Transition of VFE scheme from LAM to global model

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Introduction

We tested a new numerical scheme for calculations over the vertical grid in Integrated Forecasting System (IFS) of ECMWF. The results of initial tests are very promising. The scheme was developed for LAM ALADIN in close cooperation of RC LACE, METEO France and HIRLAM. Thanks to development of colleagues from METEO France and ECMWF the scheme can work also in global models IFS and ARPEGE.

The first version of vertical scheme that uses a finite-element discretisation method was implemented in the IFS in 2002 by Untch and Hortal. The scheme is known by the acronym VFE (vertical finite element). The VFE works very well with the vertical grid spacings currently used in the IFS, which range from tens of metres near the ground to several kilometres near the model top. But there are several reasons why the community around ALADIN/IFS code is looking to review the current VFE. The new scheme may:

- offer greater flexibility in the chosen accuracy in the vertical
- be better suited for reduced precision in calculations at the same level of accuracy, thus saving on computational cost
- be needed at future higher resolutions, at which the 'hydrostatic assumption' is no longer valid, i.e. the assumption that the decrease of pressure with height is balanced by the downward-directed gravitational pull of the Earth.

Therefore we extended the VFE scheme developed for NH model to work with HY core as well. We fix instability issues to stabilize the 2TL ICI time stepping with 5th order splines and higher. The results are presented in this poster.

Hybrid mass based coordinate definition

Finite differences model | Finite elements model

Fundamental difference between the finite difference scheme (FD) and VFE scheme is the definition of vertical hybrid coordinate η. The VFE uses differential form of definition.

FD: $\pi(\eta_{\tilde{l}}) = A(\eta_{\tilde{l}}) + B(\eta_{\tilde{l}})\pi_s(x,y)$

VFE: $m_l = \frac{\partial \pi(\eta_l)}{\partial \eta} = \frac{\partial A(\eta_l)}{\partial \eta} + \frac{\partial B(\eta_l)}{\partial \eta}\pi_s(x,y)$

The depth of layers are defined also in different way.

FD: $\delta\pi_l = \pi_{\tilde{l}} - \pi_{\tilde{l-1}}$

VFE: $m_l = \frac{\delta A_l}{\delta \eta_l} + \frac{\delta B_l}{\delta \eta_l}\pi_s \qquad \delta\eta_l = \eta_{\tilde{l}} - \eta_{\tilde{l-1}}$

The vertical integral operator is defined in the following way

FD: $\int_0^1 \frac{\partial \pi}{\partial \eta}\psi d\eta \approx \sum_{l=1}^{L} \psi_l \delta\pi_l$

η values remain implicit

VFE: $\int_0^1 \frac{\partial \pi}{\partial \eta}\psi d\eta \approx (\mathbf{K} m\psi)_L$

η requires explicit definition, operator $(\mathbf{K})$ is full level matrix and $(\mathbf{K})_l$ represent value of integral from top to level l

Mass conservation is ensured in both schemes in different way

FD: $\sum_{l=1}^{L} \delta\pi_l = \sum_{l=1}^{L} \delta A_l + \sum_{l=1}^{L} \delta B_l \pi_s$

$\pi_s = (A_{\tilde{L}} - A_{\tilde{1}}) + (B_{\tilde{L}} - B_{\tilde{1}})\pi_s$

VFE: $(\mathbf{K}m)_L = \left(\mathbf{K}\frac{\delta A}{\delta \eta}\right)_L + \pi_s \left(\mathbf{K}\frac{\delta B}{\delta \eta}\right)_L$

$\left(\mathbf{K}\frac{\delta A}{\delta \eta}\right)_L = 0 \qquad \left(\mathbf{K}\frac{\delta B}{\delta \eta}\right)_L = 1$

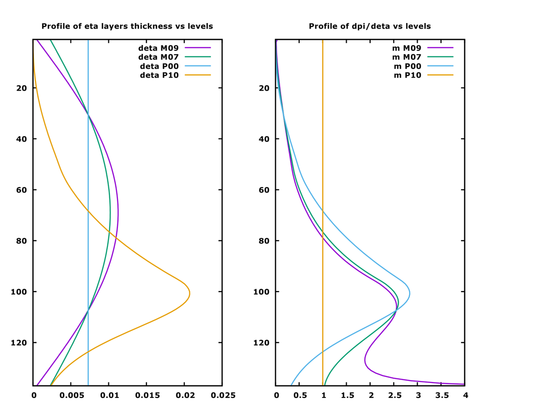
This is ensured by proper choice of BCs in FD scheme. In the VFE scheme the above integral properties are ensured by iterative adjustment of layer depths δA_l and δB_l. Once integral properties are ensured after adustment the full level A and B are computed.

