Local Semi-Implicit Scheme

Filip Váňa

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profiting also from discussions with J. Mašek, P. Smolíková, J. Vivoda and P. Bénard

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ECMWF

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- Formally it means to separate the RHS terms into two groups: $\mathcal{M} = \mathcal{L} + (\mathcal{M} - \mathcal{L})$ (assuming $\mathcal{L} >> (\mathcal{M} - \mathcal{L})$) and solve them (in SL formalism) as:

$$\frac{X_F^+ - X_O^0}{\Delta t} = \frac{1}{2}\left[\mathcal{L}_O^0 + \mathcal{L}_F^+\right] + (\mathcal{M} - \mathcal{L})_M^{\frac{1}{2}}$$

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- Spectral models are well suited for this method (being typically 3-4 times more efficient with respect to GP methods on a single processor system).

Semi-Implicit scheme in IFS

- Spectral formulation implies:
  - Linear model assumes horizontally homogenous profiles for the whole globe ($\Rightarrow$ **no orography, no gradients**)
  - To have one structure equation linear model profiles are made also **vertically uniform**
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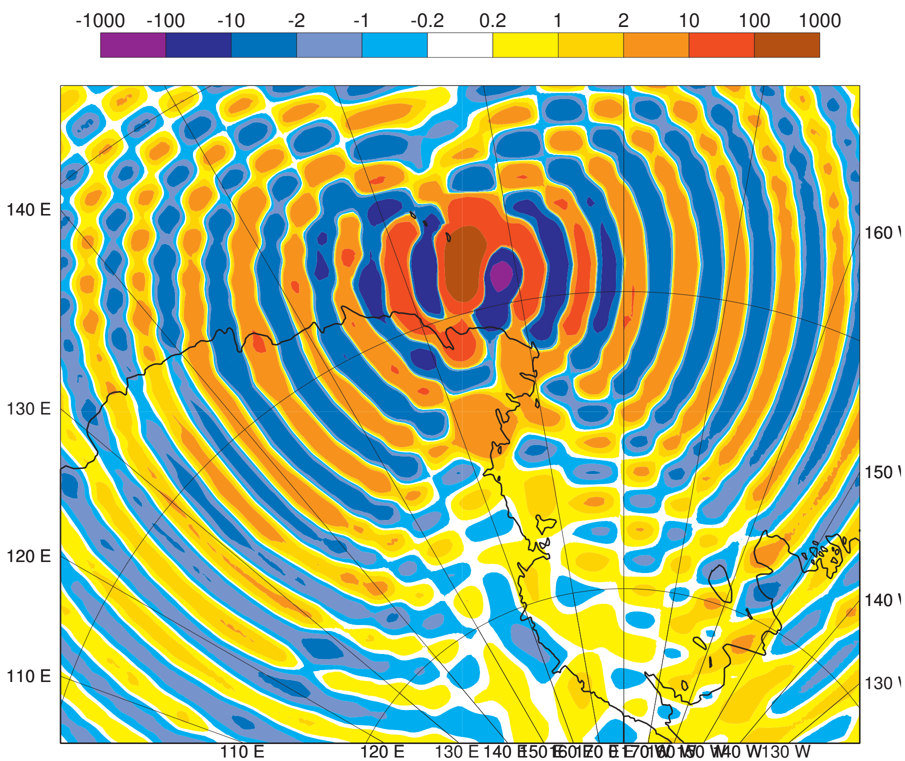
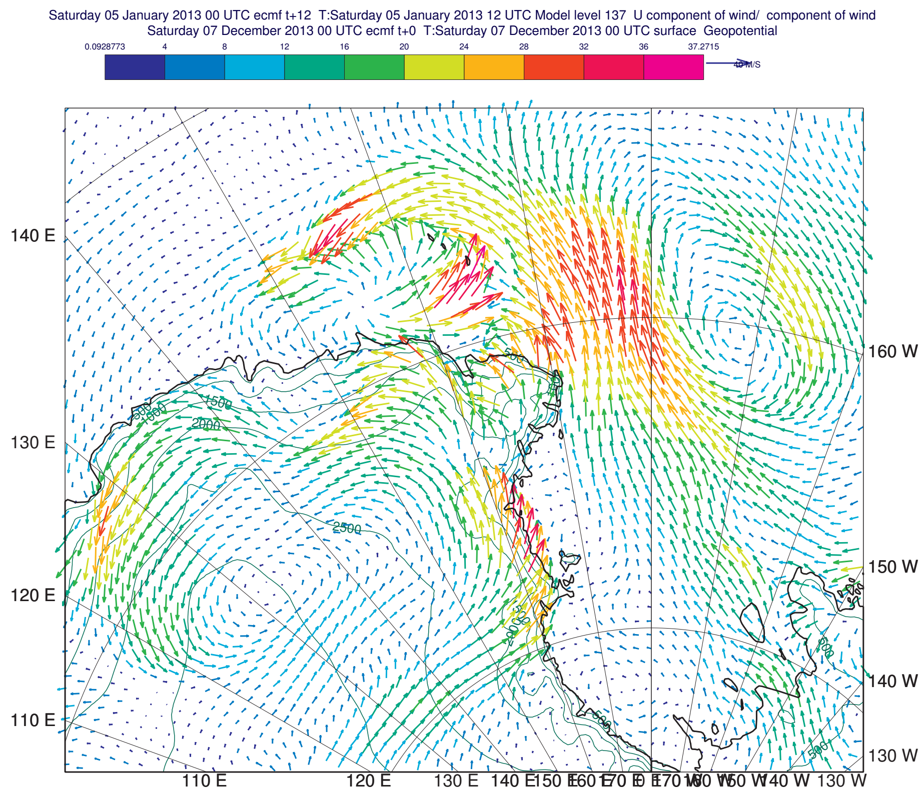
  - Physics is naturally out of the linear model
- Known problems:
  - Simple SI occasionally reported unstable $\Rightarrow$ iteration is required (near model top, steep slopes,...)
  - Convergence issues from areas with stable stratification and/or adjacent to significant orography
  - Resolutions higher than T_L399 ($\approx$50 km) are prone to a noise generation in TL/AD

