Mesoscale Atmospheric Predictability

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http://www2.uibk.ac.at/meteo
http://www.zamg.ac.at/workshop2004/

Outline

1 **Predictability**: error growth

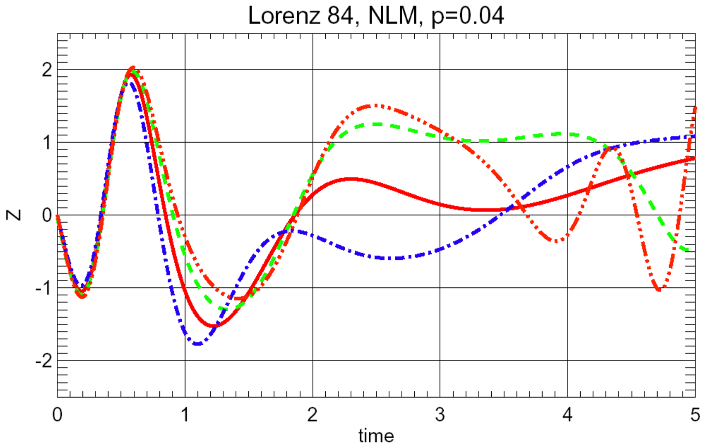
2 **Global Models:** doubling times

3 **Singular Vectors:** assessing growth

4 **Mesoscale Studies:** moist physics

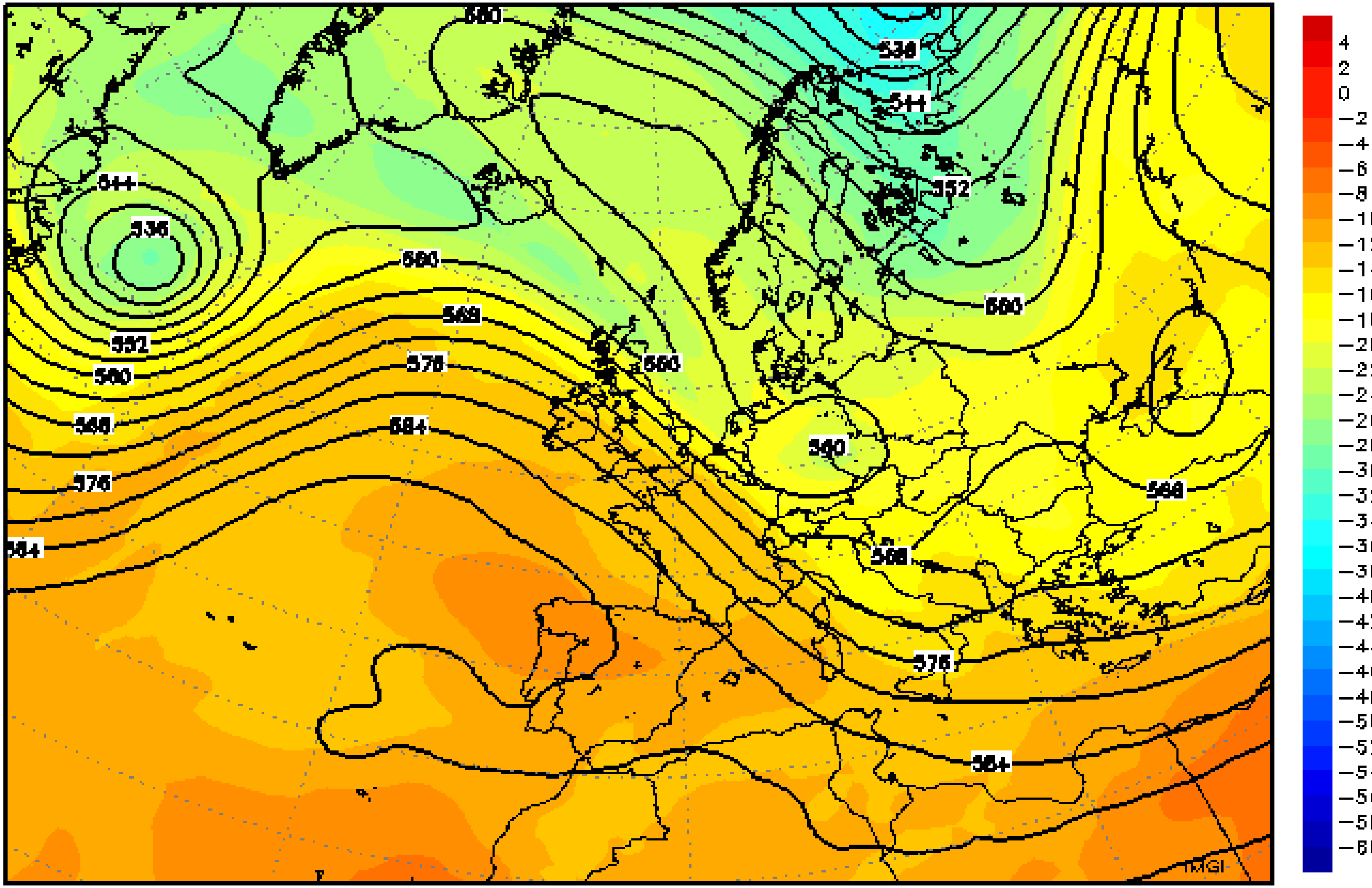
5 **Ensemble Prediction:** sampling

6 **Conclusions**



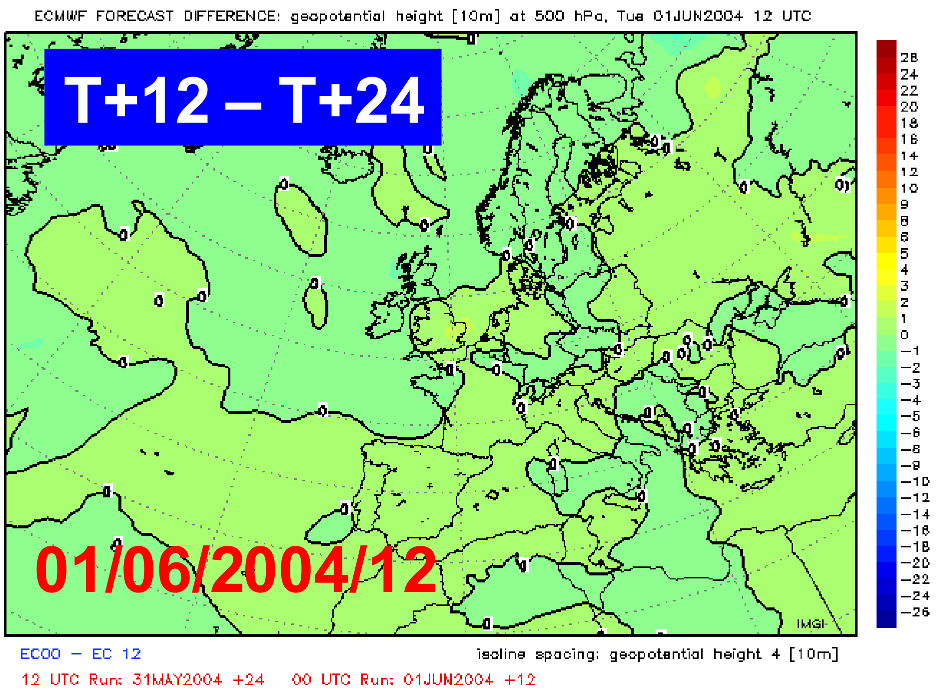
1

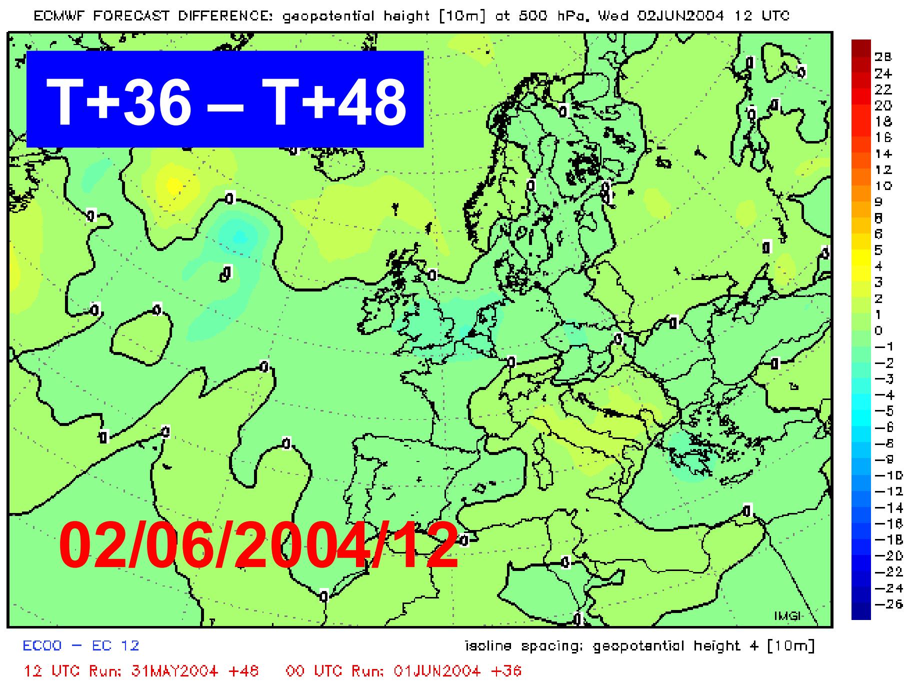
ECMWF FORECAST: geopotential height [10m], temperature [C] at 500 hPa, Wed 02JUN2004 00 UTC



12 UTC Run: 31MAY2004 +36 iso spacing: temperature 2 [C], shaded
geopotential height 4 [10m]

T+36; 02/06/2004/00


ECMWF FORECAST DIFFERENCE: geopotential height [10m] at 500 hPa, Tue 01JUN2004 12 UTC
T+12 – T+24
01/06/2004/12
ECOO – EC 12
isoline spacing: geopotential height 4 [10m]
12 UTC Run: 31MAY2004 +24   00 UTC Run: 01JUN2004 +12


ECMWF FORECAST DIFFERENCE: geopotential height [10m] at 500 hPa, Wed 02JUN2004 12 UTC
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02/06/2004/12
ECOO – EC 12
isoline spacing: geopotential height 4 [10m]
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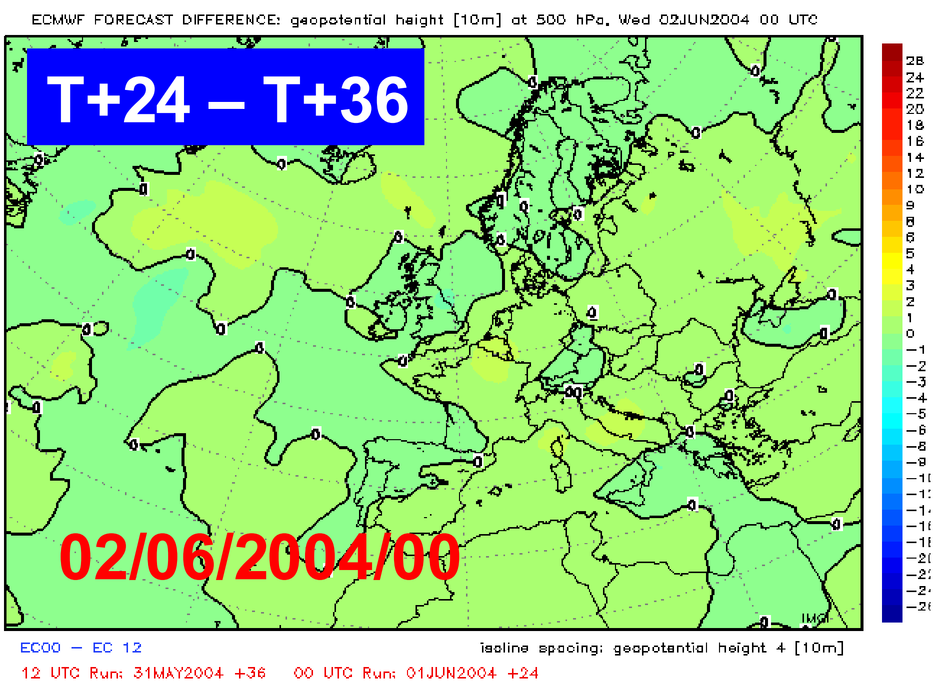
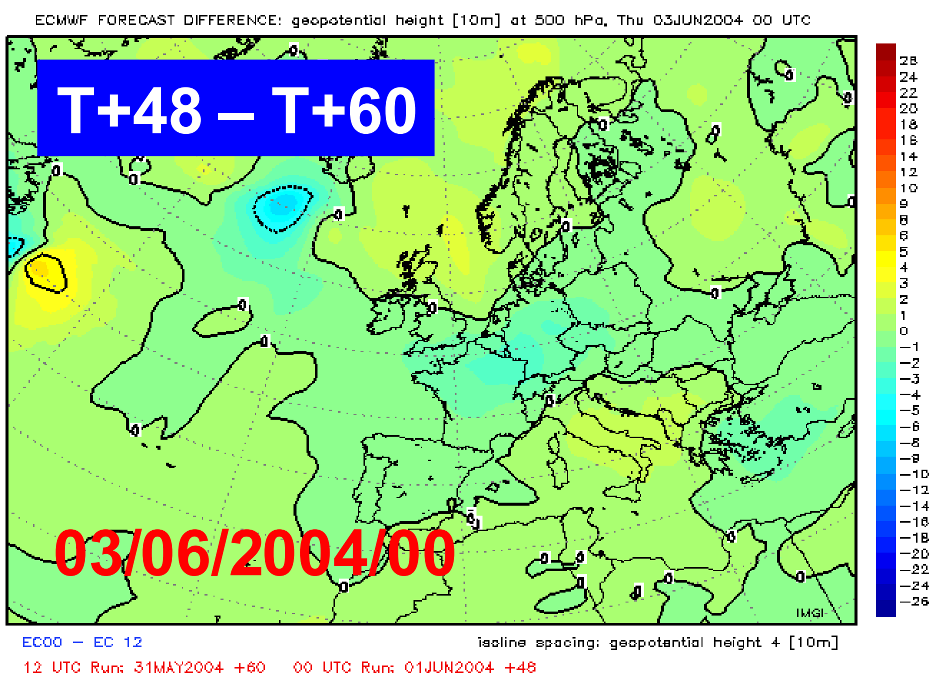
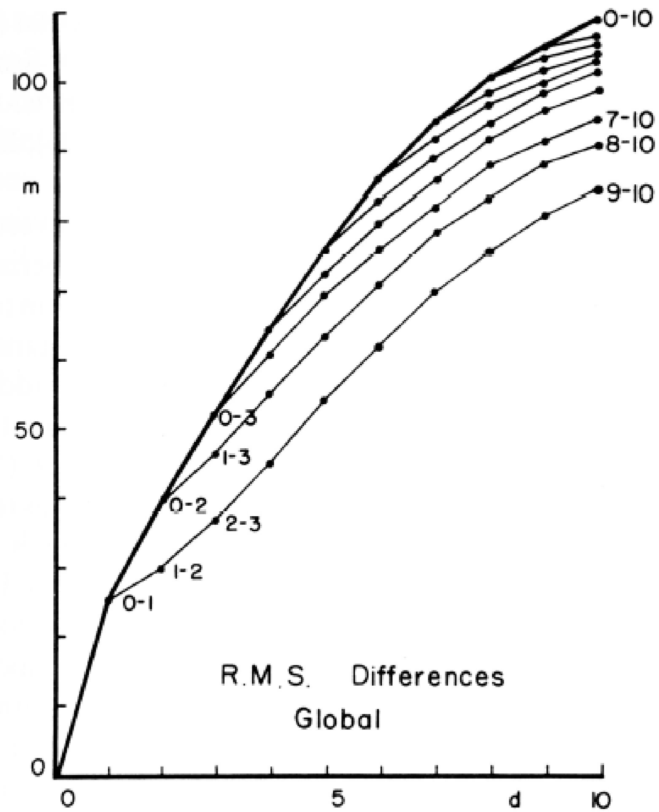

