The physics-dynamics interface

stability and accuracy

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Contents

1. stability and accuracy of the physics-dynamics interface
    - not well studied in literature (as dynamics) BUT becoming more pertinent in context of the evolution to NH
    - new project
2. a result on transparent boundary conditions that may be relevant for the ALADIN research agenda

Open questions

Lectures Jean-François, “Numerics for physics”
Kranjska Gora, Slovenia, 2002:

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“these aspects are interdependent”

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“scientific validity of the choices should be reverified from time to time”

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"predictor-corrector schemes will reopen these issues"

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Methodology: first approach

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$$\frac{\partial F}{\partial t} + U\frac{\partial F}{\partial x} + i\omega F = -\beta F + R\, e^{i[kx+\Omega]}$$

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$$\frac{\partial F}{\partial t} + U\frac{\partial F}{\partial x} + i\omega F = -\beta F + R\, e^{i[kx+\Omega]}$$

- solution 1: free solution (homogeneous Eq.)

$$F(x,t) = F_k^{free} e^{-\beta t} e^{i[kx-(\omega+kU)t]}$$

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$$\frac{\partial F}{\partial t} + U\frac{\partial F}{\partial x} + i\omega F = -\beta F + R\, e^{i[kx+\Omega]}$$

- solution 2: forced regular solution

$$F(x,t) = \frac{R}{\beta + i(\omega + kU + \Omega)} e^{i[kx+\Omega t]}$$

$$\beta + i(\omega + kU + \Omega) \neq 0$$

Methodology: first approach

- simple system, but with EXACT solutions
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$$\frac{\partial F}{\partial t} + U\frac{\partial F}{\partial x} + i\omega F = -\beta F + R\,e^{i[kx+\Omega]}$$

- solution 3: forced resonant solution

$$F(x,t) = R\,t\,e^{i[kx+\Omega t]}$$
$$\beta = 0 \quad \text{and} \quad \omega + kU + \Omega = 0$$

Simple example (2TL SL SI)

$$\frac{F_A^+ - F_D^0}{\Delta t} + \frac{i}{2}\omega^*(F_A^+ + F_D^0) + \frac{i}{2}(\omega - \omega^*)(F^{(0)} + F_D^0) =$$
$$-\beta\left(\epsilon F_A^+ + (1-\epsilon)F_D^0\right)$$

$$F^{(0)} \equiv F^0, \quad \mathrm{non-extrapolating}$$

Simple example (2TL SL SI)

$$\frac{F_A^+ - F_D^0}{\Delta t} + \tfrac{i}{2}\omega^*(F_A^+ + F_D^0) + i(\omega - \omega^*)F_D^0 = -\beta\left(\epsilon F_A^+ + (1-\epsilon)F_D^0\right)$$

$$F_A^+ = \frac{1 + \frac{i}{2}\omega^*\Delta t - i\omega\Delta t - \beta(1-\epsilon)\Delta t}{1 + \frac{i}{2}\omega^*\Delta t + \epsilon\beta\Delta t} e^{-ikU\Delta t}\, F_A^0$$

Simple example (2TL SL SI)

$$F_A^+ = \mathcal{A}\, F_A^0$$

$$\mathcal{A} = \frac{1 + \frac{i}{2}\omega^* \Delta t - i\omega \Delta t - \beta(1-\epsilon)\Delta t}{1 + \frac{i}{2}\omega^* \Delta t + \epsilon\beta\Delta t} e^{-ikU\Delta t}$$

STABILITY

$$\beta\Delta t\,[\beta\Delta t(1-2\epsilon) - 2] \leq \omega(\omega^* - \omega)\Delta t^2$$

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$$\beta\Delta t\left[\beta\Delta t(1-2\epsilon) - 2\right] \leq \omega(\omega^* - \omega)\Delta t^2$$

$$\beta = 0 \qquad (0 \leq)\omega < \omega^*$$

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STABILITY

$$\beta\Delta t\,[\beta\Delta t(1-2\epsilon) - 2] \leq \omega(\omega^* - \omega)\Delta t^2$$

$$\epsilon = 0: \quad \beta\Delta t(\beta\Delta t - 2) \leq \omega(\omega^* - \omega)\Delta t^2 \quad \beta\Delta t \leq 2$$