12 hours adiabatic forecast with T_L511



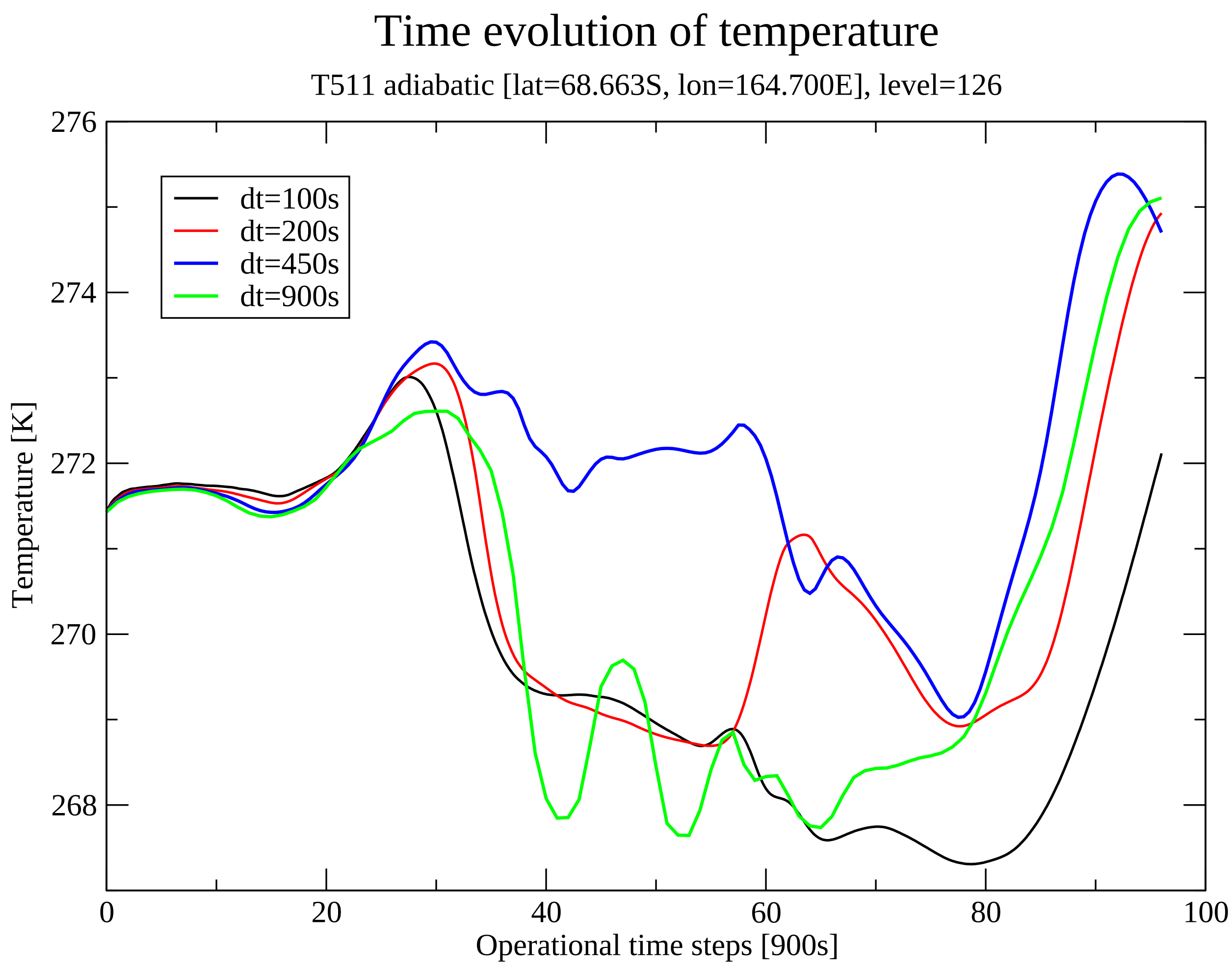
TL forecast of temperature



NL model forecast of wind

both from the lowermost model level

Known issues in IFS



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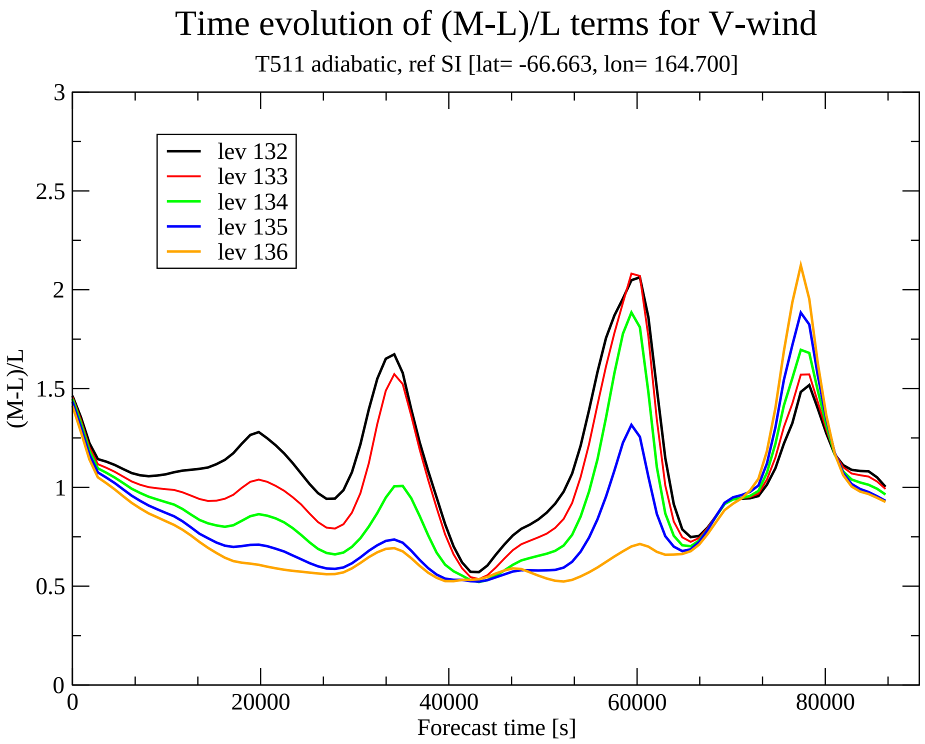