ECMWF FORECAST DIFFERENCE: geopotential height [10m] at 500 hPa, Wed 02JUN2004 00 UTC
T+24 – T+36
02/06/2004/00
ECOO – EC 12
isoline spacing: geopotential height 4 [10m]
12 UTC Run: 31MAY2004 +36   00 UTC Run: 01JUN2004 +24


ECMWF FORECAST DIFFERENCE: geopotential height [10m] at 500 hPa, Thu 03JUN2004 00 UTC
T+48 – T+60
03/06/2004/00
ECOO – EC 12
isoline spacing: geopotential height 4 [10m]
12 UTC Run: 31MAY2004 +60   00 UTC Run: 01JUN2004 +48



rms of difference between forecasts verifying at same time, but with initial states lagging by 24 hours (48 hours, ...)

→ growth of 1-day, 2-day, ... forecast errors

forecast differences grow slower than difference between model and analysis (= forecast error)

Figure 1: Error growth curves from Lorenz (1982).

- **intrinsic error growth**
- **chaotic: to extent to which model and atmosphere correspond**

Description: Error growth model Lorenz (1982)

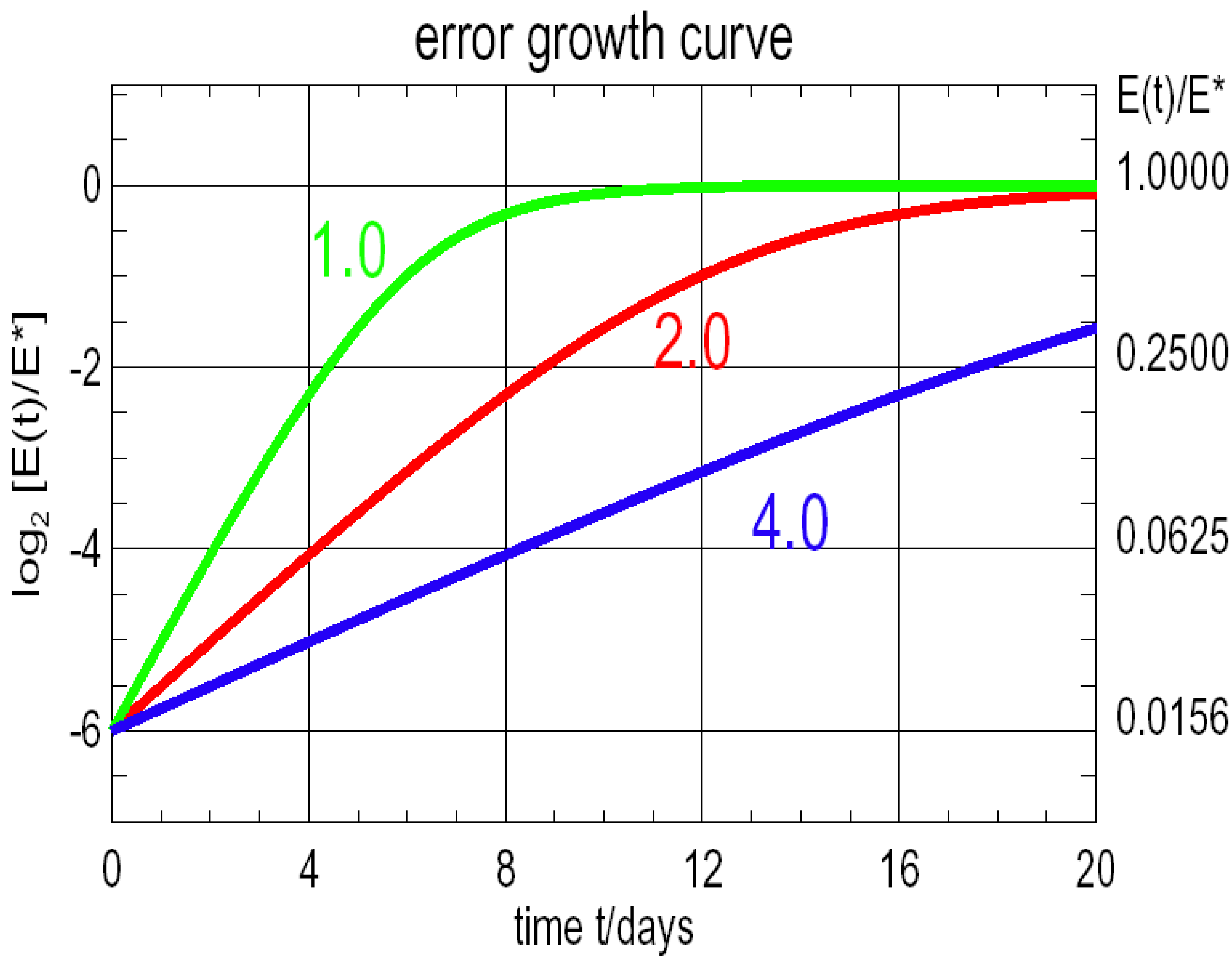
- E^* rms saturation error, E rms error
- doubling time $\tau_d = \alpha^{-1} \ln 2$

$$\boxed{\frac{1}{E}\frac{dE}{dt} = \alpha \frac{E^* - E}{E^*}} \tag{1}$$

with solution

$$E(t) = \frac{E^* \left(1 + \tanh\left[\frac{\alpha}{2}(t - t_0)\right]\right)}{\frac{E^*}{E_0}\left(1 - \tanh\left[\frac{\alpha}{2}(t - t_0)\right]\right) + 2\tanh\left[\frac{\alpha}{2}(t - t_0)\right]} \tag{2}$$

- Cutting initial state error in half $\rightarrow$ adding one doubling time in range of predictability (in exponential regime)
- $$E^* = \sqrt{2}\sigma_{clim}$$



solution with:

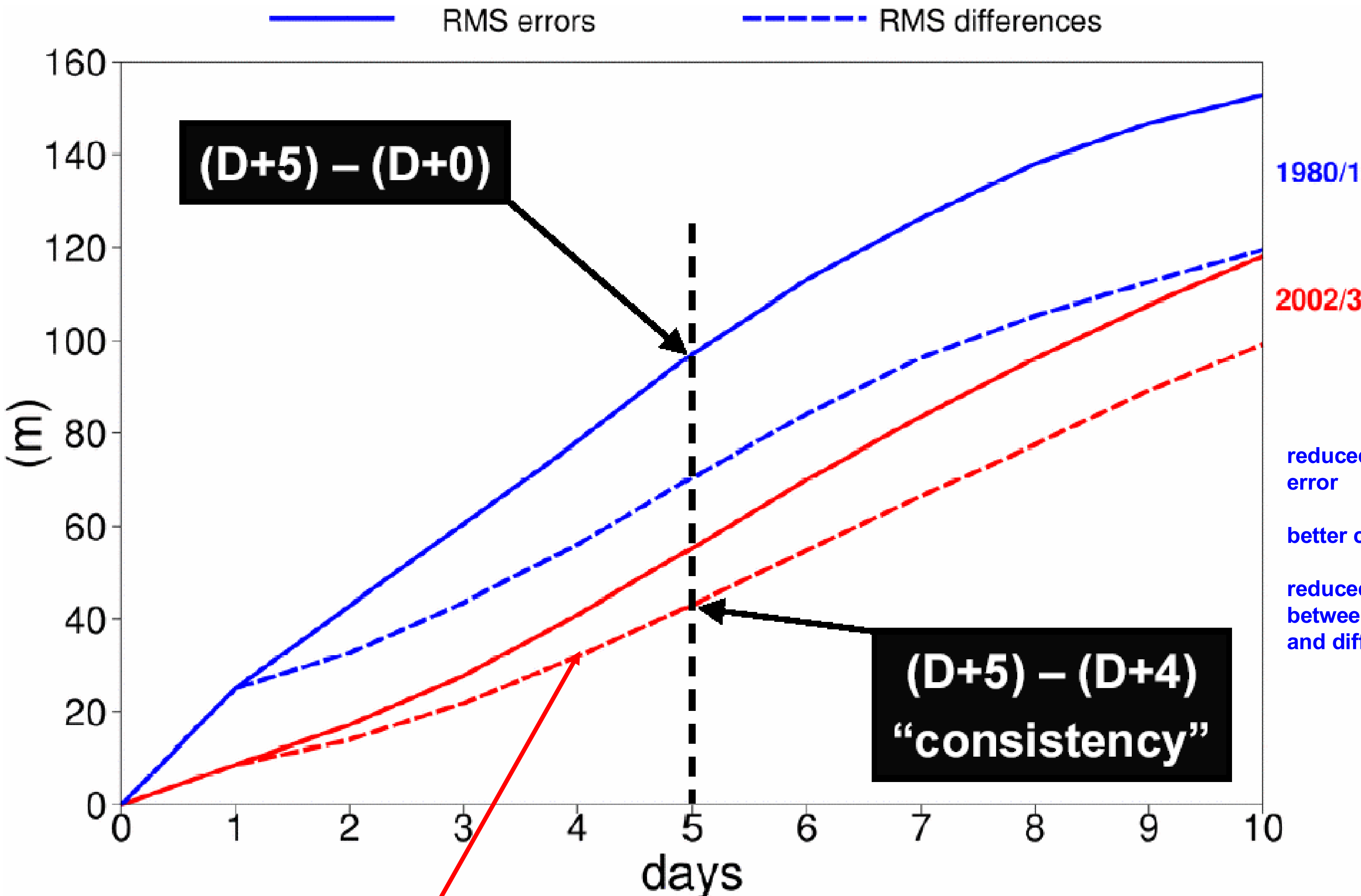
$\frac{E_0}{E^*} = 2^{-6}$

τ_d=1/2/4 days

$\rightarrow \alpha = \tau_d^{-1} \ln 2$

note:
exponential
growth
for $E \ll E^*$

R.m.s. errors and differences between successive forecasts
Northern hemisphere 500hPa height Winter