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STABILITY

$$\beta\Delta t\left[\beta\Delta t(1-2\epsilon) - 2\right] \leq \omega(\omega^* - \omega)\Delta t^2$$

$$\epsilon \geq \frac{1}{2} : \text{unconditionally} \quad \text{stable}$$

Simple example (2TL SL SI)

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ACCURACY

$$\frac{F^+}{F^0} - \frac{F^{exact}(t+\Delta t)}{F^0} =$$

$$\frac{1}{2}(\beta + i\omega)\left[\beta(2\epsilon - 1) + i(\omega^* - \omega)\right]\Delta t^2 + \mathcal{O}(\Delta t)^3$$

Total time splitting or not?

"In case of total time-splitting, the physics is done after the dynamics and starts from dynamically balanced fields of the current time step"

Total time splitting or not?

$$\frac{F_A^+ - F_D^0}{\Delta t} + \frac{i}{2}\omega^*(F_A^+ + F_D^0) + \frac{i}{2}(\omega - \omega^*)(F^{(0)} + F_D^0) = \Phi$$

Total time splitting or not?

$$
\begin{aligned}
\Phi &= (1-\epsilon^{TT})\,\Phi[F_A^0] \\
F_A^{exp} &= e^{-ikU\Delta t}\left(1-\tfrac{i}{2}\omega\Delta t\right)F_A^0 - \tfrac{i}{2}(\omega-\omega^*)\Delta t\,F^{(0)} \\
&\quad +(1-\epsilon^{TT})\,\Phi[F_A^0] \\
F_A^{dyn} &= \frac{1}{1+\frac{i}{2}\omega^*\Delta t}F_A^{exp} \\
\Phi &= \epsilon^{TT}\Phi[F_A^{dyn}] \\
F_A^+ &= F_A^{dyn} + \epsilon^{TT}\Phi[F_A^{dyn}]
\end{aligned}
$$

Total time splitting: $\epsilon^{TT} = 1$

$$\mathcal{A}^{TT} = (1 - \beta\Delta t)\frac{1 + \frac{i}{2}\omega^*\Delta t - i\omega\Delta t}{1 + \frac{i}{2}\omega^*\Delta t}$$

Total time splitting: $\epsilon^{TT} = 1$

$$\mathcal{A}^{TT} = (1 - \beta\Delta t)\frac{1 + \frac{i}{2}\omega^*\Delta t - i\omega\Delta t}{1 + \frac{i}{2}\omega^*\Delta t}$$

STABILITY

$$\frac{\beta\Delta t(\beta\Delta t - 2)}{(1 - \beta\Delta t)^2} \leq \frac{\omega(\omega^* - \omega)\Delta t^2}{1 + \frac{1}{2}(\omega^*\Delta t)^2}$$

$$\omega \leq \omega^* \quad \text{and} \quad \beta\Delta t \leq 2$$

NO Total time splitting: $\epsilon^{TT} = 0$

$$\mathcal{A} = \frac{1 + \frac{i}{2}\omega^* \Delta t - i\omega\Delta t - \beta\Delta t}{1 + \frac{i}{2}\omega^* \Delta t}$$

NO Total time splitting: $\epsilon^{TT} = 0$

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$$\omega \leq \omega^* \quad \text{and} \quad \beta\Delta t \leq 2$$

Total time splitting or not?

$$|\mathcal{A}|^2 - |\mathcal{A}^{TT}|^2 \;=\; \frac{\beta\Delta t\,[2-\beta\Delta t]\left(\frac{1}{2}\omega^* - \omega\right)^2 \Delta t^2}{1+\frac{1}{4}{\omega^*}^2\Delta t^2} \geq 0$$

$$\text{if} \quad \beta\Delta t \leq 2$$

Total time splitting or not?

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Total time splitting is more stable

Total time splitting or not?

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BUT !

Total time splitting or not?

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$$\text{if} \quad \beta\Delta t \leq 2$$

IF WE WANT GUARANTEES FOR PHYSICS AND DYNAMICS SEPERATELY IT DOESN'T MATTER!