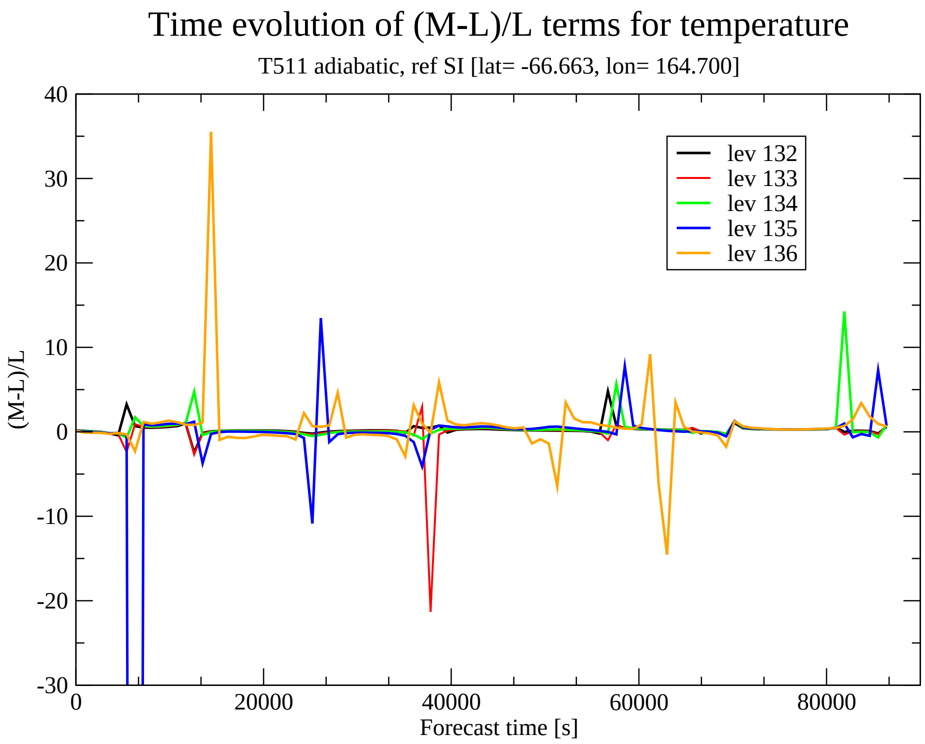

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- Following the proposal of Diamantakis (2014) the SETTLS method could be replaced by a non-extrapolating 2TL scheme:

$$\frac{X^+ - X^0}{\Delta t} = \frac{1}{2}\left[(\mathcal{M})^0\right]_O + \frac{1}{2}\left[(\mathcal{M})^0 + M'(X^*)(X^+ - X^0)\right]_F$$

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- Can't be easily inverted: requires an iterative procedure for the implicit term

Shallow water implementation

Governing equations:

$$\frac{dh}{dt} = -h\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = \boxed{-\bar{H}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)} + (\bar{H} - h)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right),$$

$$\frac{du}{dt} = \boxed{-g\frac{\partial h}{\partial x}} + fv - g\frac{\partial H_s}{\partial x} - \nu u,$$

$$\frac{dv}{dt} = \boxed{-g\frac{\partial h}{\partial y}} - fu - g\frac{\partial H_s}{\partial y} - \nu v,$$

implying then:

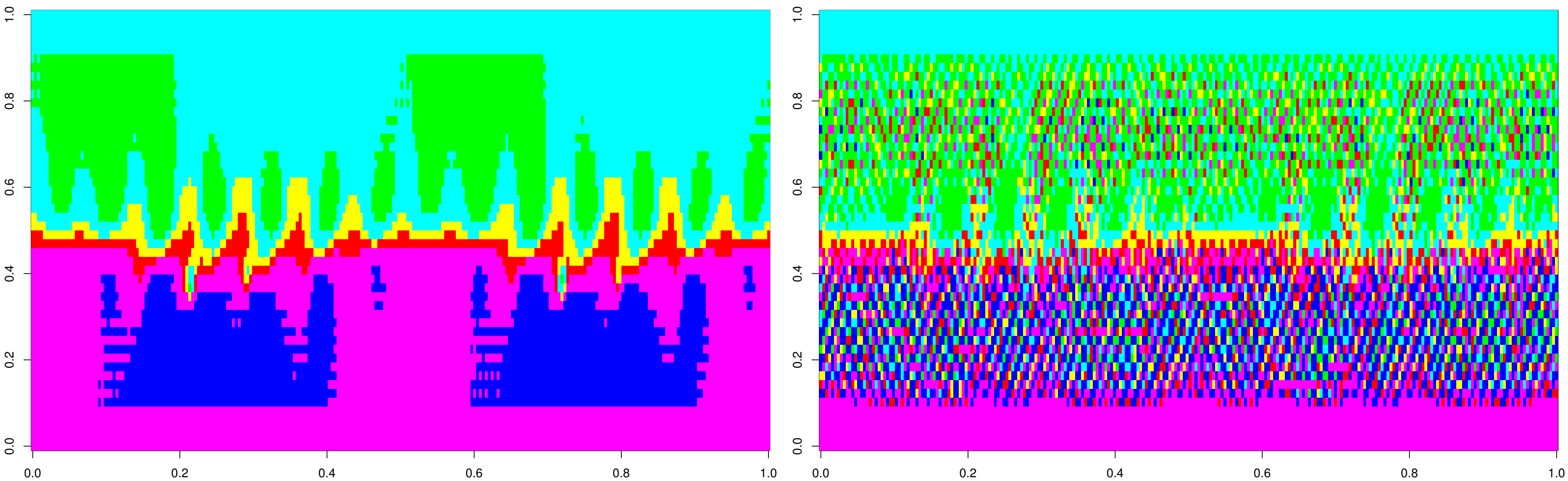
$$\mathbf{M}'(X^*)(X - X^0) = \begin{pmatrix} -\left(\frac{\partial u^*}{\partial x} + \frac{\partial v^*}{\partial y}\right)(h - h^0) - h^*\left(\frac{\partial u}{\partial x} - \frac{\partial u^0}{\partial x} + \frac{\partial v}{\partial y} - \frac{\partial v^0}{\partial y}\right) \\ f(v - v^0) - g(\frac{\partial h}{\partial x} - \frac{\partial h^0}{\partial x}) - \nu(u - u^0) \\ -f(u - u^0) - g(\frac{\partial h}{\partial y} - \frac{\partial h^0}{\partial y}) - \nu(v - v^0) \end{pmatrix}$$

Shallow water experiment setup

- SISL shallow water model with the IFS timestep organization (GP space only)
- Barotropic instability case
  - Domain 254 x 50 points.
  - $\Delta x = \Delta y = 100$ km.
  - $f = f_0 + \beta(y - y_0)$,
    with $f_0 = 0.0001 s^{-1}$ and $\beta = 1.6 \times 10^{-11} m^{-1} s^{-1}$
  - $\nu = 0$
  - Initial condition: zonal jet with geostrophic ballance + noise.
  - Formation of cyclones and anticyclones on each side of a zonal jet.
  - Forecast range 210000s.

Shallow water results

Height h



Explicit scheme with $\Delta t = 30$s (left) and $\Delta t = 70$s (right).