reduced D+1 error

better consistency

reduced gap between error and difference

A. Simmons, ECMWF

amplification of 1-day forecast error, 1.5 days

nonlinearity of dynamics

and

instability with respect to small perturbations

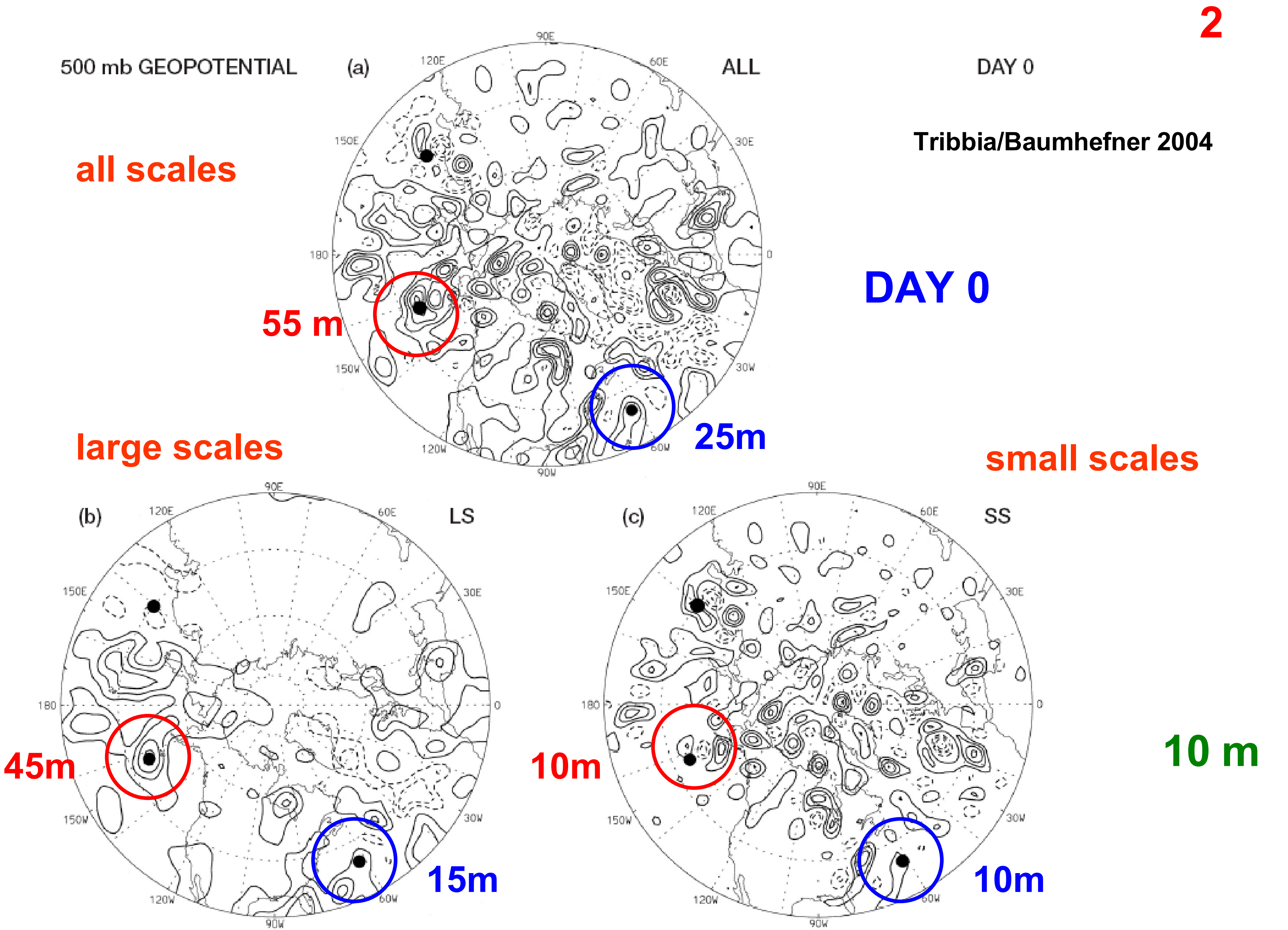
→

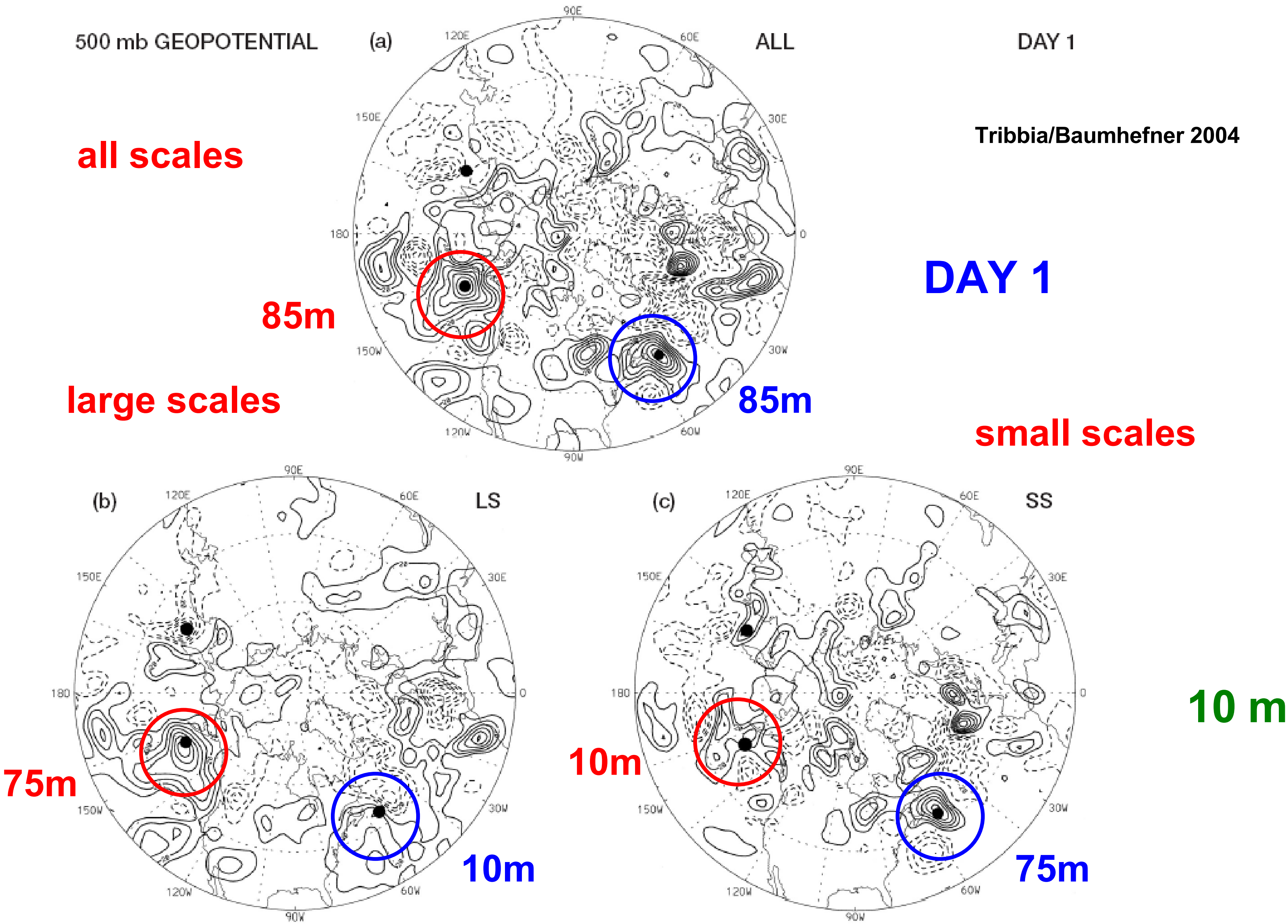
sensitive dependence on present condition

chaos

irregularity and nonperiodicity

unpredictability and error growth


2
500 mb GEOPOTENTIAL
(a)
ALL
DAY 0
Tribbia/Baumhefner 2004
all scales
55 m
DAY 0
25m
large scales
small scales
(b)
LS
45m
15m
(c)
SS
10m
10 m
10m
90E
120E
60E
150E
30E
180
0
150W
30W
120W
60W
90W

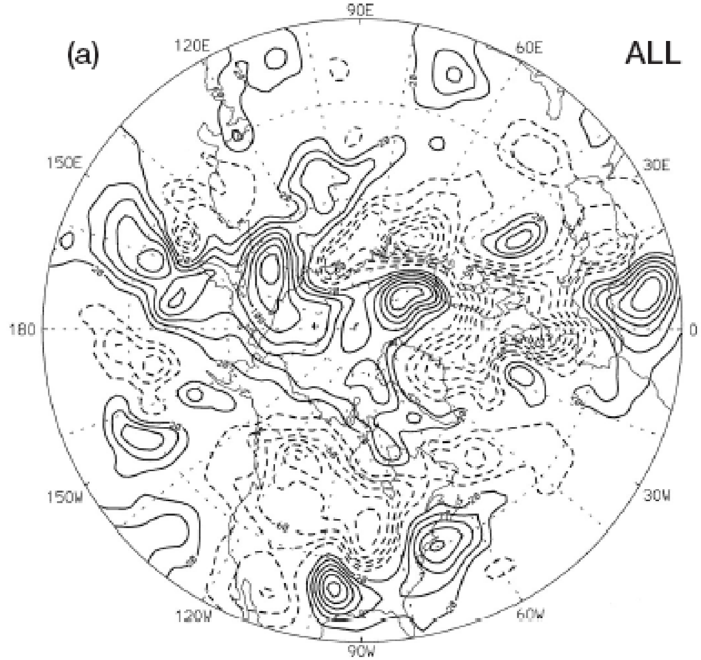

500 mb GEOPOTENTIAL
(a)
ALL
DAY 1
Tribbia/Baumhefner 2004
all scales
85m
85m
DAY 1
large scales
small scales
(b)
LS
(c)
SS
75m
10m
10m
75m
10 m