Predictor

$$
\begin{aligned}
\Phi^P &= \left(1-\epsilon^{TT}\right)\Phi^P[F_A^0] \\
F_A^{exp} &= e^{-ikU\Delta t}\left(1-\tfrac{i}{2}\omega\Delta t\right)F_A^0 - \tfrac{i}{2}(\omega-\omega^*)\Delta t\, F^{(0)} \\
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\Phi^P &= \epsilon^{TT}\Phi^P[F_A^{dyn}] \\
F_A^P &= F_A^{dyn} + \epsilon^{TT}\Phi^P[F_A^{dyn}]
\end{aligned}
$$

Corrector

$$
\begin{aligned}
\Phi^C &= (1-\xi^{TT})\,\Phi^C[F_A^0, F_A^P] \\
F_A^{exp} &= e^{-ikU\Delta t}\left(1-\frac{i}{2}\omega\Delta t\right)F_A^0 - \frac{i}{2}(\omega-\omega^*)\Delta t\,F^P \\
&\quad +(1-\xi^{TT})\,\Phi^C[F_A^0, F_A^P] \\
F_A^{dyn} &= \frac{1}{1+\frac{i}{2}\omega^*\Delta t}F_A^{exp} \\
\Phi^C &= \xi^{TT}\Phi^C[F_A^{dyn}] \\
F_A^P &= F_A^{dyn} + \xi^{TT}\Phi^C[F_A^{dyn}]
\end{aligned}
$$

Accuracy $\Phi - D$ interface PC scheme

- exact solution ($U = 0$):

$$F(t + \Delta t) = \left\{1 - (i\omega + \beta)\Delta t + \mathcal{O}[\Delta t]^2\right\} F(t)$$

Accuracy $\Phi - D$ interface PC scheme

- exact solution ($U = 0$):

$$F(t + \Delta t) = \left\{1 - (i\omega + \beta)\Delta t + \mathcal{O}[\Delta t]^2\right\} F(t)$$

- $\Phi^P = 0, \xi^{TT} = 1$:

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Open questions to be treated

- other solutions than the diffusive
- (de)centering
- other first guess than non-extrapolating
- predictor corrector
- convergence properties when iterating more than once
- partial (fractional) physics as in ECMWF (Wedi 1999)

So ...

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- start from concrete problems (cases) and use this method as a testing lab

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- start from concrete problems (cases) and use this method as a testing lab
- or try to find some structure in it to stay as general as possible

Conclusions

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- nevertheless, this type of exercise is useful
- comments, questions, problems, ... are welcome

Transparent LBCs in a spectral LAM

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Transparent LBCs in a spectral LAM

- work of A. McDonald: transparent LBCs in a LAM
- test in 3D: working
- advantage w.r.t Davies scheme: provide coupling data on a line instead of a coupling zone

A spectral shallow-water model

$$\Phi \equiv \ln \phi$$

$$\begin{aligned}
\frac{\partial u}{\partial t} + \bar{u}\frac{\partial u}{\partial x} &= -e^{\bar{\Phi}}\frac{\partial \Phi}{\partial x} + fv \\
\frac{\partial v}{\partial t} + \bar{u}\frac{\partial v}{\partial x} &= -fu \\
\frac{\partial \Phi}{\partial t} + \bar{u}\frac{\partial \Phi}{\partial x} &= -\frac{\partial u}{\partial x}
\end{aligned}$$

A spectral shallow-water model

1. compute derivatives
2. inverse FFT
3. explicit part in gridpoint space
4. FFT
5. solve Helmholtz Eq.