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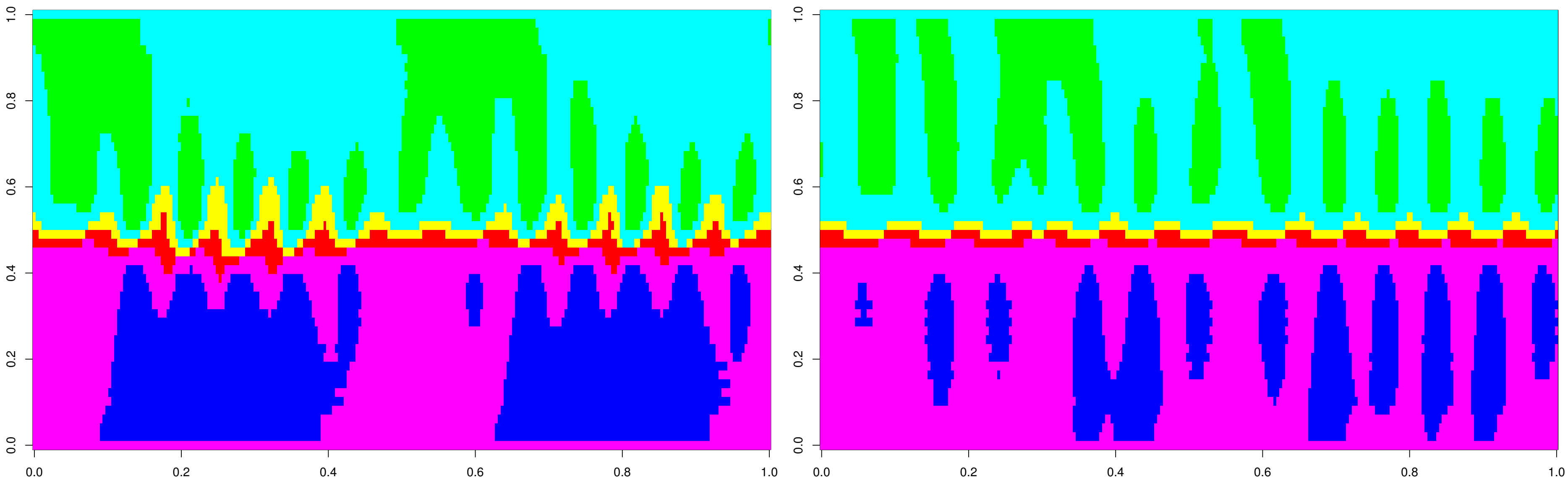

Semi-Implicit scheme with $\Delta t = 70$s (left) and $\Delta t = 300$s (right).

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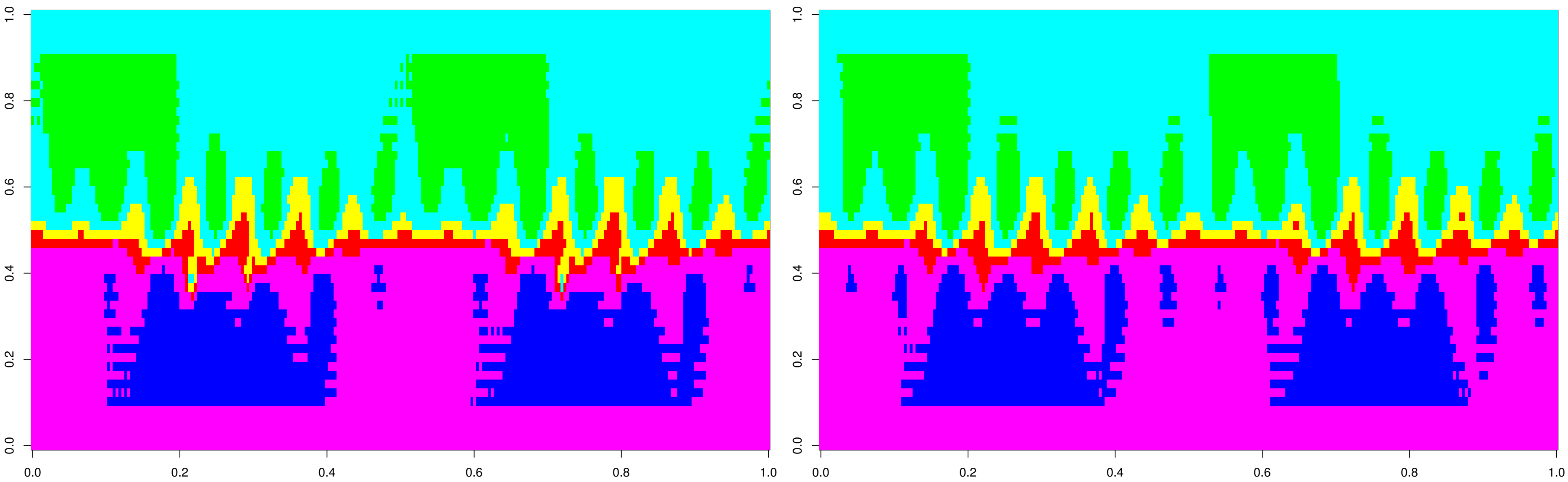

New scheme with $\Delta t = 70$s (left) and $\Delta t = 300$s (right).

