular the *65* case gives bad mspl bias, this is a subject of investigation.





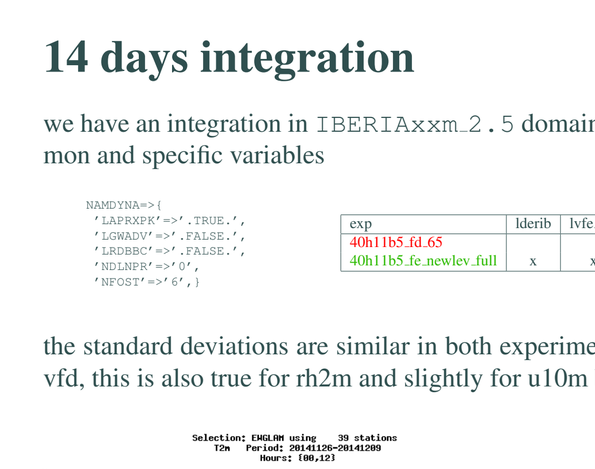
14 days integration

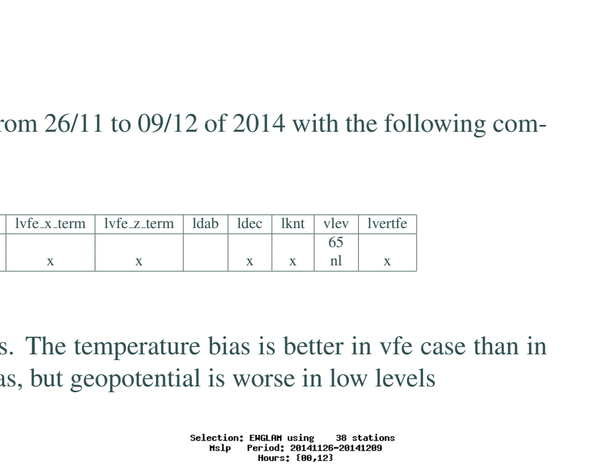
we have an integration in `IBERIAxxm_2.5` domain from 26/11 to 09/12 of 2014 with the following common and specific variables

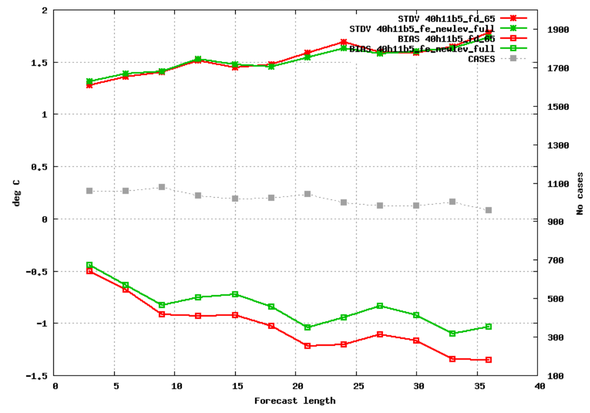
```
NAMDYNA=>{
 'LAPRXPK'=>'.TRUE.',
 'LGWADV'=>'.FALSE.',
 'LRDBBC'=>'.FALSE.',
 'NDLNPR'=>'0',
 'NFOST'=>'6',}
```

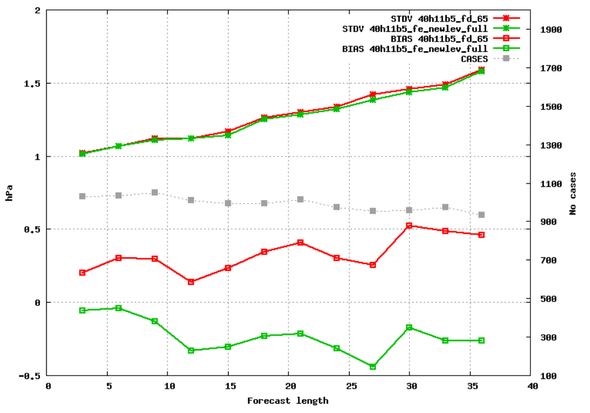
exp	lderib	lvfe_gw	lvfe_x_term	lvfe_z_term	ldab	ldec	lknt	vlev	lvertfe
40h11b5_fd_65								65	
40h11b5_fe_newlev_full	x	x	x	x		x	x	nl	x

the standard deviations are similar in both experiments. The temperature bias is better in vfe case than in vfd, this is also true for rh2m and slightly for u10m bias, but geopotential is worse in low levels









Future Work

the model gives acceptable mslp bias only in the case of a restrictive definition of levels, but ... why? we should improve if it is possible the mslp bias with an arbitray choice of `VLEV`, for example for the *65* default HIRLAM set.

there is a collaboration in this topic between P. Smolikova (CHMI), J. Vivoda (SHMU), J. Simarro (AEMET) and M. Hortal (AEMET, recently retired much to our reget) with much previous work made with them, there are several approaches to this problem which are important to explore in order to have a more solid nhvfe scheme

References

[Bubnová, Hello, Bénard, and Geleyn (1995)] Integration of the fully elastic equations cast in the hydrostatic pressure terrain-following coordinate in the framework of the arpege/aladin nwp system. *Mon. Weather Rev.*, **123:**515–535. doi: 10.1175/1520-0493(1995)123⟨0515:iotfee⟩2.0.co;2.

[de Boor (1972)] On calculating with b-splines. 1972. *J. Approx. Theory.*, **6:**50–62. doi: 10.1016/0021-9045(72)90080-9.

[Smolíková and Vivoda (2013)] Finite elements used in the vertical discretization of the fully compressible forecast model aladin-nh. *ALADIN-HIRLAM Newsletter*, **1:**31–46.

[Subías (2015)] B-splines as a Tool to Solve Constraints in Non-Hydrostatic Forecast Model *http://arxiv.org/abs/1601.03446*

[Untch and Hortal (2004)] A finite-element scheme for the vertical discretization in the semi-langrangian version of the ecmwf forecast model. *Q. J. R. Meteorol. Soc.*, **130:**1505–1530. doi: 10.1256/qj.03.173.