500 mb GEOPOTENTIAL

DAY 3

Tribbia/Baumhefner 2004

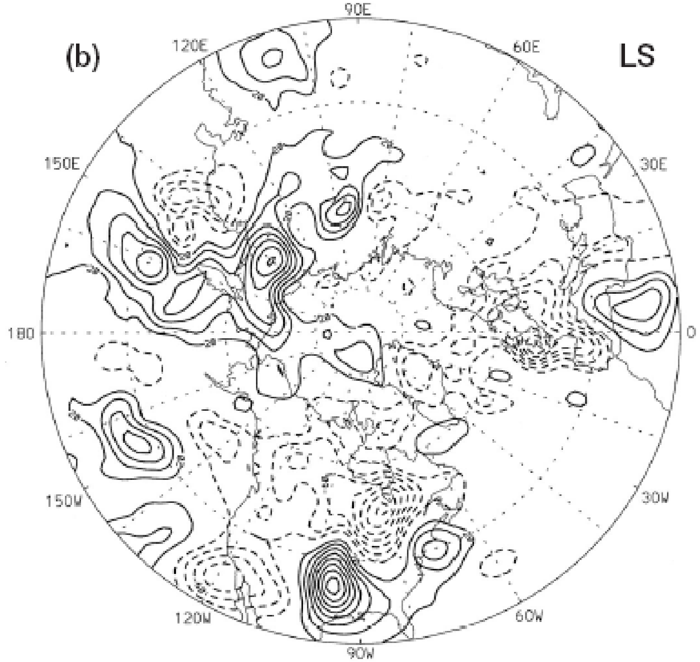
all scales

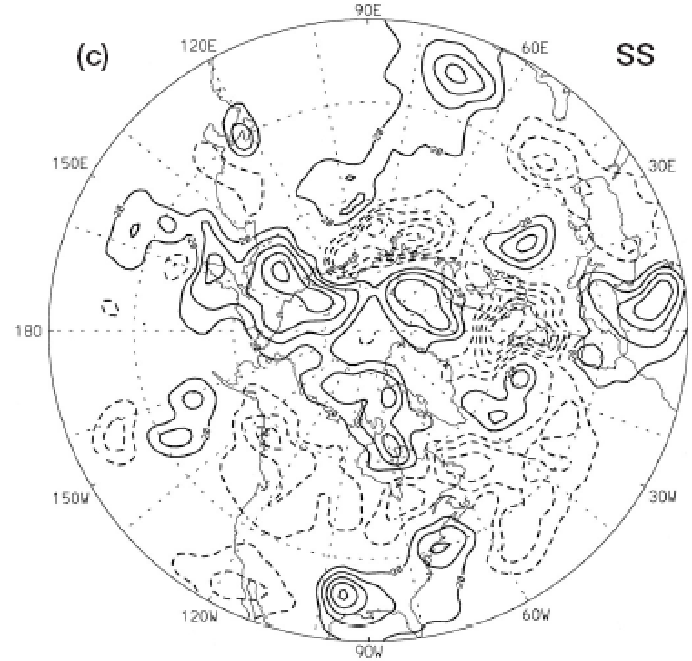


DAY 3

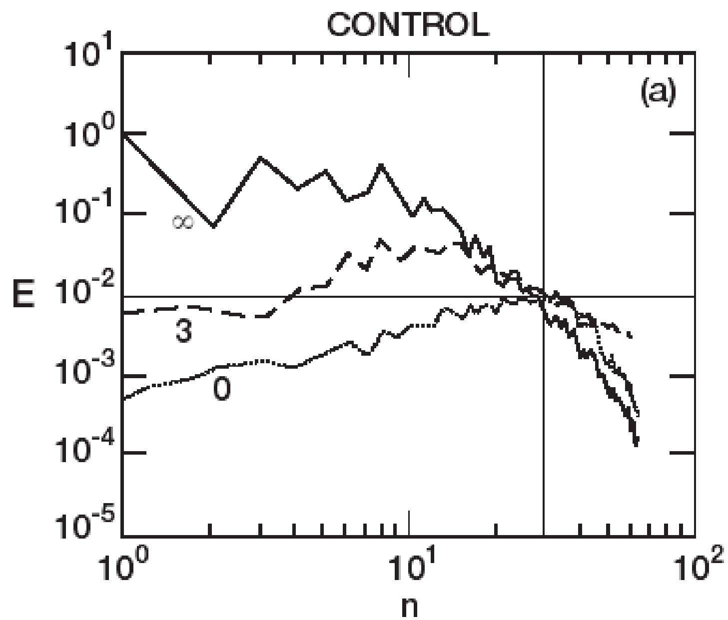
large scales

small scales



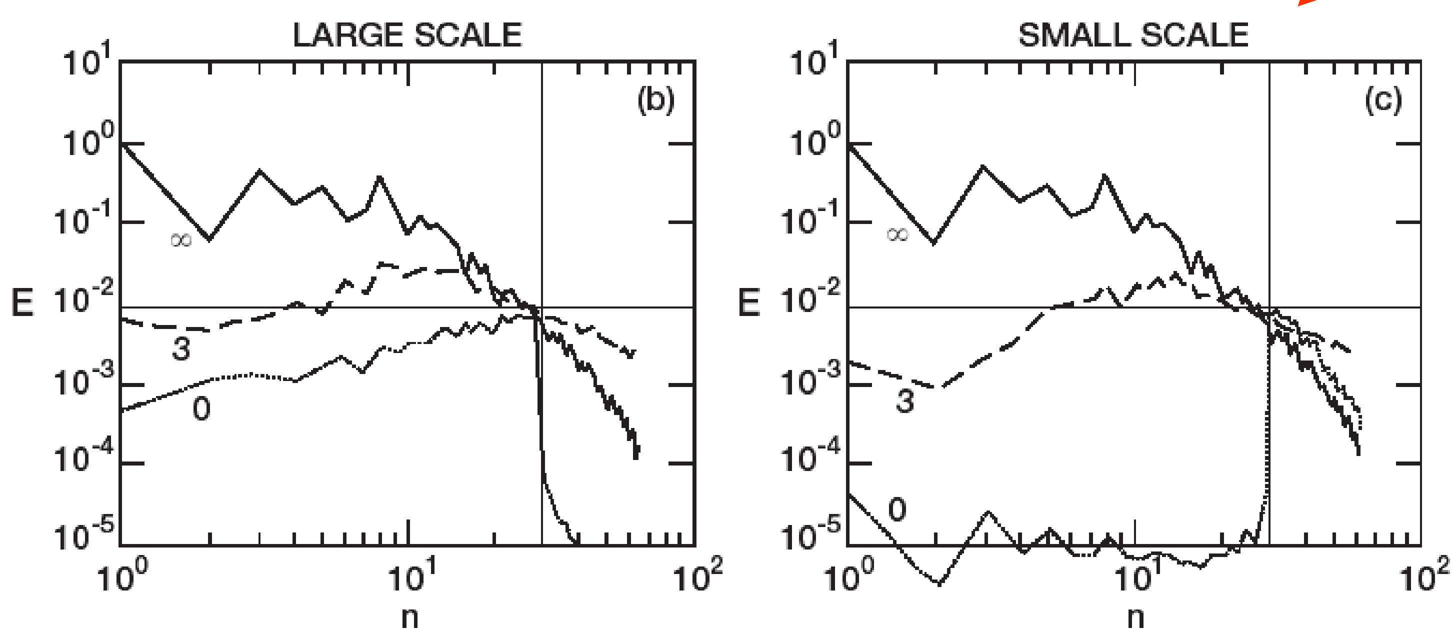


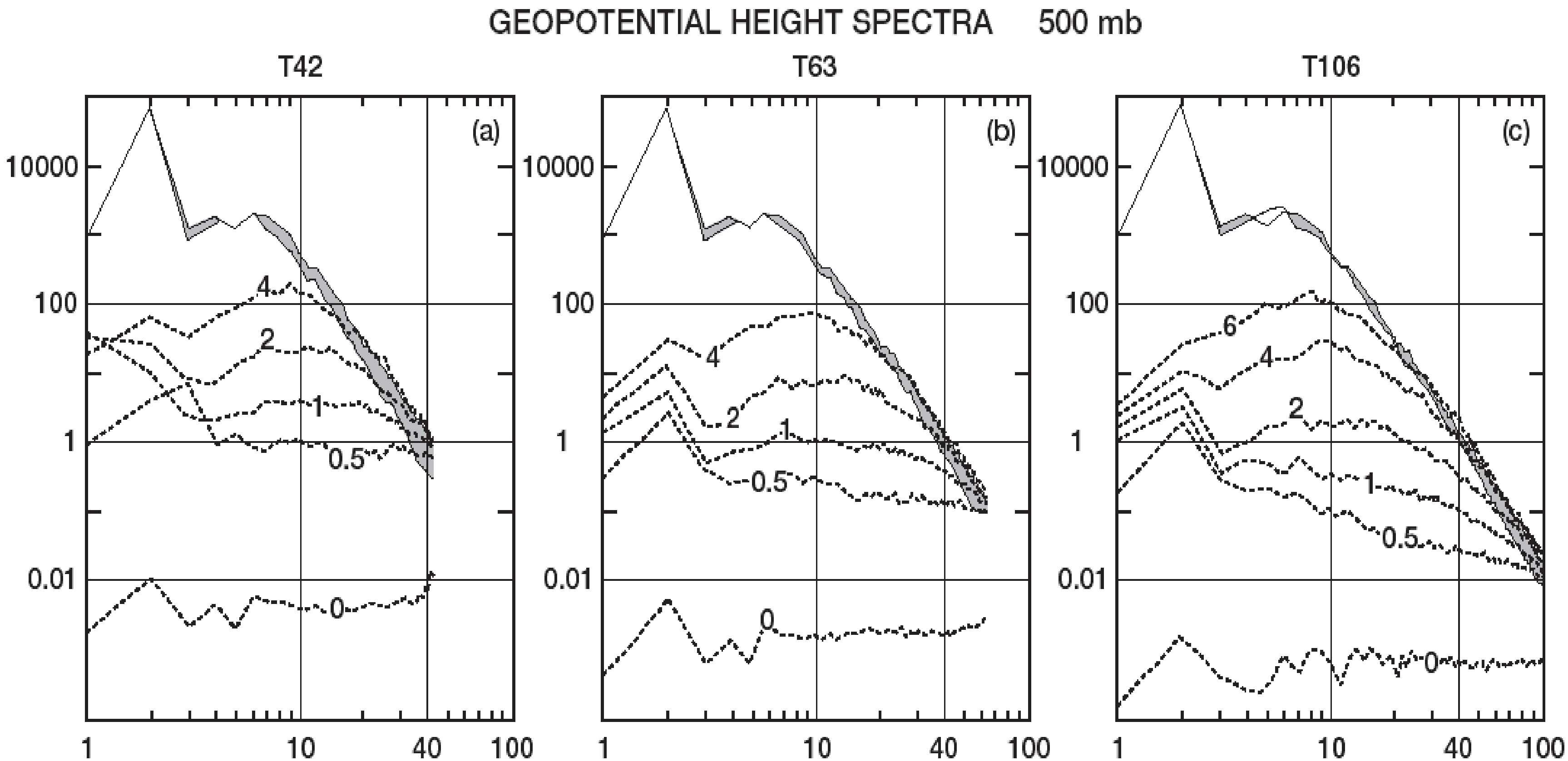
20 m



similarity of spectra at day 3
→ **spectrally local error reduction will not help**

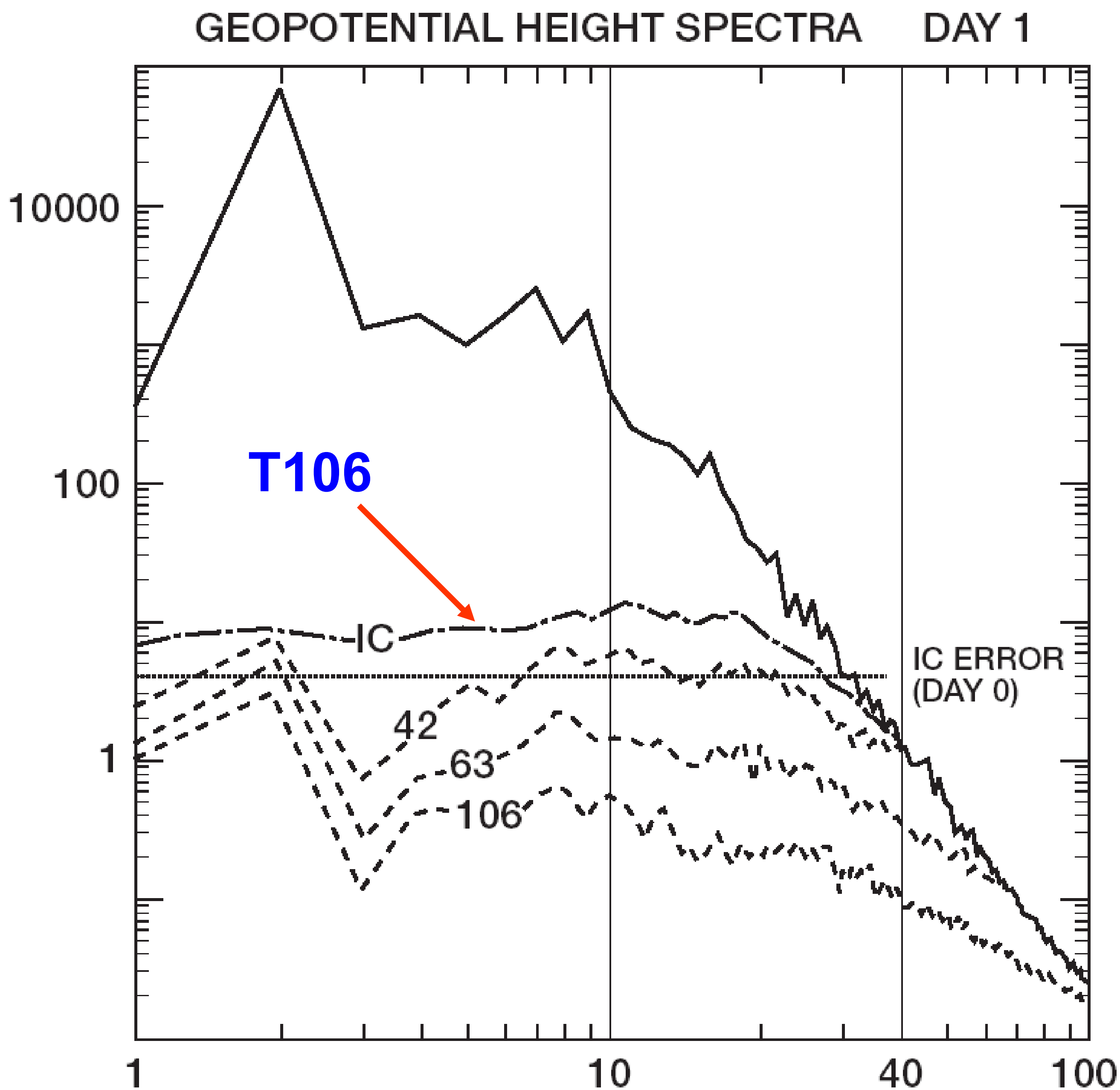
only small-scale error





error growth to due resolution differences (against T170):

D+1 error T42 = 10 x D+1 error T63 = 10 x D+1 error T106



even at T42 the D+1 truncation error growth has not exceeded D+1 IC T106 growth

T106 truncation error growth is one order of magnitude smaller than D+1 T106 IC error growth

need IC/10 before going beyond T106

[IC analysis error growth exponential]

Tribbia Baumhefner 2004

- sensitive dependence on i.c.
- preferred directions of growth

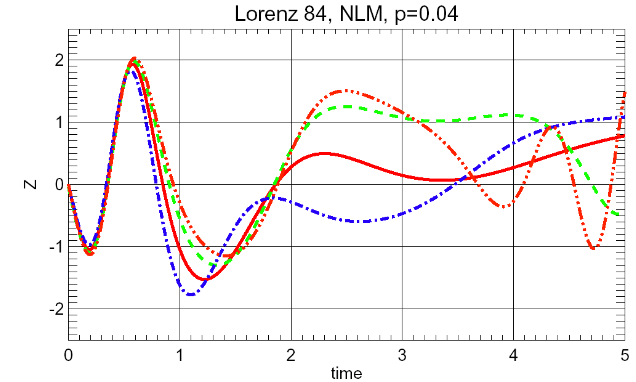
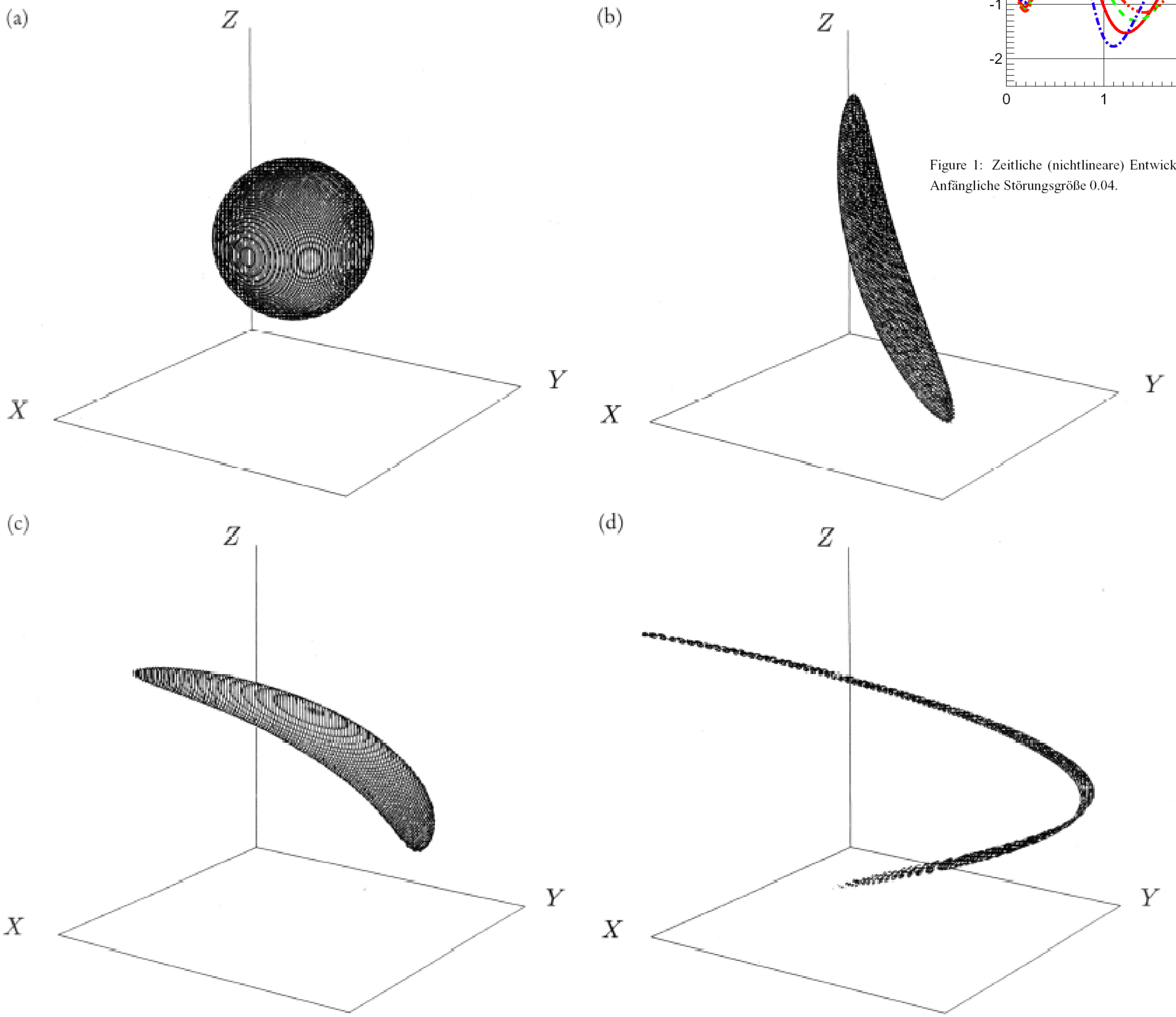


Figure 1: Zeitliche (nichtlineare) Entwicklung der Z Komponente des Lorenz (1984) Systems. Anfängliche Störungsgröße 0.04.