Shallow water results

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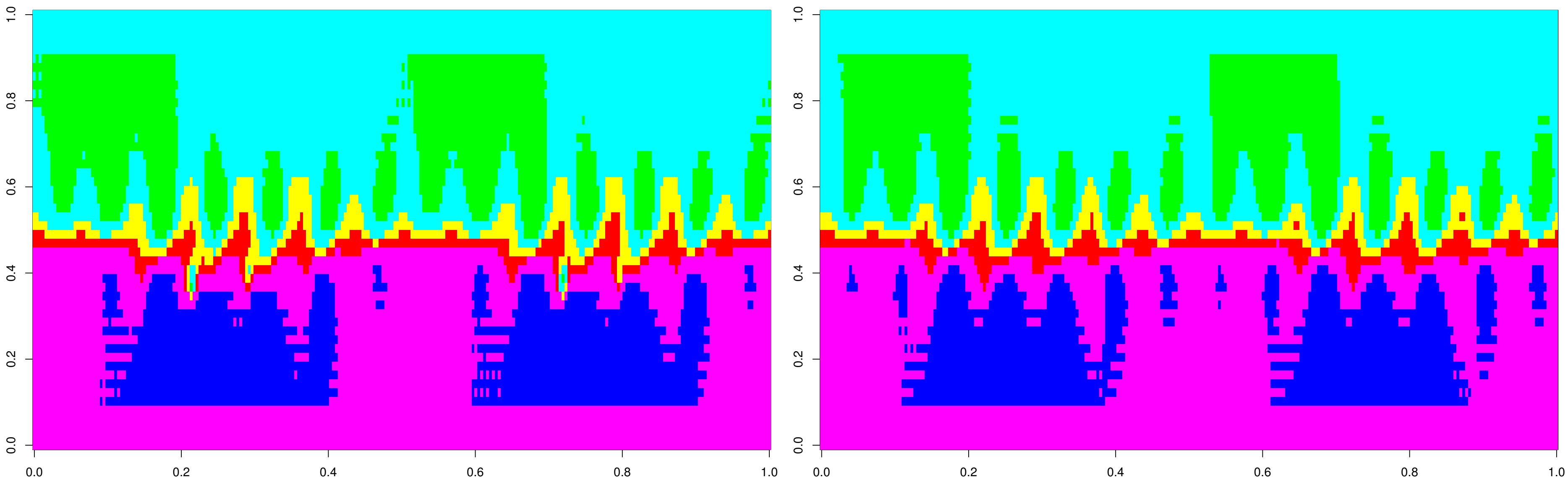
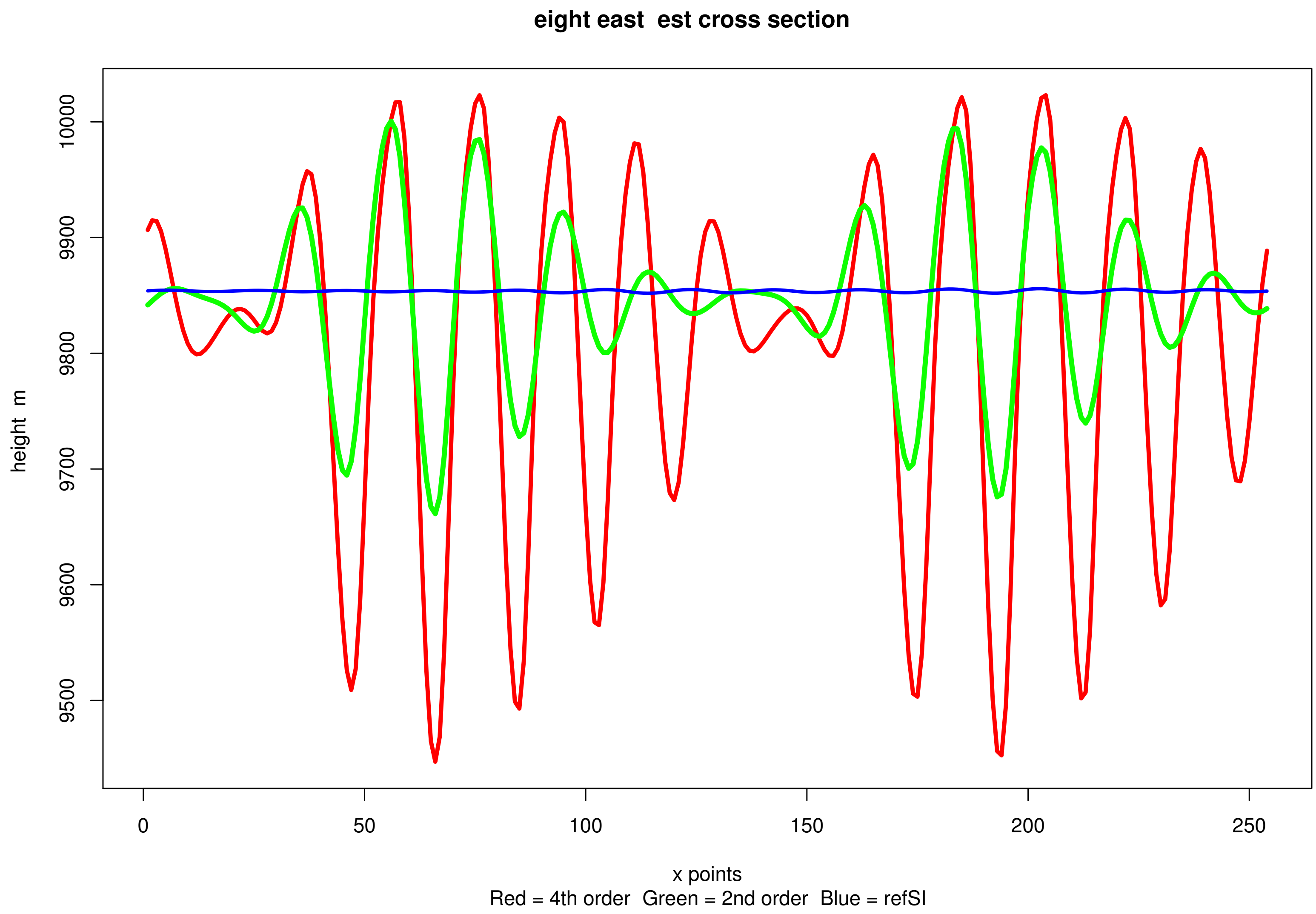


