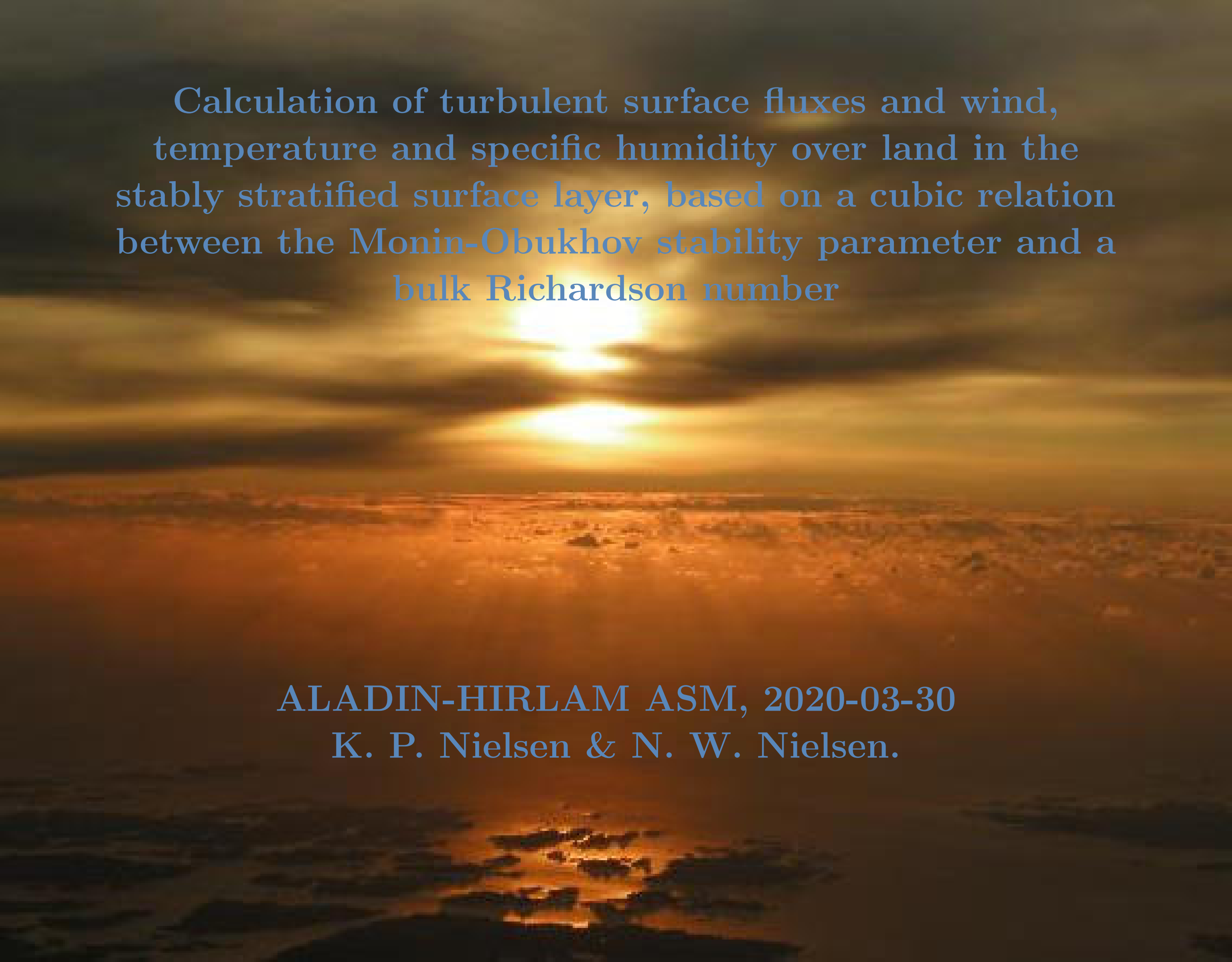

Calculation of turbulent surface fluxes and wind, temperature and specific humidity over land in the stably stratified surface layer, based on a cubic relation between the Monin-Obukhov stability parameter and a bulk Richardson number

ALADIN-HIRLAM ASM, 2020-03-30

K. P. Nielsen & N. W. Nielsen.

Coupling between the surface and the atmosphere

- Multifaceted, including the radiation flux coupling. . .
- **SURFEX computes surface variables.**
- **These include T_{2m}, q_{2m} and U_{10m} from the lowest model level and the surface variables.**
- T_{2m} and q_{2m} are assimilated, which affects the surface variables.