Explicit definition of η

As already mentioned. in FD η is implicit, in VFE it must be explicitly defined. When current definition of η is used (LREGETA true and false as well), the time stepping becomes unstable for VFE scheme constructed with splines order 5 and higher when tested with 137 levels. Stabilisation require higher density of levels close to BCs in η space. We choose following definition

$$\eta_k = (1-\beta)\frac{k}{L} + \beta\left[\frac{1}{2} - \frac{1}{2}cos(\pi\frac{k}{L})\right]$$

We found experimentally that $\beta \approx 0.5$ is stable for high order operators and the eigenvalues of linear SI model are pure imaginary one as required (abort in SUSI, SUNHSI routines).



Profiles of layer thickness $\delta\eta$ and $\frac{\partial\pi}{\partial\eta}$ vs levels index for various definitions of η coordinate (P00 - LREGETA=.T., P10 - LREGETA=.F., M07 - new def. with $\beta = 0.7$, M09 - new def. with $\beta = 0.9$)

Analysis of stability of 2TL scheme with VFE scheme

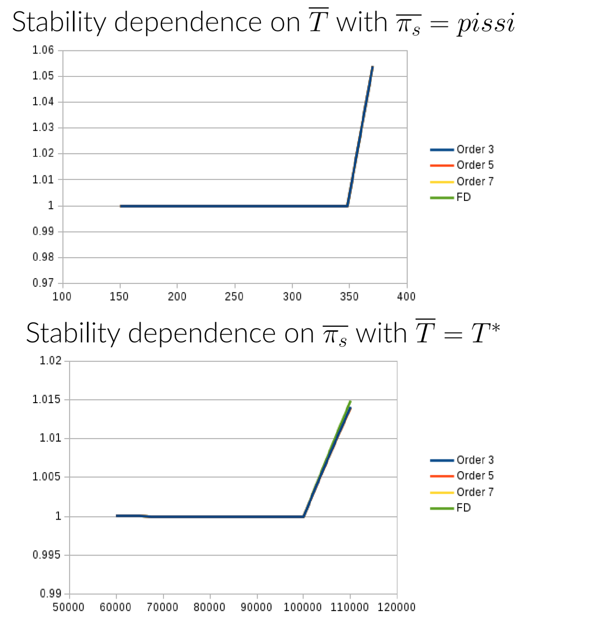
We implemented into model setup the analysis of stability exactly as in Simmons,1978. We assume isothermal resting atmosphere fully described by 2 parameters $(\overline{T}, \overline{\pi_s})$. The corresponding SI reference state has same properties and is fully determined by 2 parameters $(T^*, \pi_s{}^*)$. Such atmosphere evolved numerically with 2TL SI scheme. We analyze the stability properties of small amplitude waves under such conditions when nonlinear part evolves according $\overline{\mathbf{L}}$ linearized model and SI part uses tradition linear model $\mathcal{L}$
The 2TL ICI scheme then gives

$$\frac{X^{t+\delta t(0)} - X^t}{\delta t} = RX^t + \frac{\mathcal{L}.X^{t+\delta t(0)} + \mathcal{L}X^t}{2}$$
$$\frac{X^{t+\delta t(n)} - X^t}{\delta t} = \frac{RX^{t+\delta t(n-1)} + RX^t}{2} + \frac{\mathcal{L}.X^{t+\delta t(n)} + \mathcal{L}.X^t}{2}$$

with explicitly treated residual part $RX = \overline{\mathbf{L}}X - \mathcal{L}X$. The scheme can be written in matrix form taking into account that X is vector with $dim(X) = 4L+1$ for each horizontal eigenvalue. We then write for NESC scheme

$$M_p X^{t+\delta t(0)} = (I - \tau\mathcal{L})^{-1}(2\tau R + I + \tau\mathcal{L})X^t$$
$$M_c X^{t+\delta t(1)} = (I - \tau\mathcal{L})^{-1}(\tau R M_p + \tau R + I + \tau\mathcal{L})X^t$$

Largest eigenvalues of matrix M_c are showed. Stability properties of VFE scheme in examined conditions are the same as with FD scheme for all spline orders.



