ALADIN-HIRLAM « Dynamics Day »:

[*Energy* / *Enthalpy* / θ] *equations derived from the* *First principle* *only?*

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Motivations

- A study of the thermal equations for IFS + comparisons with those for Meso-NH / ARPEGE / ALADIN / ALARO
- Is it possible to avoid the use the entropy equation for deriving the [Energy / Enthalpy / θ] equations?
- This could reinforce the physical founding of these equations...

Grabowski (1998) … Smolarkiewicz … Wedi …
… Kurowski *et al.* (2014) : ***Dynamics?***

$$\phi = \left[\, c_{pd\,?}\,\theta_b(z)\,\pi' \;\; ; \;\; c_{pd\,?}\,\theta_0\,\pi' \, \right]$$

$$\frac{d\vec{u}}{dt} = -\,\Theta\,\vec{\nabla}(\phi) - \Upsilon_b \left(\frac{\theta'_{v\,?}}{\theta_b(z)} \right) \vec{g} + \vec{F}_E$$

$$\Theta = \left[\, \underbrace{1}_{anelastic} \;\; ; \;\; \underbrace{\frac{\theta_{v\,?}(\vec{x},t)}{\theta_0}}_{compressible} \, \right]$$

$$\Upsilon_b = \left[\, 1 \;\; ; \;\; \frac{\theta_b(z)}{\theta_0} \, \right]$$

- a link with moisture (θ_v and r_v) via the “conversion term”?
- a link between “kinetic / thermal” energies?
- which is the underlying “concervation law”?

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Grabowski (1998) … Smolarkiewicz … Wedi …
… Kurowski *et al.* (2014) : ***Physics?***

$$\frac{d\,\theta'}{dt} = -\vec{u}\,.\,\vec{\nabla}(\theta_a) + H \quad \rightarrow \quad H = ?$$

- The *dry-air* entropy equation:

$$s \approx c_{pd} \ln(\theta) + Cste \;\Rightarrow\; \frac{ds}{dt} \approx \frac{c_{pd}}{\theta}\,\frac{d\,\theta}{dt} \approx \frac{\dot{Q}}{T}$$

$$\theta = T \left(\frac{p_{00}}{p}\right)^{R_d/c_{pd}} \;\Rightarrow\; \frac{d\,\theta}{dt} \approx \frac{\theta}{T}\,\frac{\dot{Q}}{c_{pd}} \approx \frac{\dot{Q}}{\Pi\, c_{pd}} = H$$

Grabowski (1998) … Smolarkiewicz … Wedi …
… Kurowski *et al.* (2014) : ***Physics?***

$$\frac{d\,\theta'}{dt} = -\,\vec{u}\,.\,\vec{\nabla}(\theta_a) + H \;\longrightarrow\; H = ?$$

- The *moist-air* entropy equation / Grabowski (1998, ...)

$$s \approx c_{pd}\ln(\theta_E) + Cste \;\Rightarrow\; \frac{ds}{dt} \approx \frac{c_{pd}}{\theta_E}\,\frac{d\,\theta_E}{dt} \approx \frac{\dot{Q}}{T}$$

- Based on the equivalent potential temperature (Betts, 1973):

$$\theta_E \approx \theta\,\exp\left(\frac{L_v\,q_v}{c_{pd}\,T}\right) \;\Rightarrow\; \frac{d\,\theta}{dt} \approx \frac{1}{\Pi\,c_{pd}}\left(-\,L_v\,\frac{dq_v}{dt} + \dot{Q}\right) = H$$

- But θ_E (Betts 73, like others) + "ds/dt = ..." all approximated

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Grabowski (1998) … Smolarkiewicz … Wedi …
… Kurowski *et al.* (2014) : ***Physics?***

$$\frac{d\,\theta'}{dt} = -\vec{u}\,.\,\vec{\nabla}(\theta_a) + H \quad \longrightarrow \quad H = ?$$

- Another moist-air entropy equation in Meso-NH:

$$s \approx c_{pd}\ln(\theta_{eiv}) + Cste$$

- Potential temp. Dutton (1976) + Tripoli & Cotton (1981)

$$\theta_{eiv} \approx \theta\ \exp\left(\frac{L_v^0\, r_v}{c_{pd}\, T} - \frac{L_f^0\, r_i}{c_{pd}\, T}\right)$$

$$-\left(\frac{R_m}{c_{pm}} - \frac{R_d}{c_{pd}}\right)\left(\frac{\theta}{p}\right)\frac{dp}{dt} + \frac{1}{\Pi\ c_{pm}}\left(L_f^0\,\frac{d_i r_i}{dt} - L_v^0\,\frac{dr_v}{dt} + \dot{Q}\right) = H$$

The link with MESO-NH ?