Monin-Obukhov similarity theory

$$\overline{u}(z) = \frac{u_*}{k}(\ln(\frac{z + z_0}{z_0}) - \psi_m) \qquad (1)$$

$$\overline{\theta}(z) - \overline{\theta}(z_{0\theta}) = \frac{\theta_*}{k_\theta}(\ln(\frac{z + z_{0\theta}}{z_{0\theta}}) - \psi_h) \qquad (2)$$

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$$\psi_m(\zeta) = \int_{\zeta_0}^{\zeta}(1 - \phi_m(\zeta))dln\zeta \quad (3)$$

$$\psi_\theta(\zeta) = \int_{\zeta_{0\theta}}^{\zeta}(1 - \phi_h(\zeta))dln\zeta \quad (4)$$

$$\zeta = \frac{z}{L} \quad \zeta_0 = \frac{z_0}{L} \quad \zeta_{0\theta} = \frac{z_{0\theta}}{L} \quad L = \frac{u_*^2}{kb\theta_*} \quad b \approx \frac{g}{\overline{\theta}(z)}$$

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DMI

High Resolution Hirlam Limited Area Model

Hirlam modification of the Geleyn (1988) solution

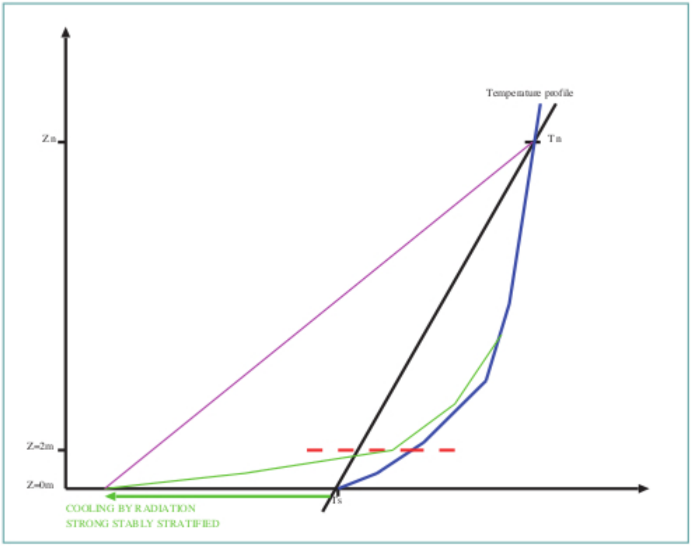

Figure 1: Illustration of temperature profiles under very stable conditions acoording to the current formulas by J.-F. Geleyn (quasi-linear profiles) and possible (imagined) non-linear profiles occurring under non-stationary conditions dominated by radiative cooling at the surface

Hirlam diagnostic formula modification (Sass & Woetmann Nielsen 2008).