A spectral shallow-water model

$$\left[\left(1 - \tfrac{\Delta t}{2}\mathcal{L}\right) X\right]_F^+ = \rho_D$$

$$\begin{aligned}
\rho_u^{grid} &= u^0 + \tfrac{\Delta t}{2}\left(f v^0 - e^{\bar{\Phi}}\frac{\partial \Phi^0}{\partial x}\right), \\
\rho_v^{grid} &= v^0 - \tfrac{\Delta t}{2} f u^0, \\
\rho_\Phi^{grid} &= \Phi^0 - \tfrac{\Delta t}{2}\frac{\partial u^0}{\partial x},
\end{aligned}$$

A spectral shallow-water model

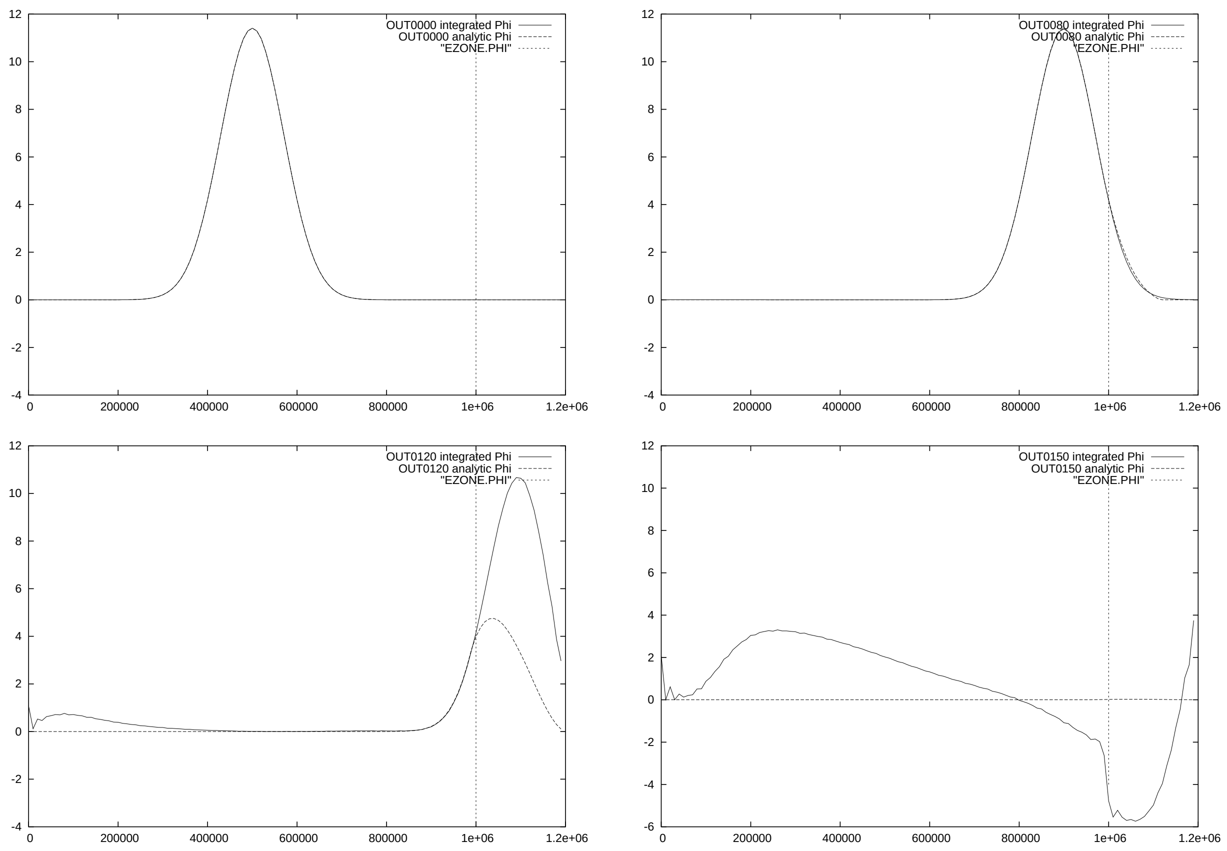
$$\left[\left(1 - \tfrac{\Delta t}{2}\mathcal{L}\right) X\right]_F^+ = \rho_D$$

$$\begin{aligned}
\rho_{u,1}^{grid} &= u_1^0 + \frac{\Delta t}{2}\left[f v_1^0 - e^{\bar{\Phi}}\left(\frac{\Phi_2^0 - \Phi_1^0}{\Delta x}\right)\right],\\
\rho_{v,1}^{grid} &= v_1^0 - \frac{\Delta t}{2} f u_1^0,\\
\rho_{\Phi,1}^{grid} &= \Phi_1^0 - \frac{\Delta t}{2}\frac{u_2^0 - u_1^0}{\Delta x}.
\end{aligned}$$

A spectral shallow-water model

1. compute derivatives
2. inverse FFT
3. overwrite the “incoming” LBC values that have to be impose
4. explicit part in gridpoint space + extrapolate the “outgoing” values
5. make fields periodic
6. FFT
7. solve Helmholtz Eq.