Lorenz 1984 model

Ehrendorfer 1997

Singular Vectors

Maximize the L2 norm: $N = \mathbf{y}'^{\dagger}\mathbf{N}\mathbf{y}'$

Given the TLM: $\mathbf{y}' = \mathbf{M}\mathbf{x}'$

Constraint: $1 = C = \mathbf{x}'^{\dagger}\mathbf{C}\mathbf{x}'$

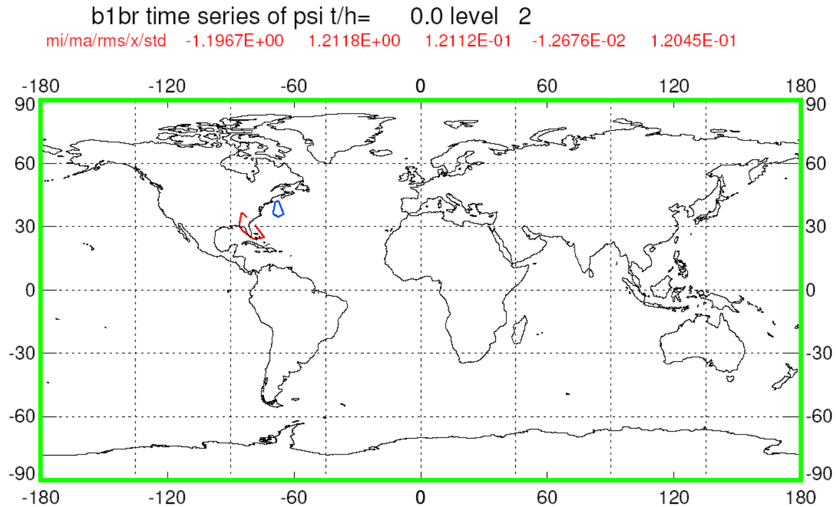
Solution:

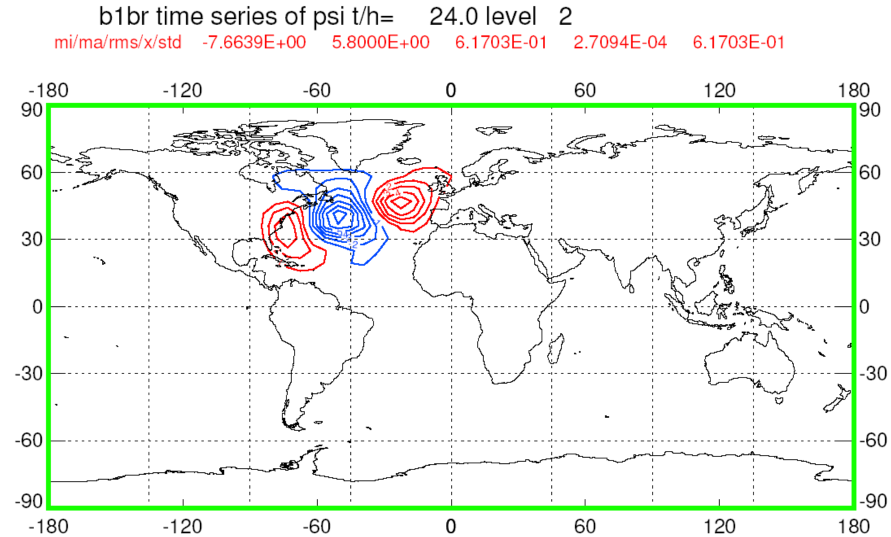
$$\mathbf{x}' = \mathbf{C}^{-\frac{1}{2}}\mathbf{z}$$

$$\lambda^2\mathbf{z} = \mathbf{C}^{-\frac{1}{2}}\mathbf{M}^{\dagger}\mathbf{N}\mathbf{M}\mathbf{C}^{-\frac{1}{2}}\mathbf{z}$$

R.M. Errico

SVs / HSVs -> fastest growing directions:
account for in initial condition
stability of the flow

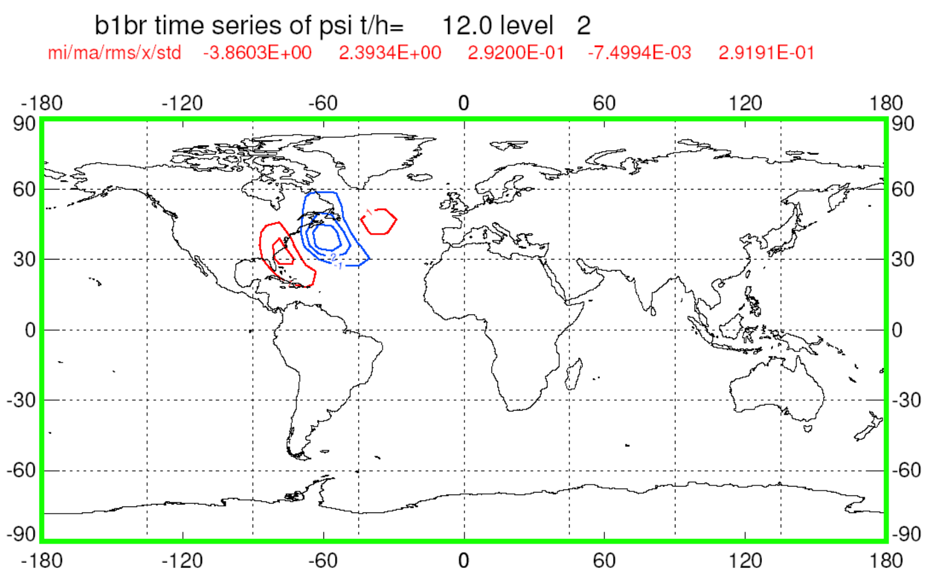


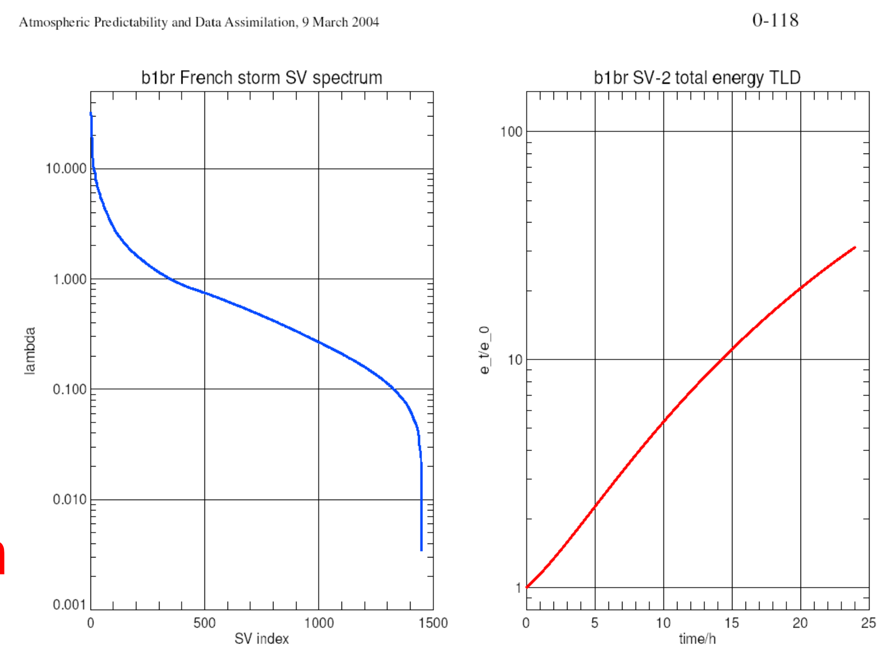


psi´ at 500 hPa

Optimized TL error growth

data assimilation, stability, error dynamics





French storm

tau_d = 4.9 h

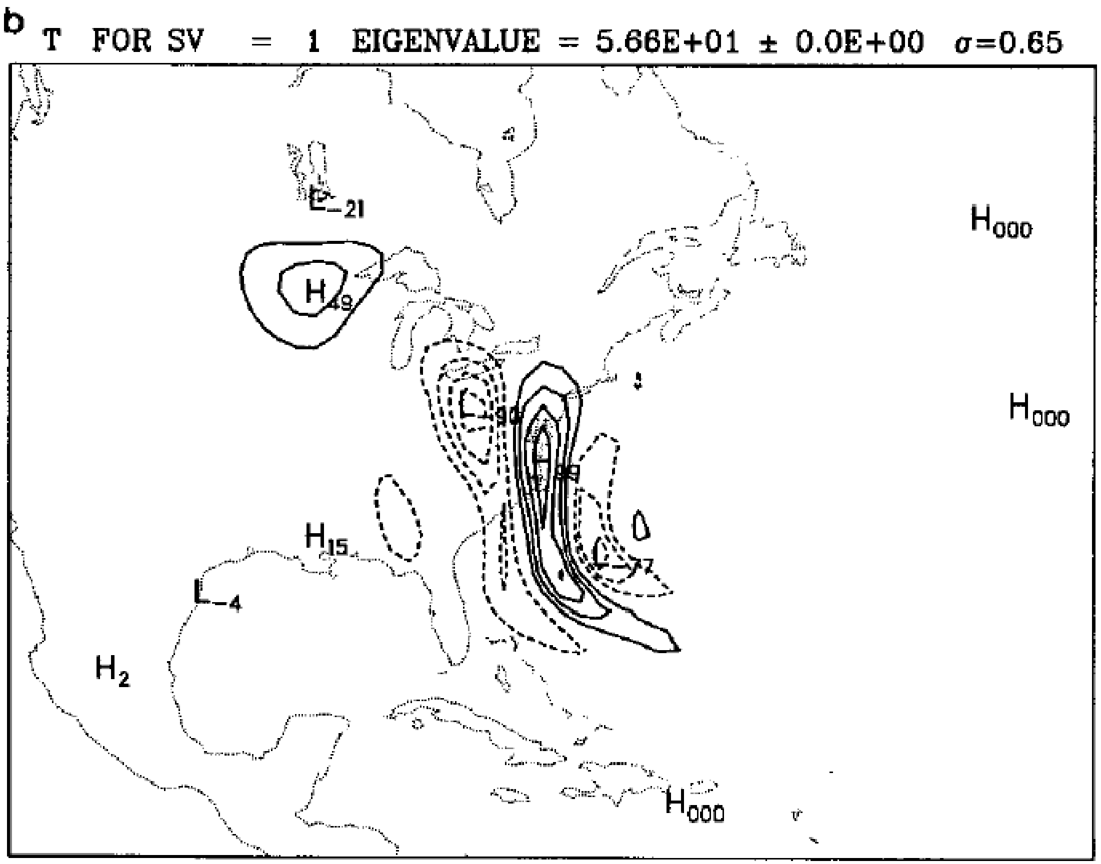
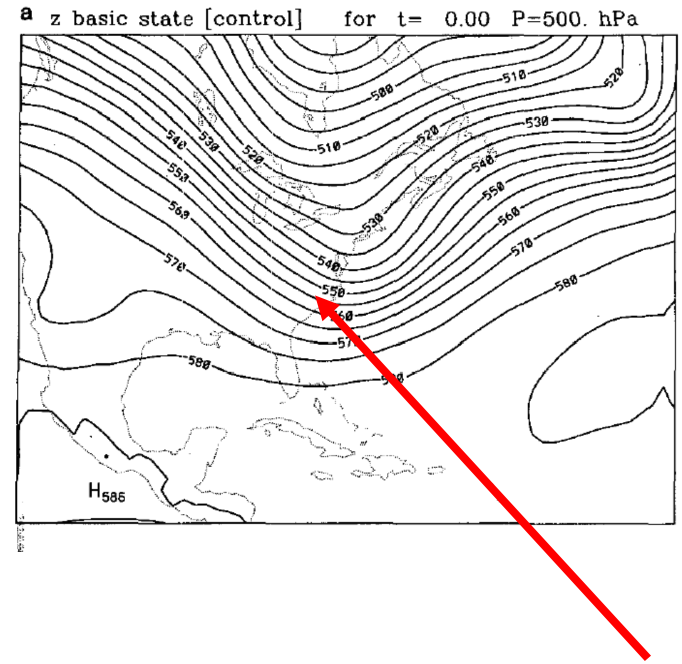


FIG. 5. First TE-norm SV for case 1 in terms of T at (a) $\sigma = 0.35$ (approximately 350 hPa) and (b) $\sigma = 0.65$ (approximately 650 hPa). Units and CIs: (a) 10^{-4} K, 0.01 K; (b) 10^{-3} K, 0.02 K. Zero contours are suppressed.