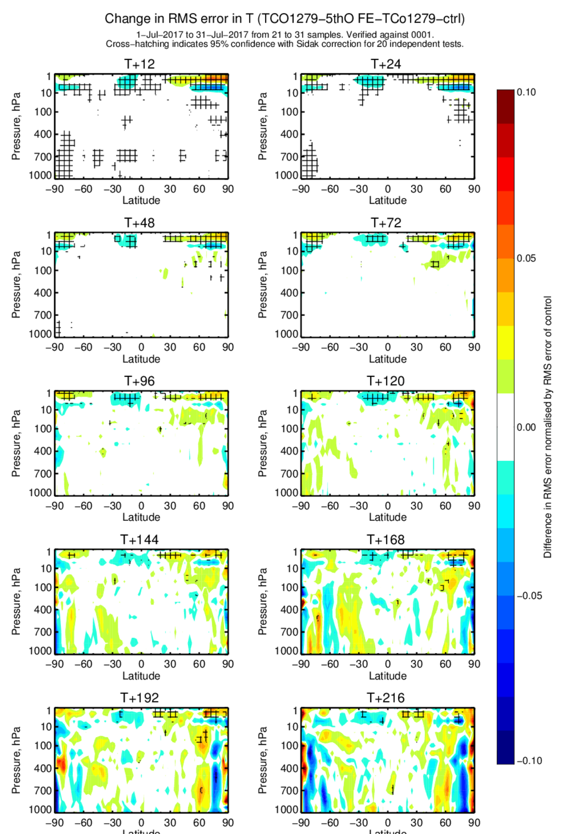
Growth rate per time step for single horizontal wave number $\triangle = -\frac{n(n+1)}{a^2}$ for $n = 1000$ and $dt = 600s$ for FD and VFE scheme with various orders of splines with 137 levels and fixed $T^* = 350K, T_a{}^* = 50K and \pi_s{}^* = 100000Pa$ and with definition of η with $\beta = 0.5$.

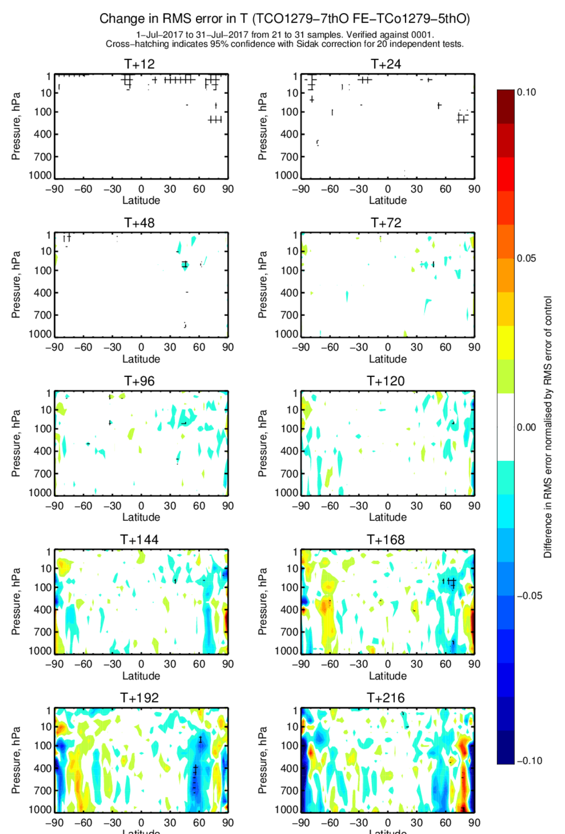
Real experiments with IFS - HY model

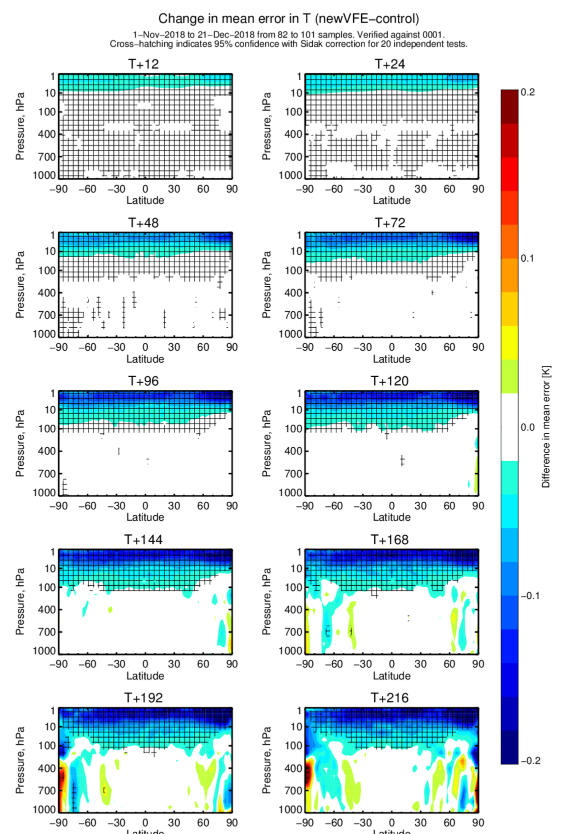
We did experiment of operational 3rd order ECMWF VFE scheme against new VFE scheme with 5th order basis using HY dynamics TCO1279, 137 levels and $\beta = 0.5$ in η definition. Change of RMS error in temperature with respect to operational RMS error is shown on left figure. New VFE scheme has statistically significant cooling effect in the stratosphere. This is good result as ECMWF has positive temperature bias in this region. We run experiments for whole July 2017.