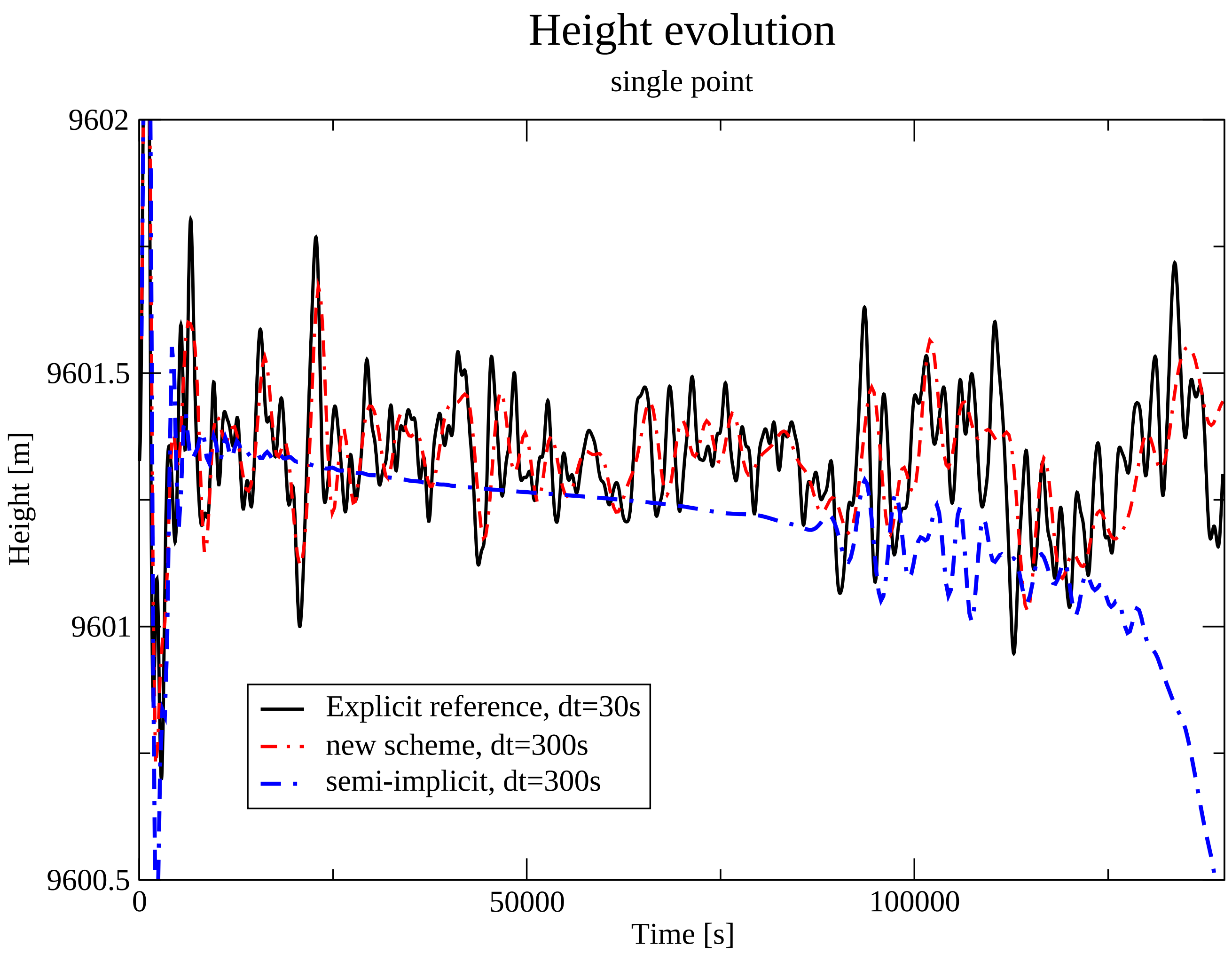
Explicit scheme with $\Delta t = 30$s (left) and the new scheme with $\Delta t = 300$s (right).

Shallow water results - II.

Longitudinal cross-section from the central area ($\Delta t = 400$ s)



Placing there some orography...



Methods to speed-up the iterative process

- Second order accuracy to define $\mathcal{L}$:

$$\mathcal{L}(X) = \mathcal{M}(X^0) + \mathbf{M}'(X^0)(X - X^0) + \frac{1}{2}\mathbf{M}''(X^0)(X - X^0)^2$$

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- 2TL method vs SETTLS
$\Rightarrow$ Minimum speedup (around 6%), still 2TL is used as the new default.

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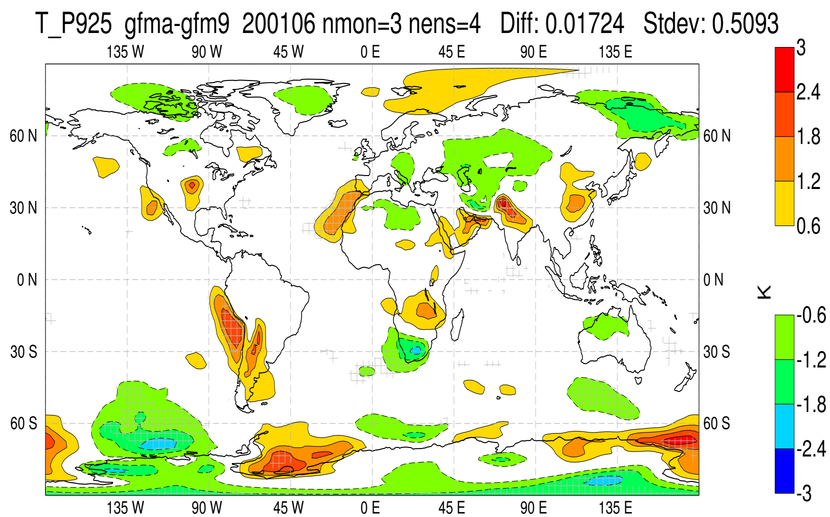