lambda = 56.6

tau_d = 4.1 h

Ehrendorfer/Errico 1995

MAMS - DRY

TE-Norm

Mesoscale Adjoint Modeling System
MAMS2

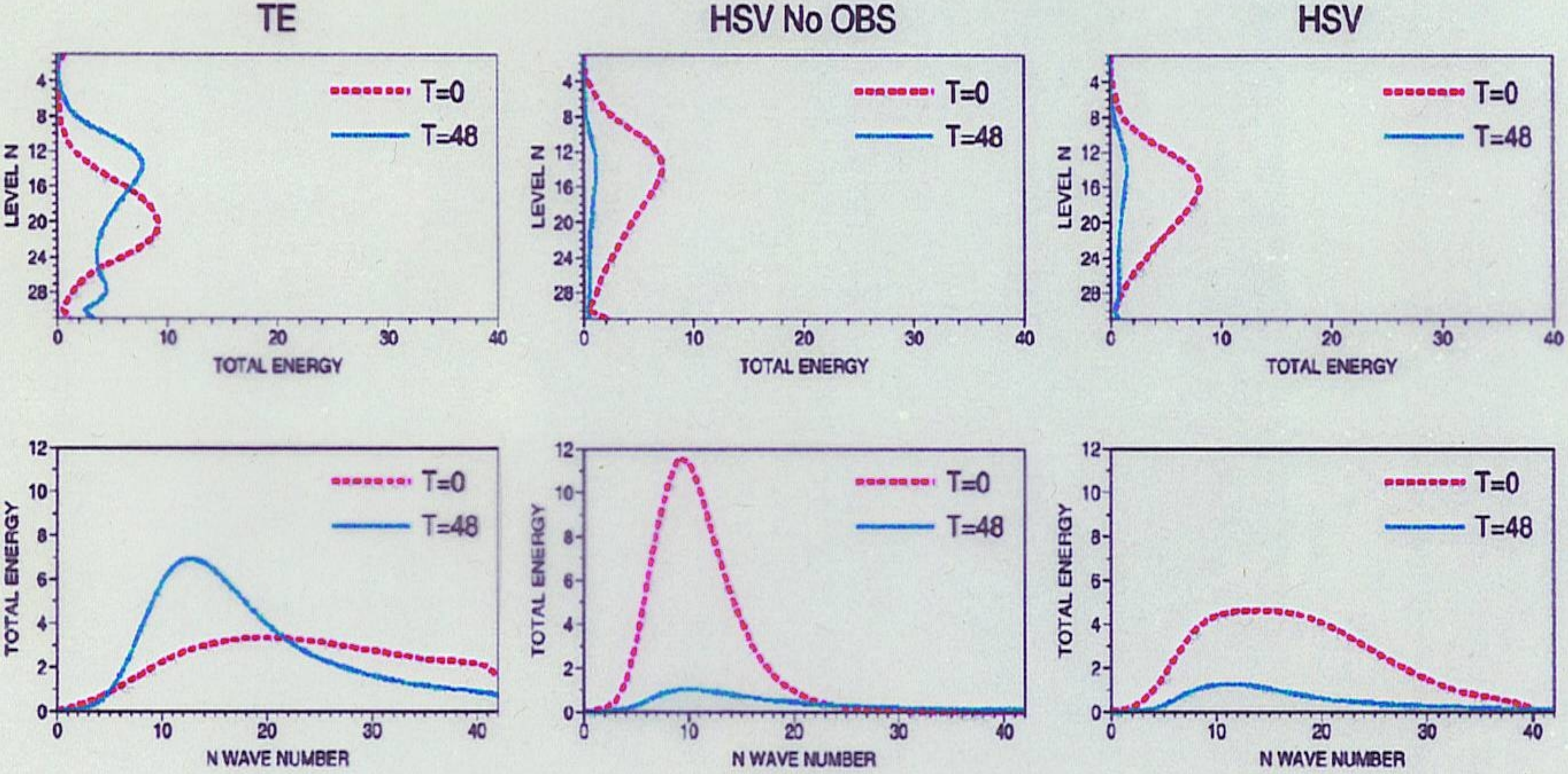
PE with water vapor (B grid)
Bulk PBL (Deardorff)
Stability-dependent vertical diffusion (CCM3)
RAS scheme (Moorthi and Suarez)
Stable-layer precipitation
Dx=80 km
20-level configuration (d sigma=0.05)
Relaxation to lateral boundary condition

12-hour optimization for SVs
4 synoptic cases

moist TLM (Errico and Raeder 1999 QJ)

Energy Profiles and Wavenumber Spectra: HSVs and TESVs

Northern Hemisphere 25 SVs Autumn 1999



Jan Barkmeijer, ECMWF

Lorenz 1969

Tribbia/Baumhefner 2004

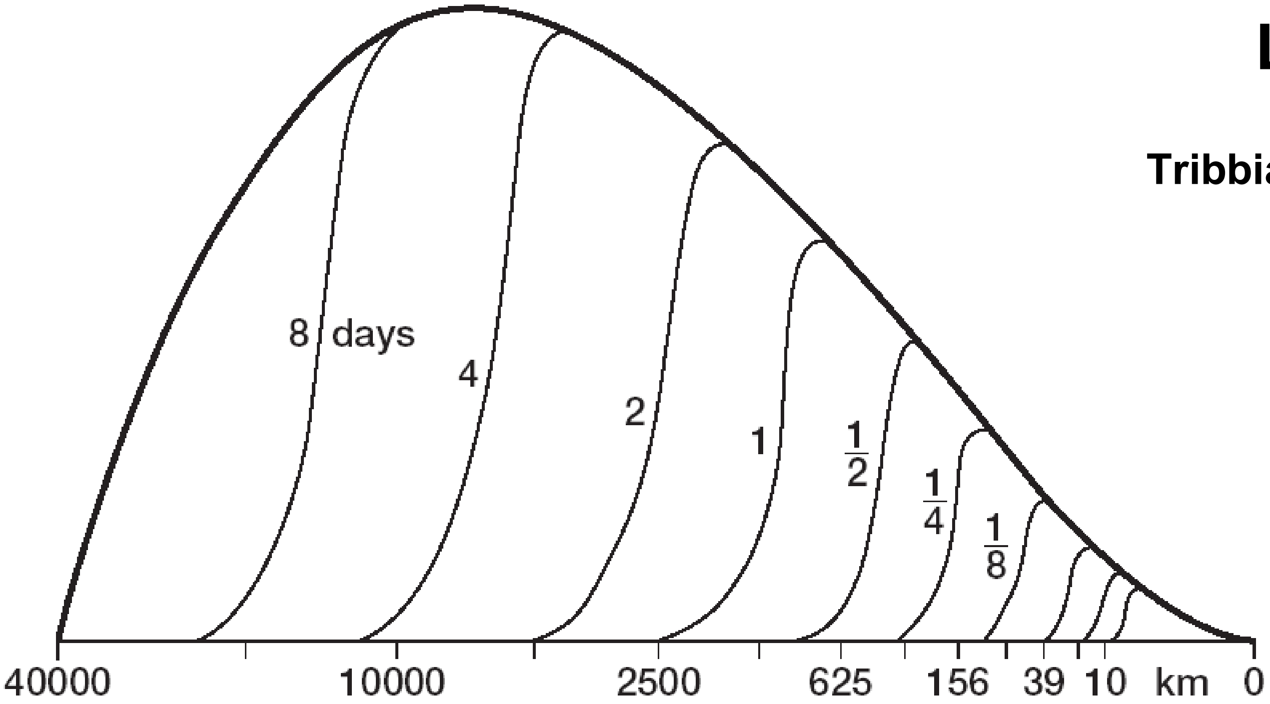


Figure 1: Growth of errors initially confined to smallest scales, according to a theoretical model (taken from a paper by E. Lorenz presented in AIP Conf. Proceedings #106). Horizontal scales on bottom; full atmospheric motion spectrum = upper curve.

errors in small scales propagate upscale … in spectral space

small-scale errors grow and … contaminate … larger scale

Adjoint Sensitivity Analysis
The Problem

Given a differentiable scalar measure $J = J(\mathbf{y})$

and a model $\mathbf{y} = \mathcal{M}(\mathbf{x})$

determine $\partial J / \partial x_i$

grad_x J = M^T grad_y J

such that $J' = \sum_i \frac{\partial J}{\partial x_i} x_i'$

approximates $\Delta J = J(\mathbf{x} + \mathbf{x}') - J(\mathbf{x})$

The solution is:

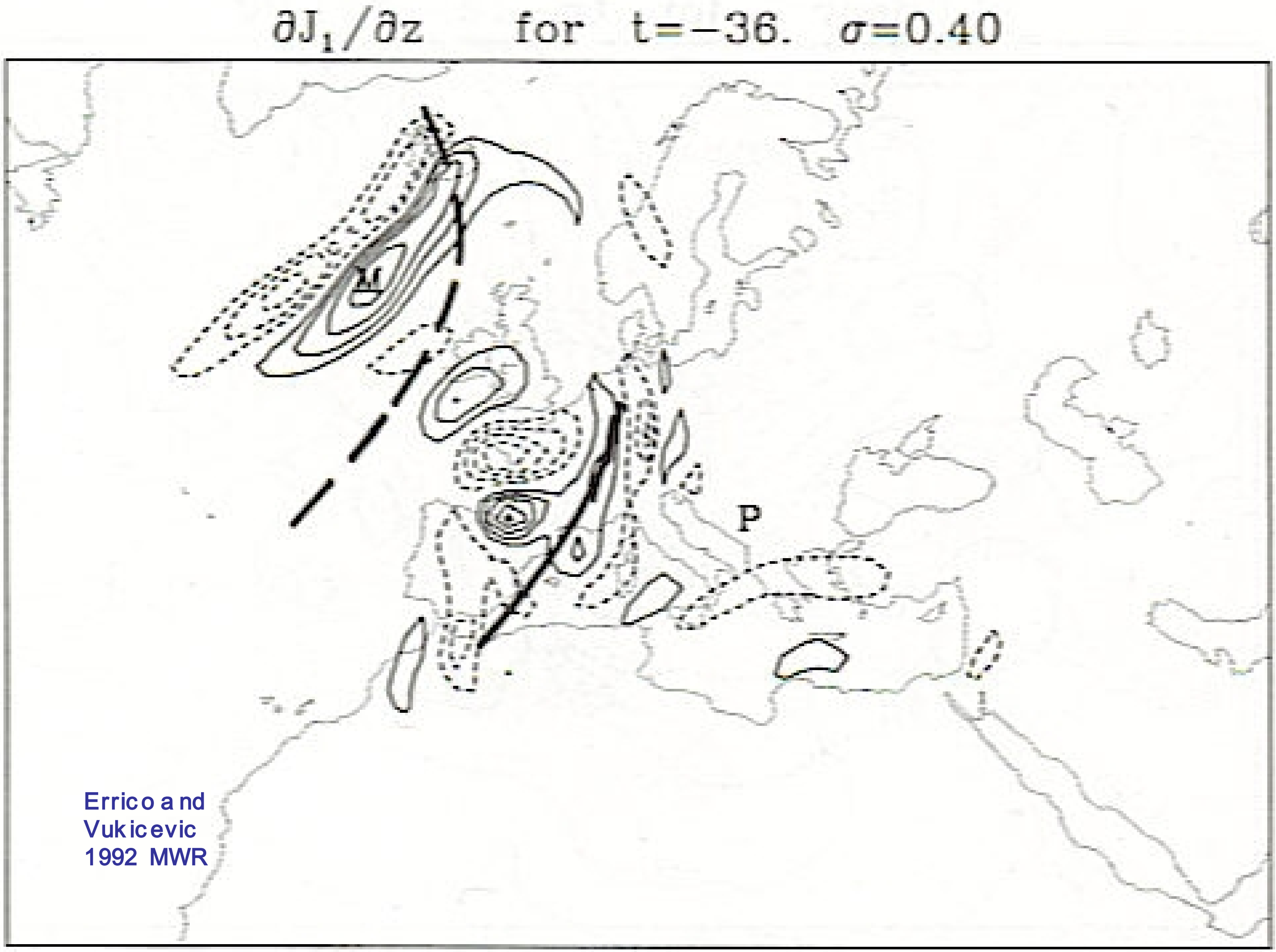
$$\frac{\partial J}{\partial x_i} = \sum_j \frac{\partial y_j}{\partial x_i} \frac{\partial J}{\partial y_j}$$

M

Contrast with: $J' = \sum_i \frac{\partial J}{\partial y_i} y_i' \qquad y_i' = \sum_j \frac{\partial y_i}{\partial x_j} x_j'$

R.M. Errico

Example Sensitivity Field



Contour interval 0.02 Pa/m   M=0.1 Pa/m

36-h sensitivity of surface pressure at P to Z-perturb.

10m at M → 1 Pa at P

4 MOIST PHYSICS

Errico et al. 2004 QJ

initial- and final-time norms

E -> E

E_m -> E

V_d -> E

V_d -> P

V_m -> E

V_m -> P

Norms Considered

Energy Norm:

E
$$E = \frac{1}{N_w}\sum_{i,j,k}\Delta\sigma_k\left(u'^2_{i,j,k} + v'^2_{i,j,k}\right) + \frac{C_p}{T_r N_t}\sum_{i,j,k}\Delta\sigma_k T'^2_{i,j,k} + \frac{RT_r}{p^2_{sr} N_t}\sum_{i,j} p'^2_{s\,i,j} \quad (1)$$

Moist Energy Norm (dry fields zero):

E_m
$$E_m = \frac{L^2}{C_p T_r N_t}\sum_{i,j,k}\Delta\sigma_k q'^2_{i,j,k} \quad (2)$$

Precip. Rate Norm (used only as end–time norm; non–convective + convective precip.):

P
$$P = \frac{1}{N_t}\sum_{i,j} R'^2_{t\,i,j} \quad (3)$$

Dry and Moist Variance–weighted norm (penalize large q high up):

V_d
$$V_d = \frac{1}{N_w}\sum_{i,j,k}\Delta\sigma_k\left(\frac{u'^2_{i,j,k}}{V_{u\ k}} + \frac{v'^2_{i,j,k}}{V_{v\ k}}\right) + \frac{1}{N_t}\sum_{i,j,k}\Delta\sigma_k\frac{T'^2_{i,j,k}}{V_{T\ k}} + \frac{1}{N_t}\sum_{i,j}\frac{p'^2_{s\ i,j}}{V_p} \quad (4)$$

V_m
$$V_m = \frac{1}{N_t}\sum_{i,j,k}\frac{q'^2_{i,j,k}}{V_{q\ k}}\Delta\sigma_k \quad (5)$$

A larger value of E can be produced with an initial constraint V_m=1 compared with V_d=1.