We also compare new VFE scheme with 7th order basis against new VFE with 5th order with same experimental setting as before. Again the change of RMS error in temperature is shown on middle figure. The scores are neutral. This would suggest that using higher than 5th order splines in VFE operators construction has no positive impact.

We have to run low res IFS model in order to perform long term evaluation of the scheme. We run experiments for the whole year 2018. The change in RMS error in temperature is presented. We clearly see very strong cooling that is statistically significant (cross hatching indicates 95



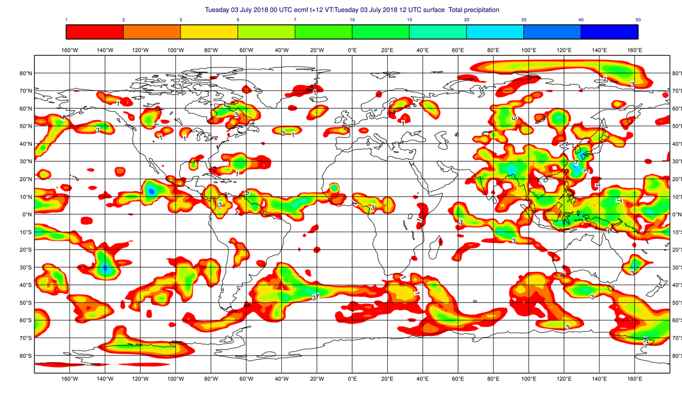




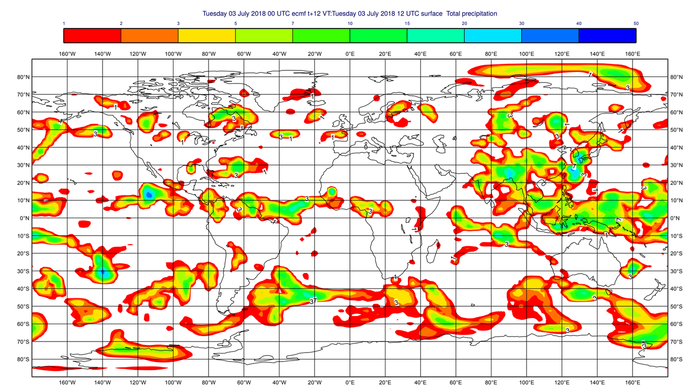
Real experiments with IFS - NH model experiment

We have tested new formulation of VFE with $\beta = 0.5$ and 3th order of splines also with NH version of IFS dynamical core. The test was technical one and results are shown on figures below. We see that VFE core performes comparably to FD one, but we did not perform verification due to lack of time.

FD reference



VFE exp



Transformation of gw to d_4 - Idealized experiments with small planet

The vertical momentum treatment in NH ALADIN requires transformation of gw to d_4 and back. Having atmosphere in steady state the local change for any prognostic quantity myst satisfy $\frac{\partial X}{\partial t} = 0$. Every time step we perform transformation

at time t	at time $t \tilde{+} dt$ (explicit guess)
$gw^t = gw_s^t + \mathbf{T_i}\left[\frac{mRT}{p}(d_4 - \mathcal{X})\right]^t$	$d_4{}^{t\tilde{+}dt} = \left[\frac{p}{mRT}\mathbf{T_d}(gw) + \mathcal{X}\right]^{t\tilde{+}dt}$

These must not influence steady state $\Rightarrow d_4{}^{t\tilde{+}dt} = d_4{}^t$ and therefore $\mathbf{T_i}\mathbf{T_d} = \mathbf{I_d}$ shall be valid.