Dian, CMHI, revised Kuhlmann (2009) solution

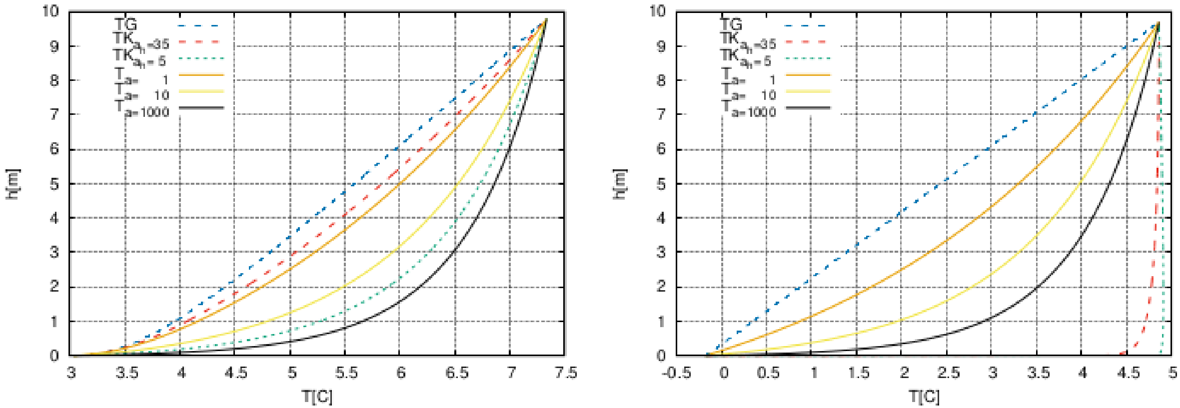

Figure 4.3: Vertical temperature profile as function of height z: +25h (left); +31h (right). Blue dashed line: Geleyn formula (2.12). Red dashed line: Kullmann formula (2.15) for $a_K = 35$. Green dashed line: Kullmann formula (2.15) for $a_K = 5$. Orange, yellow and black solid lines: New revised formula (3.2) for $a = 1$, $a = 10$ and $a = 1000$ respectively.

Dian (2016)

DMI

Empirical ϕ-functions

$$\phi_m(\zeta) = 1 + \frac{a_m}{k}\zeta \tag{5}$$

$$\phi_h(\zeta) = 1 + \frac{a_1\zeta + a_2\zeta^2}{1 + a_3\zeta}(1 + \frac{k}{R_\infty\zeta}). \tag{6}$$

To a good approximation (6) can be replaced by the quadratic form

$$\phi_h(\zeta) = 1 + \frac{a_{h1}}{k}\zeta + \frac{a_{h2}}{k^2}\zeta^2, \tag{7}$$

Woetmann Nielsen (2017). (5) and (6) have been proposed by Zilitinkevich et al. (2013).

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Analytical solution for the stability parameter ζ

$$\zeta^3 + A\zeta^2 + B\zeta + C = 0 \tag{8}$$

$$A = \frac{k \cdot a_{h1} - k \cdot k_\theta (\frac{a_m}{k})^2 Ri_b}{a_{h2}} \tag{9}$$

$$B = \frac{k^2(\alpha + \beta) - 2k \cdot k_\theta \frac{a_m}{k} \alpha Ri_b}{a_{h2}} \tag{10}$$

$$C = -\frac{k \cdot k_\theta \alpha^2 Ri_b}{a_{h2}} \tag{11}$$

$$\alpha \equiv \ln\left(\frac{z}{z_0}\right) \qquad \beta \equiv \ln\left(\frac{z_0}{z_{0\theta}}\right)$$

The condition for one and only one positive solution to (8) is that $z_{0\theta} > \frac{z_0}{39.81}$ (Woetmann Nielsen 2017).