(hypothetical norm comparison:
“larger E with E_m=1 compared with V_d=1“)

MAMS - MOIST

t_d = 2.5 h

Norm	W1	W2	S1	S2
$E \rightarrow E$	26.9	50.6	25.9	59.3
$E_m \rightarrow E$	27.8	28.8	4.9	39.3
$V_d \rightarrow E$	51.6	135.0	121.0	60.2
$V_d \rightarrow P$	1.5	3.9	2.6	2.9
$V_m \rightarrow E$	241.0	162.0	149.0	201.0
$V_m \rightarrow P$	0.8	4.8	4.0	9.1

t_d = 2 h

moisture perturbations more effective than dry perturbations to maximize E

larger than E_m-> E and V_d->E

has no vertical scaling

Doubling time: t_d= OTI * ln 2 / ln λ

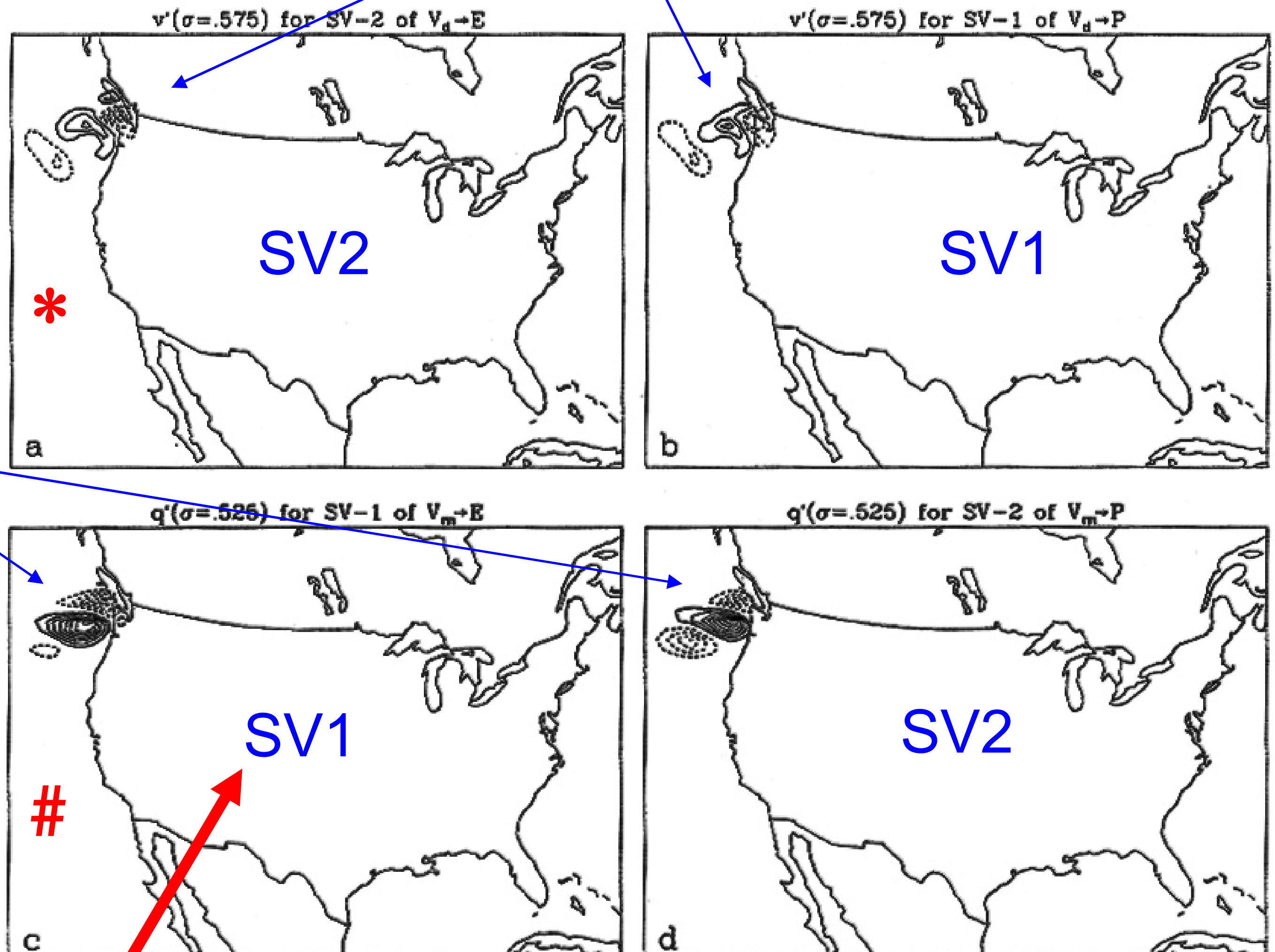
initial time
case S2

-> E   r= 0.81   -> P

V_d ->   v´

r=0.76

V_m ->   q´



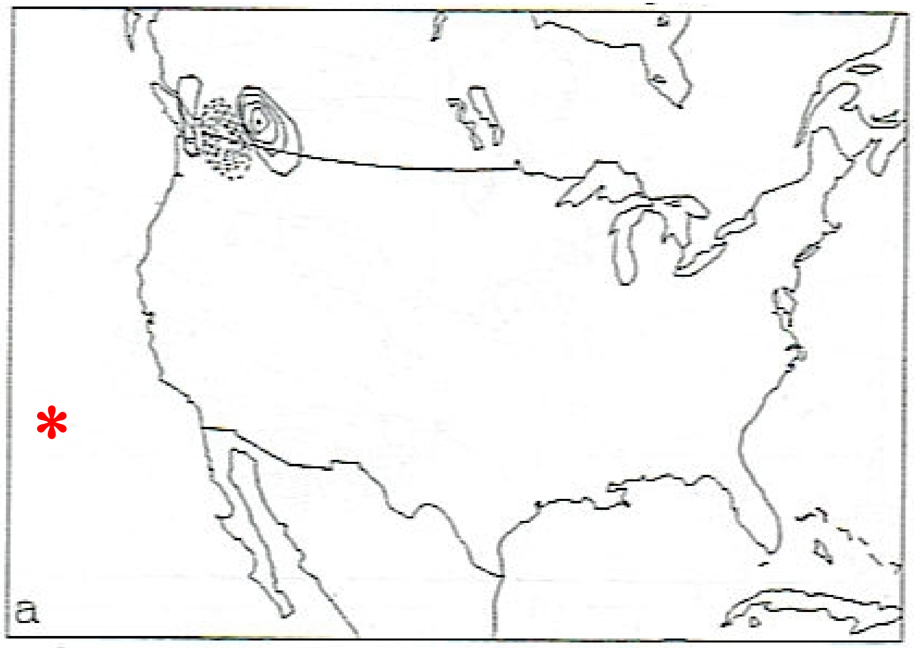
highly correlated with
T´ of SV2 for V_d -> E

large r: similar structures are optimal for maximizing both E and P

Perturbations in Different Fields Can Produce the Same Result

12-hour v TLM forecasts

Initial u, v, T, ps Perturbation



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Initial q Perturbation

#

Errico et al.
QJRMS 2004

H=c_p T + L q

condensational heating

summarizing comments on moist-norm SV-study:

- moisture perturbations alone may achieve larger E than dry perturbations

- given same initial constraint, similar structures can be optimal for maximizing E and P
  in most cases however: structures are different

- dry-only and moist-only SVs may lead to nearly identical final-time fields (inferred dependence on H);
  q converts to T (diabatic heating) through nonconvective precipitation

[- nonlinear relevance: TLD vs NLD may match closely (2 g/kg)]

[- sensitivity of non-convective precipitation not universally dominant]

5 Ensemble Prediction

- generate perturbations from (partial) knowledge of analysis error covariance P^a

- methodology on the basis of SV

- “SV-based sampling technique“

Uncertainty Prediction: Hessian SVs

The HSVs Z_0 solving the eigenvector problem:

$$\boxed{\mathsf{M}^{\mathrm{T}}\mathsf{C}^{\mathrm{T}}\mathsf{C}\mathsf{M}\mathsf{Z}_0 = (\mathsf{P}^{\mathsf{a}})^{-1}\mathsf{Z}_0\Lambda \qquad \text{s.t.} \quad \mathsf{Z}_0^{\mathrm{T}}(\mathsf{P}^{\mathsf{a}})^{-1}\mathsf{Z}_0 = \mathsf{I}} \tag{5.0.1}$$

are, when time–evolved, eigenvectors of P^{f}, because:

$$\underbrace{\left(\mathsf{C}\,\mathsf{M}\mathsf{P}^{\mathsf{a}}\right)\mathsf{M}^{\mathrm{T}}}_{\equiv \mathsf{P}^{\mathsf{f}}}\,\mathsf{C}^{\mathrm{T}}\,\underbrace{\mathsf{C}\mathsf{M}\mathsf{Z}_0}_{\equiv \mathsf{Z}_t} = \left(\mathsf{C}\mathsf{M}\mathsf{P}^{\mathsf{a}}\right)(\mathsf{P}^{\mathsf{a}})^{-1}\mathsf{Z}_0\Lambda \quad \rightarrow \quad \boxed{\left(\mathsf{C}\mathsf{P}^{\mathsf{f}}\mathsf{C}^{\mathrm{T}}\right)\mathsf{Z}_t = \mathsf{Z}_t\Lambda} \tag{5.0.2}$$

The evolved HSVs Z_t *are the eigenvectors of* $\mathsf{C}\mathsf{P}^{\mathsf{f}}\mathsf{C}^{\mathrm{T}}$ *– which is the forecast error covariance in the "final–time norm"* C. Note the final time orthogonality relationship:

$$\mathsf{Z}_t^{\mathrm{T}}\mathsf{Z}_t = \left(\mathsf{C}\mathsf{M}\mathsf{Z}_0\right)^{\mathrm{T}}\left(\mathsf{C}\mathsf{M}\mathsf{Z}_0\right) = \mathsf{Z}_0^{\mathrm{T}}\,\underbrace{\mathsf{M}^{\mathrm{T}}\mathsf{C}^{\mathrm{T}}\mathsf{C}\mathsf{M}\mathsf{Z}_0}_{} = \overbrace{\mathsf{Z}_0^{\mathrm{T}}(\mathsf{P}^{\mathsf{a}})^{-1}\mathsf{Z}_0}^{}\Lambda = \Lambda \quad \rightarrow \quad \boxed{\mathsf{Z}_t^{\mathrm{T}}\mathsf{Z}_t = \Lambda} \tag{5.0.3}$$

Uncertainty Prediction: The SV–Decomposition of $\mathsf{P^a}$

- Because the initial time SVs satisfy (5.0.1), it is true that $\mathsf{P^a}$ can be written as:

$$\mathsf{P^a} = \mathsf{Z_0 Z_0^T} \tag{5.0.4}$$

  This is a special square root for P^a (different from eigendecomposition and also not lower–triangular) $\rightarrow$ the **SV–decomposition of $\mathsf{P^a}$**

- Under linear dynamics this SV decomposition becomes the eigendecomposition of the forecast error covariance matrix, because (5.0.5) is the same as (5.0.2) together with (5.0.3):

$$\left(\mathsf{CM}\right)\mathsf{P^a}\left(\mathsf{CM}\right)^{\mathrm{T}} = \left(\mathsf{CM}\right)\mathsf{Z_0 Z_0^T}\left(\mathsf{CM}\right)^{\mathrm{T}} \quad \rightarrow \quad \boxed{\mathsf{CP^f C^T} = \mathsf{Z_t Z_t^T}} \tag{5.0.5}$$

- SV–decomposition implemented at ECMWF for generation of initial–time perturbations in the Ensemble–Prediction–System (only partly operational)

Multinormal Sampling Based on SV–Decomposition of P^a

- Transforming random variables

$$\mathbf{q} \sim \mathcal{N}(0, \mathsf{I}) \qquad \Rightarrow \qquad \mathbf{x} = \mathbf{x}_0^c + \mathsf{V}^{1/2}\mathbf{q} \qquad \rightarrow \qquad \mathbf{x} \sim \mathcal{N}(\mathbf{x}_0^c, \mathsf{V}) \tag{5.0.6}$$

- Use **SV–decomposition of** P^a (possibly truncated to N SVs) in (5.0.5) – to describe square–root of P^a – in process of generating initial–time perturbed states $\mathbf{x}$:

$$(\mathsf{P}^a)^{1/2} = \mathsf{Z}_0{}^{(N)} \tag{5.0.7}$$

$$\mathbf{x}_i = \mathbf{x}_0^c + \mathsf{Z}_0^{(N)}\mathbf{q}_i \qquad i = 1, 2, ..., M \quad \Rightarrow \quad \boxed{\mathbf{x} \sim \mathcal{N}\left(\mathbf{x}_0^c, (\mathsf{P}^a)^{(N)}\right)} \tag{5.0.8}$$