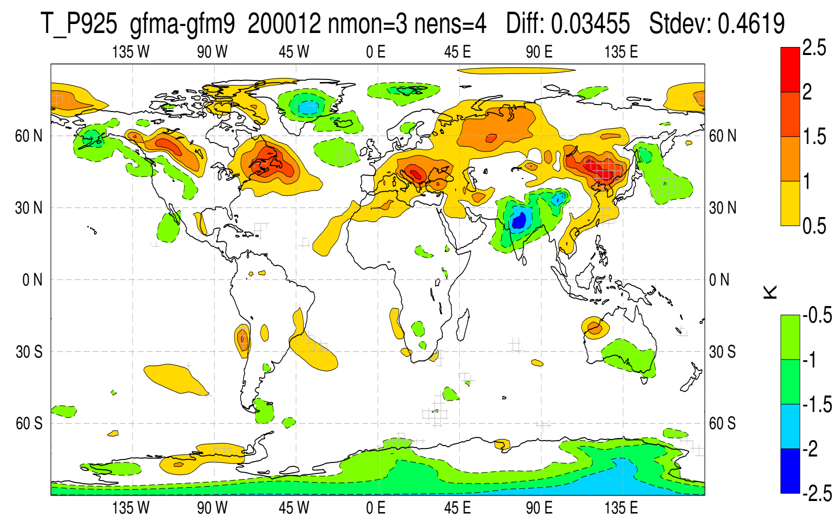

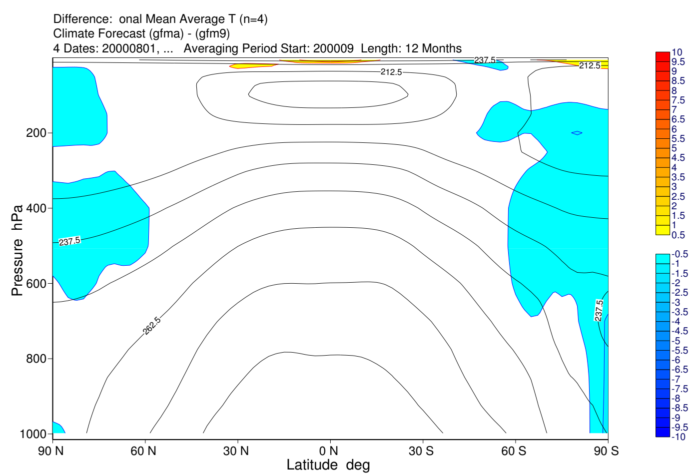

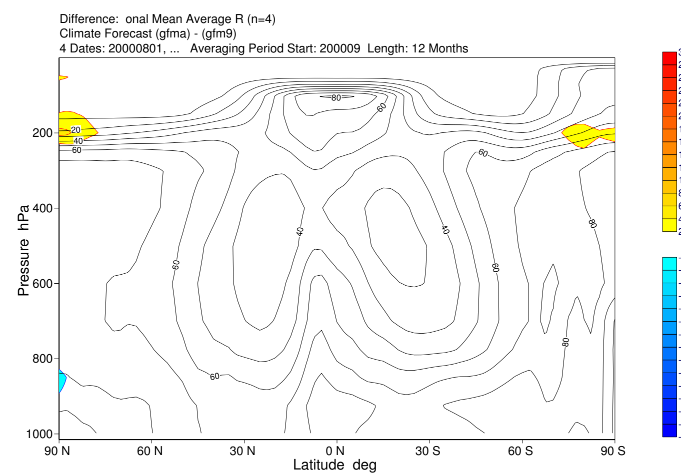

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- Having the SI and derivatives computed in grid-point space there is only little point to keep spectral space computation (I/O, filtering)
- Exclusively grid-pont version of IFS was designed with local communications only (SL comms and Atlas).
  - Fairly general linear model (extensible to any set of prognostic variables)
  - Iterative procedure is inexpensive provided the scheme is converging
  - Quality and stability strongly depends on derivatives computation (with 2^{nd} derivatives it allows $\approx$ 50-70% of the original timestep)

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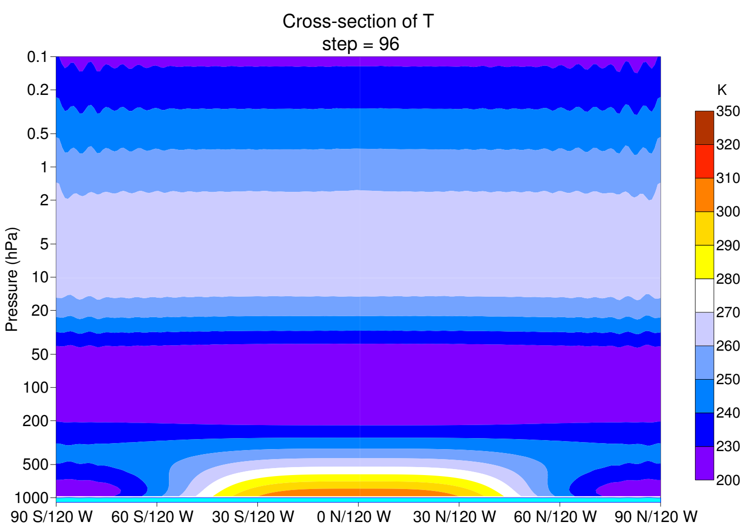

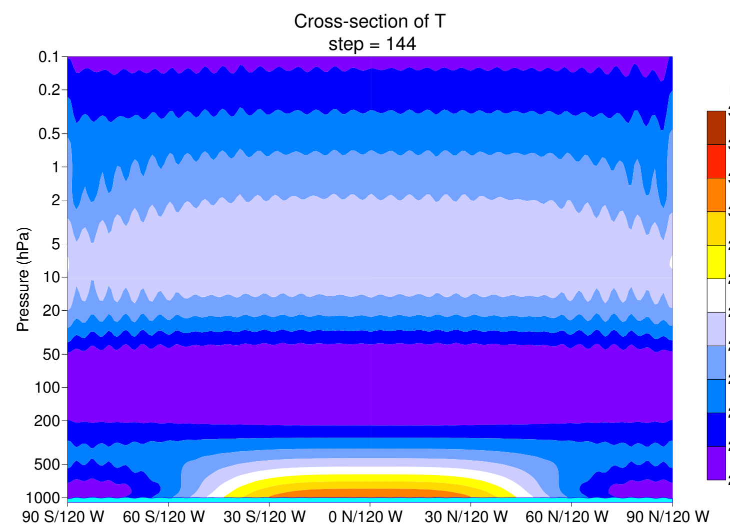

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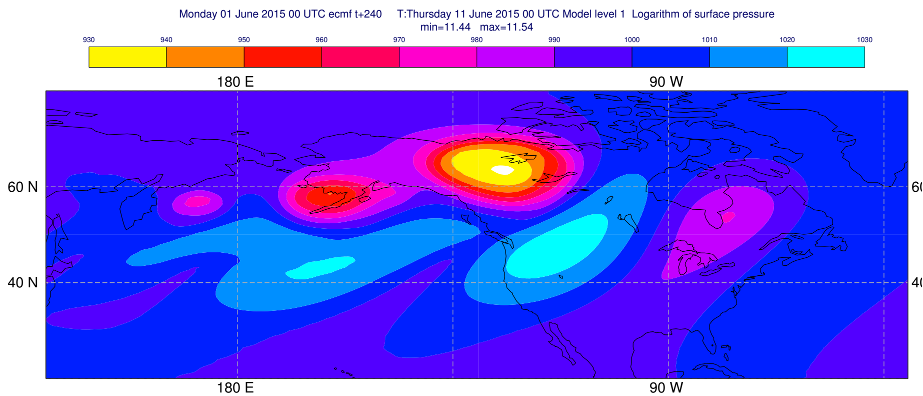
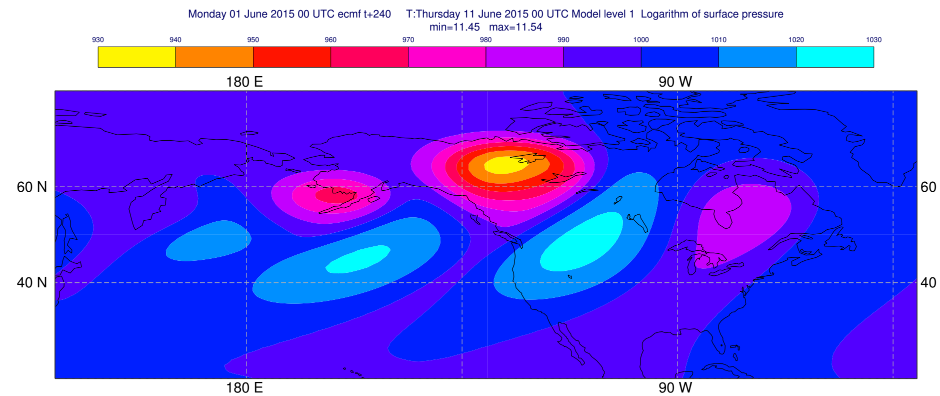
- Using derivatives of δT results in systematic cooling (better results obtained with derivatives of $\delta\Theta$ or $\delta(T - \alpha \log p_s)$
  $\rightarrow$ indicates there are probably better alternatives for the temperature related prognostic variable.