Meso-NH
mesoscale non-hydrostatic model

Part I: Dynamics **2.8 Thermodynamic equation**

$$\frac{\partial}{\partial t}(\rho_{deff}\theta) + \boldsymbol{\nabla}\cdot(\rho_{deff}\theta\,\boldsymbol{U}) = \rho_{deff}\left[\frac{R_d + r_v R_v}{R_d}\frac{C_{pd}}{C_{ph}} - 1\right]\frac{\theta}{\Pi_{ref}} w \frac{\partial \Pi_{ref}}{\partial z}$$

$$+\frac{\rho_{deff}}{\Pi_{ref} C_{ph}}\left[L_m \frac{D(r_i + r_s + r_g + r_h)}{Dt} - L_v \frac{Dr_v}{Dt} + \mathcal{H}\right].$$

The terms on the right-hand side represent respectively the moist correction in absence of any phase change (derived from the conservation of total energy, with some approximations), the effects of phase changes, and the other diabatic effects (radiation and diffusion).

$$\theta_{eiv} \Rightarrow -\left(\frac{R_m}{c_{pm}} - \frac{R_d}{c_{pd}}\right)\left(\frac{\theta}{p}\right)\frac{dp}{dt} + \frac{1}{\Pi\ c_{pm}}\left(L_f^0 \frac{d_i r_i}{dt} - L_v^0 \frac{dr_v}{dt} + \dot{Q}\right)$$

- Thus all term can be "explicated" by the use of $\boldsymbol{\theta_{eiv}}$
- But $\boldsymbol{\theta_{eiv}}$ and "ds/dt = ..." <u>approximated</u> & what about "d_i/dt"?

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What about θ_s^* or θ_s / Marquet (1993, 2011, 2015)?

Catry et al. (2007) ; Degrauwe et al. (2016)

- (1993): exact, but with two moist constants:

$$s = c_p^*(r_t) \, \ln(\theta_s^*) + C^*(r_t)$$

- (2011, 2015): exact, and with two real constants:

$$s = c_{pd} \, \ln(\theta_s) + Cste$$

- However: after many attempts, my feeling is that the corresponding entropy equations cannot serve as a starting point to derive [Energy / Enthalpy / θ] equations...

A focus on the ***Enthalpy*** equation only

- The enthalpy equation: $\dfrac{dh}{dt} = -\dfrac{1}{\rho}\dfrac{dp}{dt} + \dot{Q}_h - D$
- de Groot and Mazur (1962, 1986, ...):

$$h = \sum_k q_k\, h_k \qquad c_p = \sum_k q_k\, c_{pk} \qquad d\,h_k = c_{pk}\, dT$$

- The temperature equation:

$$c_p \frac{dT}{dt} = \underbrace{-\frac{1}{\rho}\frac{dp}{dt}}_{conversion} \underbrace{-\sum_k h_k \frac{d_i q_k}{dt}}_{changes\ of\ states} + \underbrace{\dot{Q}}_{radiation} - \underbrace{D}_{diss.+diff.}$$

$$\left(c_p \frac{dT}{dt} + \frac{1}{\rho}\frac{dp}{dt} \right) = -\sum_k h_k \frac{d_i q_k}{dt} + \dot{Q} - D$$

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Two ***Derived*** equations

- The temperature equation:

$$\left(c_p \frac{dT}{dt} + \frac{1}{\rho} \frac{dp}{dt} \right) = -\sum_k h_k \frac{d_i q_k}{dt} + \dot{Q} - D$$

- A first derived equation (Meso-NH / IFS?):

$$\frac{d\,\theta}{d\,t} = -\left(\frac{R}{c_p} - \frac{R_d}{c_{pd}} \right) \left(\frac{\theta}{p} \right) \frac{dp}{dt} + \frac{\theta}{c_p T} \left(c_p \frac{dT}{dt} + \frac{1}{\rho} \frac{dp}{dt} \right)$$

- A second derived equation (ARPEGE / ALADIN / ALARO):

$$\frac{d\,(c_p T)}{d\,t} = c_p \frac{dT}{dt} + T \left(\sum_k c_{pk} \frac{d_i q_k}{dt} \right) + T \left(\sum_k c_{pk} \frac{d_e q_k}{dt} \right)$$

Is there any improvements?

- The temperature equation:

$$\left(c_p \frac{dT}{dt} + \frac{1}{\rho} \frac{dp}{dt} \right) = -\sum_k h_k \frac{d_i q_k}{dt} + \dot{Q} - D$$

- no use of the *Gibbs functions*: $h_k = \mu_k + T\, s_k$
  (no hypothesis of constant μ_k in change of phases)

- this demonstrates more clearly the possibility to take into account supercooled water and over- or under-saturations: no use of s_k nor "equilibrium" or "reversible" assumptions

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Conclusions

- It is possible to avoid the use of the entropy equation for deriving the [Energy / Enthalpy / θ] equations
- This is likely valid for meso-NH / IFS / ARPEGE / ALADIN / ALARO models
- This may reinforce the physical founding of the equations
- It is possible to derive the equations or budget for the other entropy potential temperatures θ_E ; θ_l ; θ_s (if needed)
- An interest to use θ_s: for mixing and turbulence (Let $\neq$ 1)

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