DMI

High Resolution Hirlam Limited Area Model

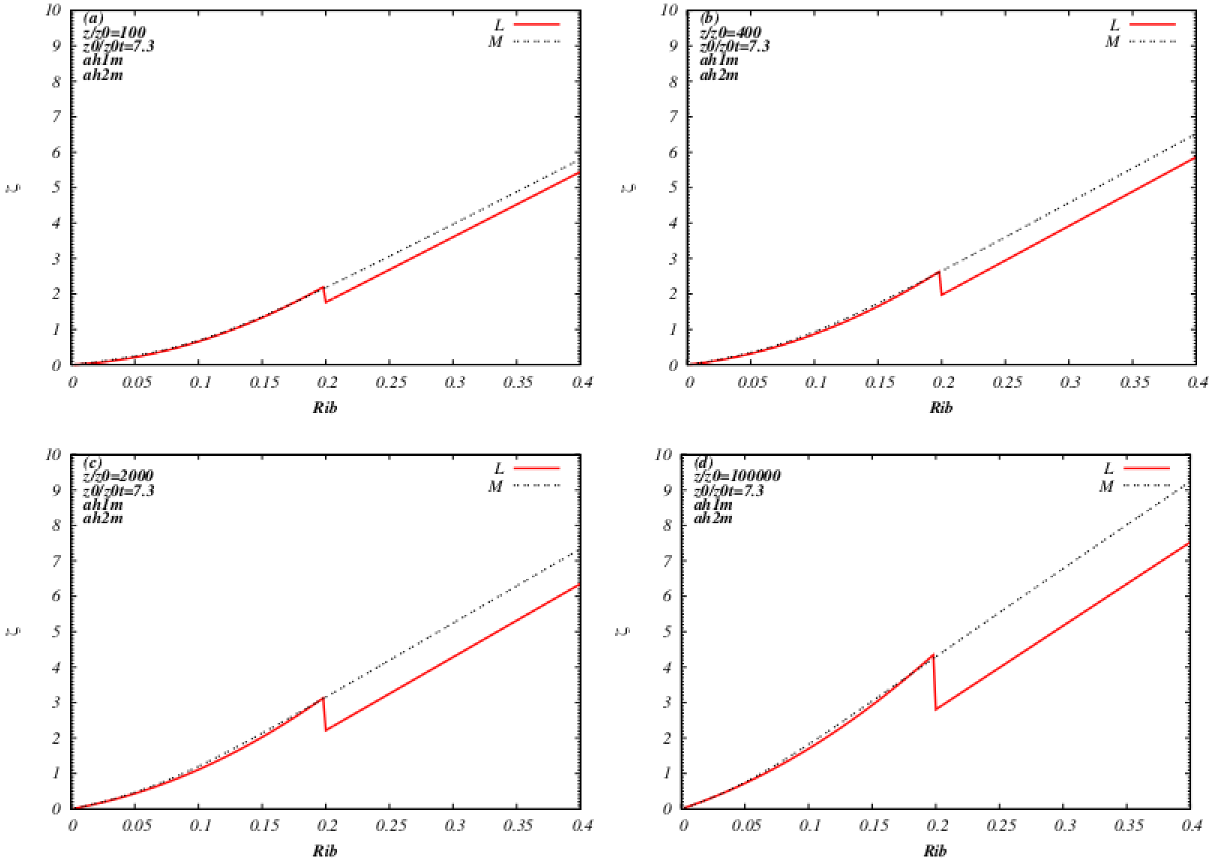


Figure 2: The M-O stability parameter, ζ, as function of the bulk Richardson number, Ri_b, for $\frac{z_0}{z_{z_{0\theta}}} = 7.3$ and (a) $\frac{z}{z_0} = 100$, (b) $\frac{z}{z_0} = 400$, (c) $\frac{z}{z_0} = 2000$, and (d) $\frac{z}{z_0} = 100000$. Curve L hows results by Li2010 and curve M are the solutions of the cubic equation (15) with the modifications a_{h1m} and a_{h2m} of a_{h1} and a_{h2}, respectively.

Computing u_* and θ_*

$$u_* = \frac{k\overline{V}(z)}{\alpha - \psi_{mBH(\zeta)}} \tag{12}$$

$$\theta_* = \frac{k_\theta(\overline{\theta}(z) - \overline{\theta}(z_{0\theta}))}{\alpha + \beta - \psi_{hBH(\zeta)}}, \tag{13}$$

$$-\psi_{mBH} = a\zeta + b(\zeta - \frac{c}{d})exp(-d\zeta) + \frac{bc}{d}, \tag{14}$$

$$-\psi_{hBH} = (1 + \frac{2}{3}a\zeta)^{3/2} + b(\zeta - \frac{c}{d})exp(-d\zeta) + \frac{bc}{d} - 1, \tag{15}$$

with $a = 1$**,** $b = 0.667$**,** $c = 5$ **and** $d = 0.35$ **(Beljaars & Holtslag 1991; Li 2010).**