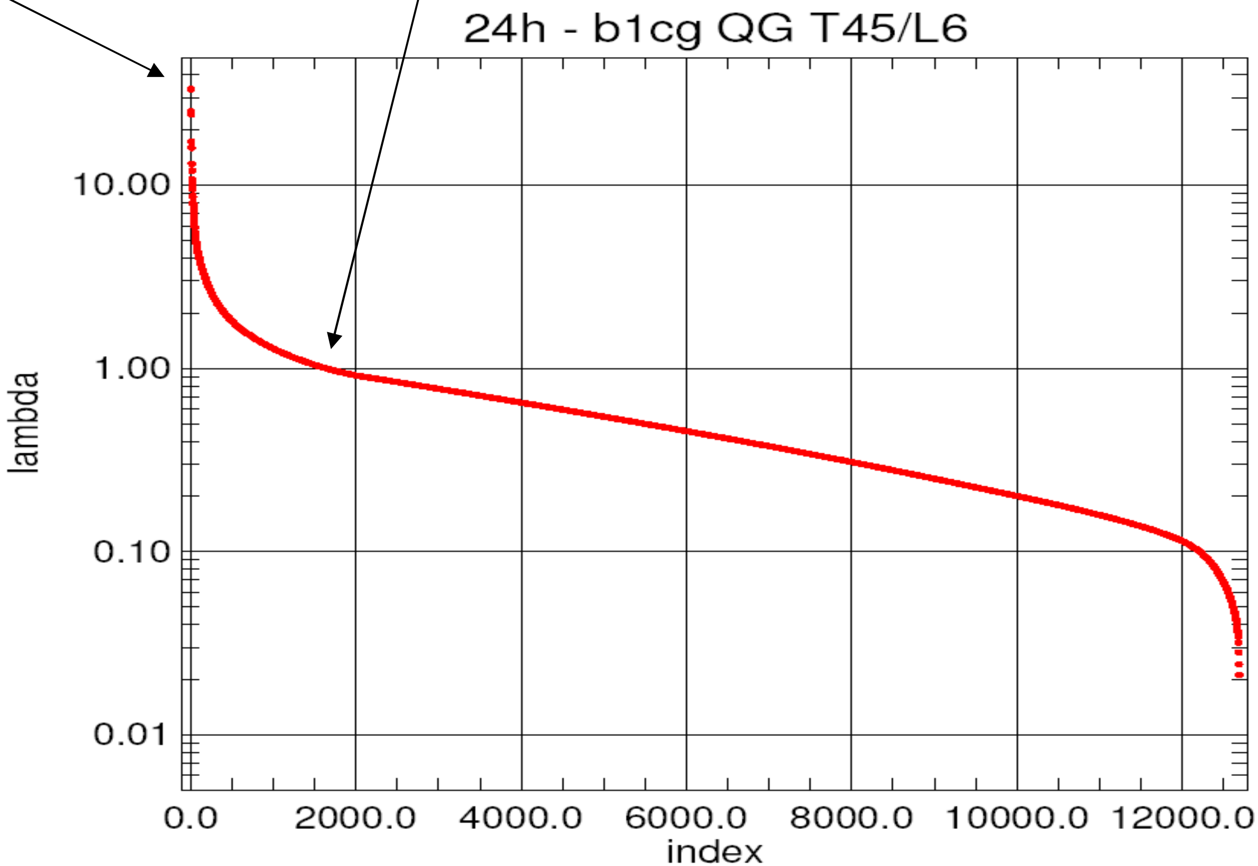
- Generating perturbations consistent with P^a knowledge based on N SVs
- Assumes normally distributed analysis errors; • non eigendecomp. $\rightarrow$ eigendecomp.
- Taking SV properties into nonlinear regime
- Strong similarity to operational *rotation* at ECMWF
- free parameters: N and M

QG TE SV spectrum

T45/L6

lambda_1=33.47

1642 = 13%



lambda= 0.0212

Figure 42: b1cg spectrum. OTI 24.0 h; $\lambda_1 = 0.334732129161647E+02$, $\lambda_{12690} = 0.212031927289761E-01$, $\lambda_{1642} = 0.100006329799706E+01$.

Table 1. *Description of experiments and some results*

Case no.	Case label	Physics	Norm	N	I	I_c	N_g	λ_1^2	$\lambda_{I_c}^2$	Sum
1	W1	dry	E	79354	1000	406	—	54.9	1.53	1552.3
2	W1	wet	E	79354	600	257	—	322.0	2.23	2387.7
3	W1	dry	R	24940	600	265	270	38.1	1.01	791.6
4	W1	dry	TR	4830	390	180	169	28.2	0.95	504.1
5	W1	wet	TR	4830	750	416	175	39.5	0.35	762.2
6	W2	dry	TR	4830	400	191	171	19.6	0.90	521.8
7	W2	wet	TR	4830	400	197	178	59.2	0.90	633.4
8	S1	dry	E	65080	600	258	—	64.9	1.53	1117.3
9	S1	wet	E	65080	600	276	—	2547.9	1.55	6627.4
10	S1	dry	TR	3969	400	213	104	20.4	0.44	381.9
11	S1	wet	TR	3969	400	211	115	368.2	0.47	1126.3
12	S2	dry	TR	3969	400	196	124	11.1	0.62	354.7
13	S2	wet	TR	3969	400	206	134	81.1	0.61	659.2

N is the size of the (sub-) space measured by the norm. I is the number of iterations of the Lanczos algorithm performed. I_c is the number of converged λ^2. N_g is the number of $\lambda_k > 1$. Sum is $\sum_{k=1}^{I_c} \lambda_k^2$.

169 SVs growing out of 4830 (dry balanced norm)

i.e. 3.5% of phase space

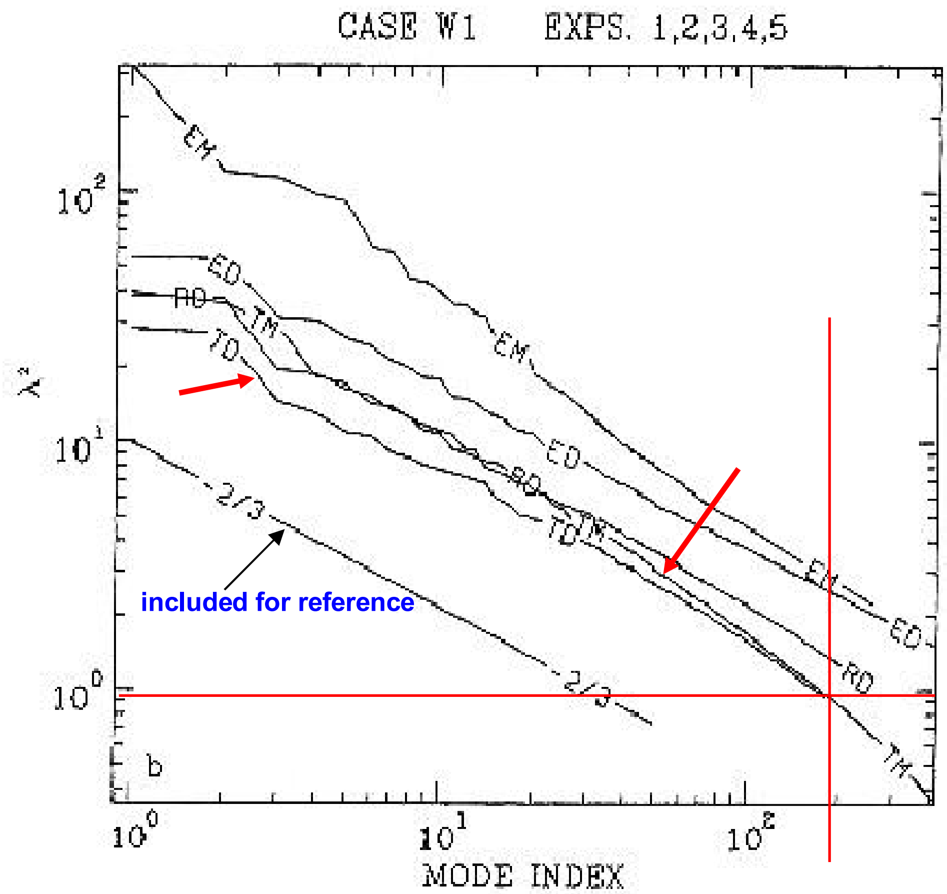


Fig. 3. Converged λ_k for the five sets of SVs determined for case W1. *E*, *R*, and *T* denote results for the energy, rotational mode, and truncated rotational mode norms, respectively. M and D denote results for moist and dry TLM, respectively. A line for which $\lambda_k^2 \propto k^{-2/3}$ is also indicated.

TD and TM curves:
169 (dry) and 175 (wet) growing SVs

(Errico et al. 2001)

height correlations 500 hPa derived from ensemble Integrations (D+4)

operational EPS (N=25)

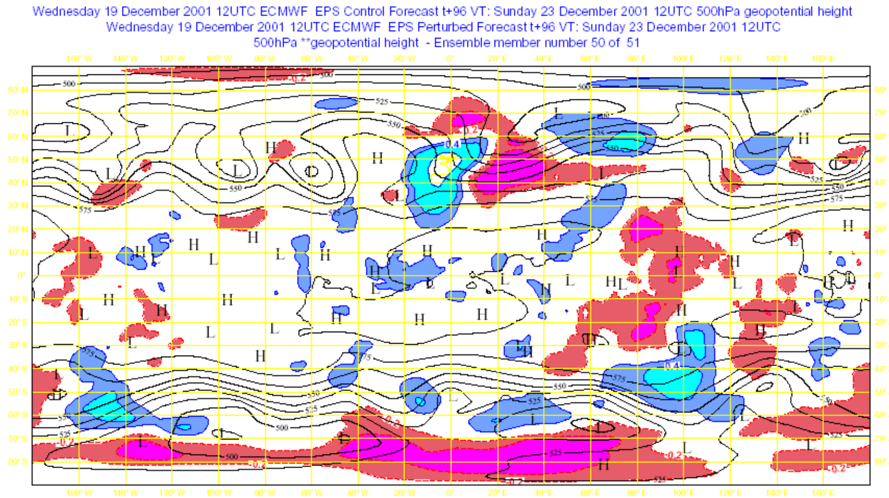


Figure 8.7: Day–4 geopotential height correlations at 500hPa obtained from ECMWF operational forecast starting from 2001121912. Also shown is Z500 height field from control forecast (black contours).

sampling, N=25, M=50

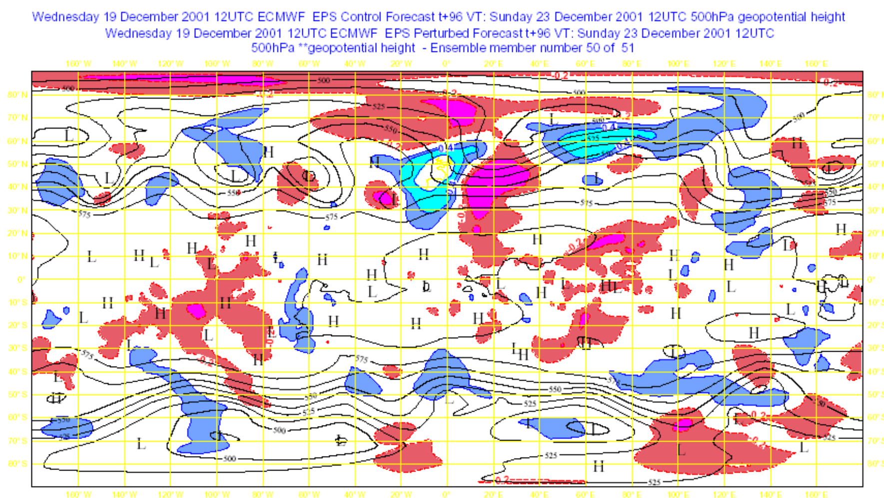


Figure 8.8: as Fig. 8.7, but for sampling experiment $M = 50/N = 25$ starting from 2001121912.

sampling, N=50, M=100

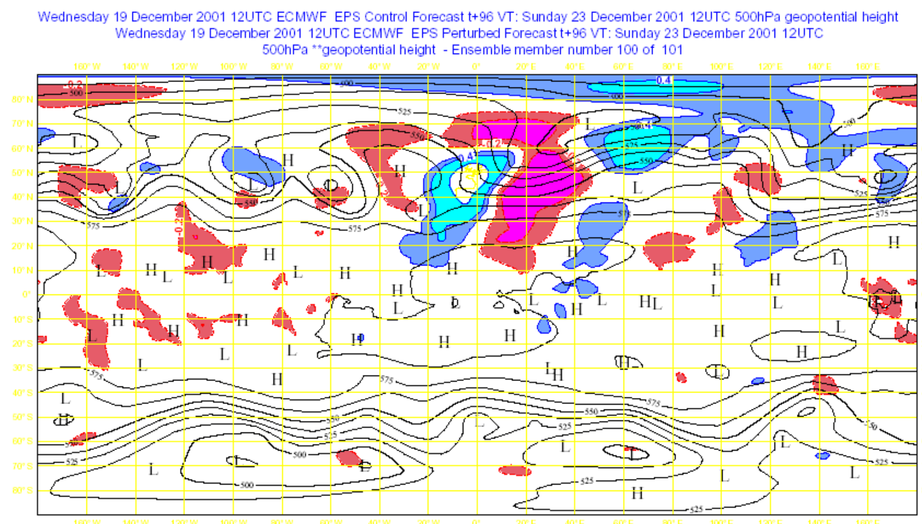


Figure 8.9: as Fig. 8.7, but for sampling experiment $M = 100/N = 50$ starting from 2001121912.

(Beck 2003)

Summary

Intrinsic Error Growth

limited predictability (nonlinearity)
presence of analysis error

Predictability

rapid doubling of analysis error
account for fastest error growth: SV dynamics
importance of lower troposphere
insight into growth mechanisms
initial moisture perturbations

Ensemble Prediction

generation of perturbations: sampling
SV relation to analysis error: nonmodal IC growth