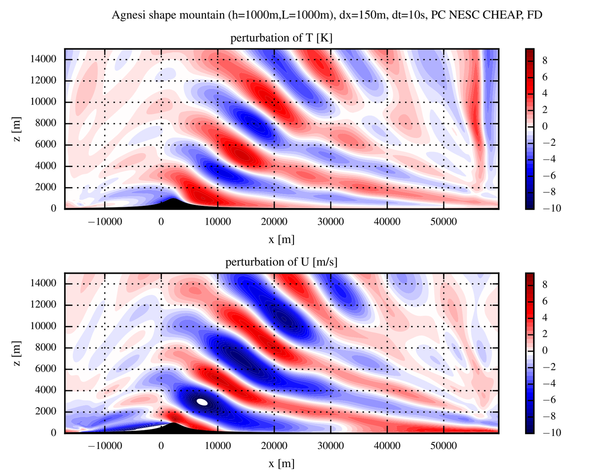
When we define integral and derivative operators using VFE that satisfy conditon above, the results from 2D idealized flow over Agnesi mountain were very noisy. To remove noise we decided to require less strict condition $\mathbf{T_i}\mathbf{T_d} = \mathbf{I_d} + \epsilon$. We also find experimentally that the noice was very sensitive to the choice we compute spline knots. These are the points that fully determines spline basis functions. New compact definition was implemented. The knots is obtained from already known full levels values of η as

$$t_i = (1-w)\eta_k + w\eta_{k+1}$$

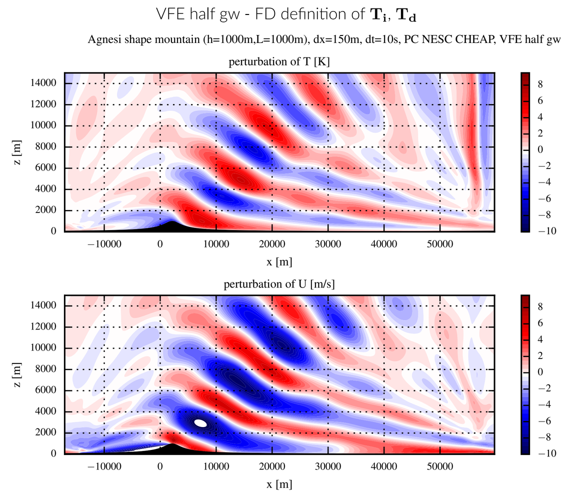
Here t_i is i-th know, full level index $k = i - int(\alpha)$ with alpha being symmetric shift from BCs toward internal part of domain defined as $\alpha = \frac{1}{2}(L - N + C)$. (L-number of levels, N-number of internal knots, C-order of spline basis). The interpolation weight is defined as $w = \alpha - int(\alpha)$. Using DeBoor's algorithm we compute B-spline basis using C-time multiple knots at boundaries. . The above definition of knots was used in all experiments presented on this poster.

We tested VFE scheme with VFE definition of $\mathbf{T_i}$, $\mathbf{T_d}$ with 2D small planet. In order to force NH flow regimes, the size of earth diameter of earth is adjusted in order to shrink grid to size when NH effects plays significant role (small planet experiment). We carried out experiment with elliptic Agnesi mountain (h=1000m, L=1000m) with resolution $dx = 150m, dt = 10s, N = 0.02s^{-1}, U = 10ms^{-1}$.

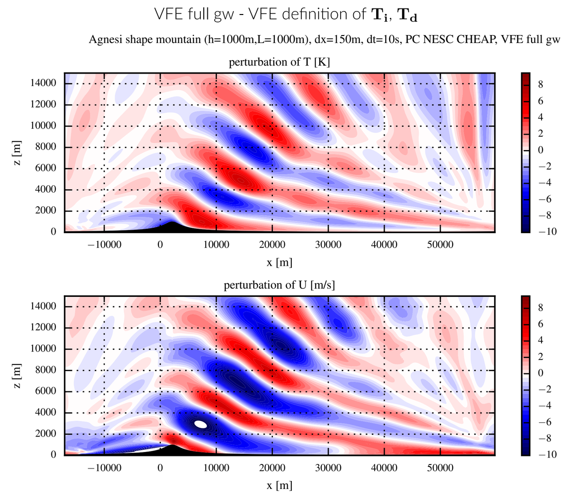
FD - reference



VFE half gw - FD definition of $\mathbf{T_i}$, $\mathbf{T_d}$



VFE full gw - VFE definition of $\mathbf{T_i}$, $\mathbf{T_d}$



This results is noise free, but it must be studied in full 3D context.

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