Computing the diagnostic U, T and q

$$\overline{V}(z_{10}) = \frac{\alpha(z_{10}) - \psi_{mBH}(\zeta_{10})}{\alpha(z) - \psi_{mBH}(\zeta)}\overline{V}(z), \tag{16}$$

$$\zeta_{10} = \zeta\frac{10}{z}.$$

$$\Delta\overline{\theta}(z_2) = \frac{\alpha(z_2) + \beta - \psi_{hBH}(\zeta_2)}{\alpha(z) + \beta - \psi_{hBH}(\zeta)}\Delta\overline{\theta}(z), \tag{17}$$

$$\Delta\overline{q}(z_2) = \frac{\alpha(z_2) + \beta - \psi_{hBH}(\zeta_2)}{\alpha(z) + \beta - \psi_{hBH}(\zeta)}\Delta\overline{q}(z), \tag{18}$$

In (17) and (18) are $\Delta\overline{\theta}(z) = \overline{\theta}(z) - \overline{\theta}_s$ and $\Delta\overline{q}(z) = \overline{q}(z) - \overline{q}_s$ provided by the model and $\Delta\overline{\theta}(z_2) = \overline{\theta}(z_2) - \overline{\theta}_s$ and $\Delta\overline{q}(z_2) = \overline{q}(z_2) - \overline{q}_s$, where $\overline{\theta}_s$ and $\overline{q}_s$ are surface skin values (Woetmann Nielsen 2020).

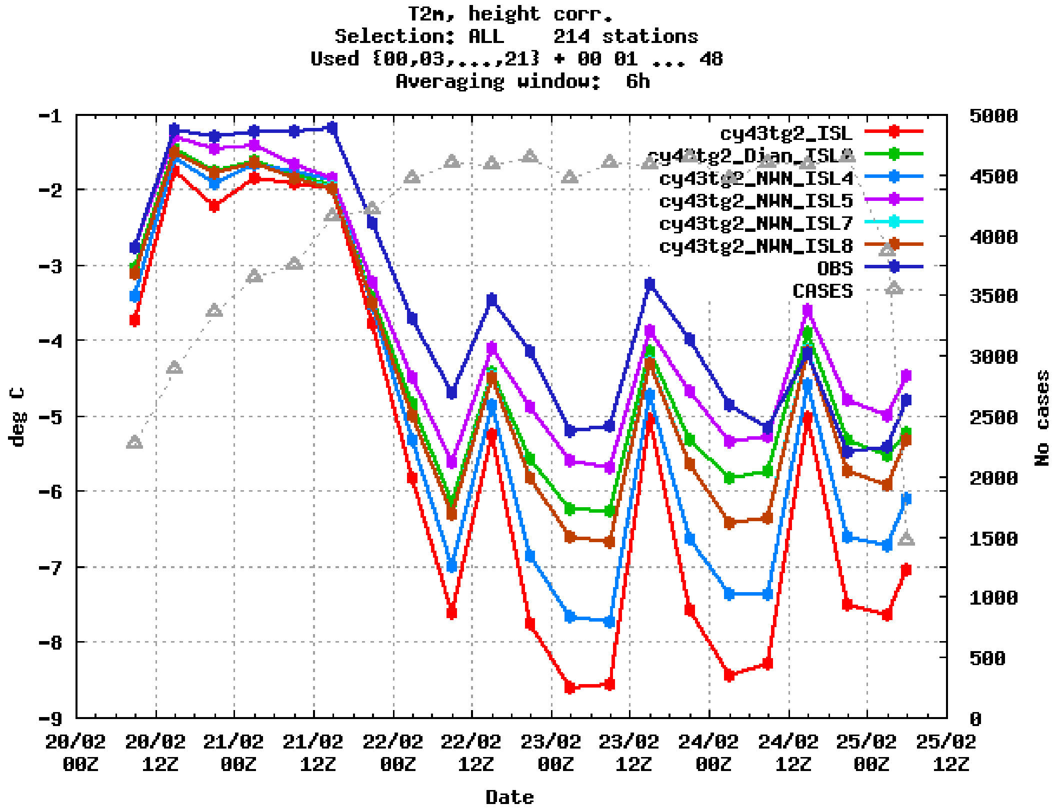
Summing up the proposed method

- **An analytical solution for the stability parameter ζ is proposed.**
- **From this u_*, θ_*, V_{10m}, ΔT_{2m} and Δq_{2m} can be calculated.**
- **This can be done without the use of surface exchange and drag coefficients.**