Test: bell shape, no periodicity



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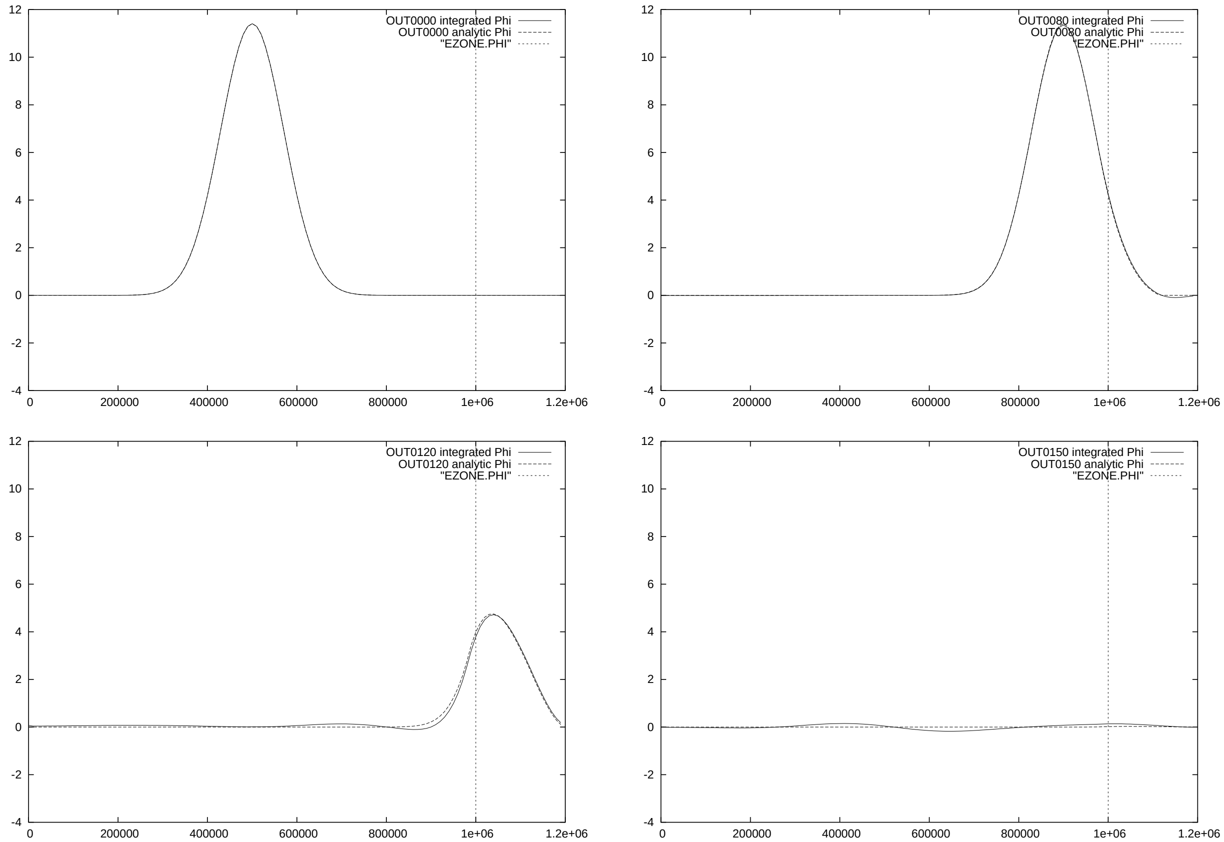

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- a possible solution would be to make the fields periodic at all time steps just before calling the FFT
- reopen the issue of the periodicity: cheap algoritm