Baroclinic wave test



IFS_ref Tco159/L139 $\Delta t = 1800$s



newSI Tco159/L139 $\Delta t = 900$s

Jablonowski and Williamson(2006) DCMIP

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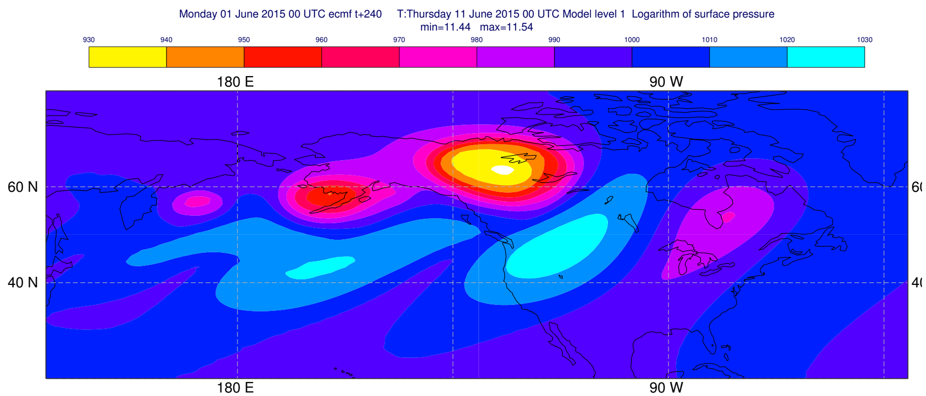

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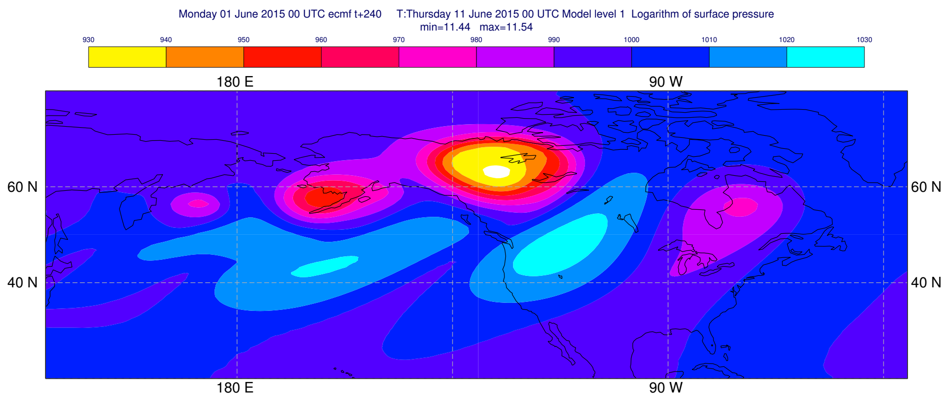
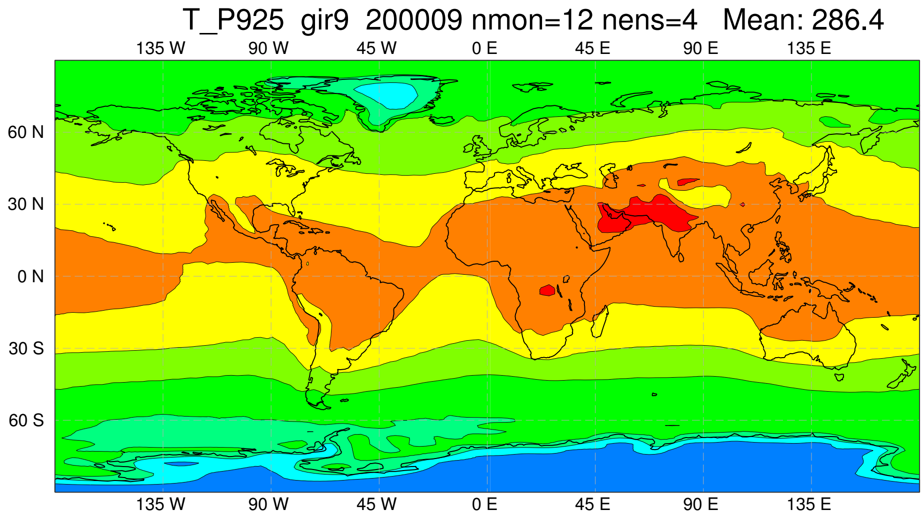


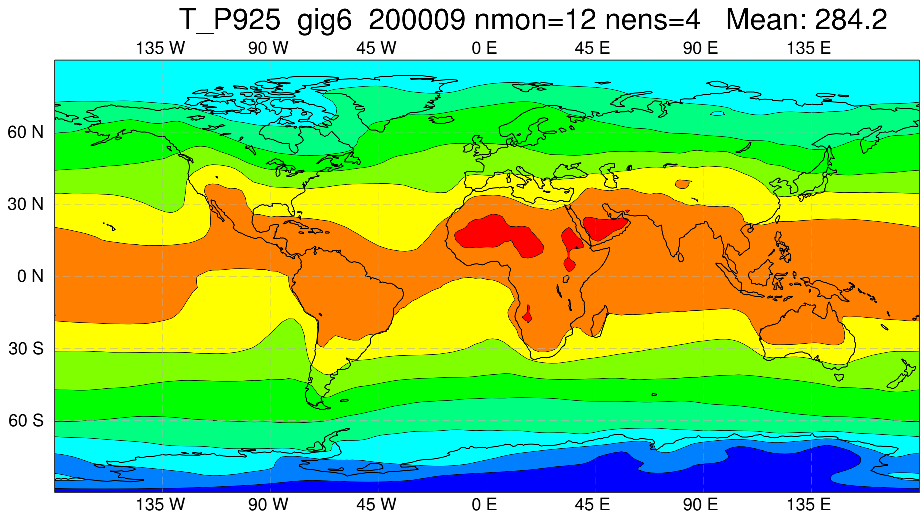
newSI_SLHD Tco159/L139 $\Delta t = 900$s

Jablonowski and Williamson(2006) DCMIP

Grid-point IFS with 2^{nd} order derivatives



New SI scheme, no spectral space



Reference IFS

Annual climate of temperature at 925 hPa (T_L255/L137)

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- TL/AD extension challenging but perfectly doable.