Challenge #1: Computation of the surface Richardson number

- **In SURFEX (.../surface_ri.F90) the bulk Richardson number for the surface layer is computed from the virtual temperature.**
- **Thus, it is affected by the surface "skin" humidity.**
- **In the MEB part of SURFEX and elsewhere, the bulk Richardson number for the surface layer is computed from the potential temperature, and is thus unaffected by the skin humidity.**

Challenge #2: Choosing proper test cases



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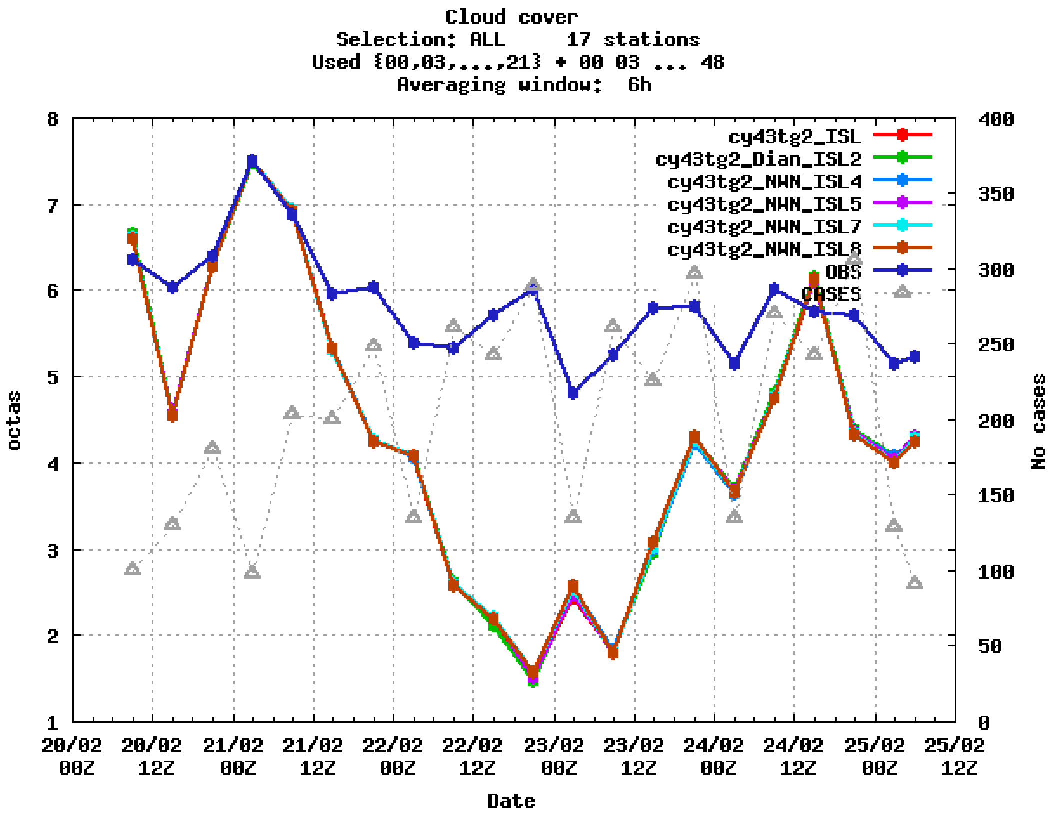

Challenge #3: A consistent implementation of the method is needed

Conclusions (so far. . .)

- **We question the use of a surface layer bulk Richardson number based on virtual temperatures.**
- **Care should be taken to study cloud free cases only when trying to optimize the stable surface layer parametrization.**
- **The new scheme needs to be tested with a consistent implementation.**

Thank you for your attention!