Regional Cooperation for
Limited Area Modeling in Central Europe

LACE
nwp central europe

DYNAMICS IN LACE

Petra Smolíková (CHMI)
with contribution from other colleagues

DHMZ

ZAMG

ROMANIA ANM

Summary

1. **Finite element method in vertical discretization of NH model** (see Alvaro Subías poster)

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3. **Physical tendency of vertical velocity in NH**

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2. **ENO technique for SL interpolations**
3. **Physical tendency of vertical velocity in NH**
4. **Testing 3D turbulence in academic and real simulations** (with 2D contribution from dynamics and vertical contribution from TOUCANS)

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2. **ENO technique for SL interpolations**
3. **Physical tendency of vertical velocity in NH**
4. **Testing 3D turbulence in academic and real simulations** (with 2D contribution from dynamics and vertical contribution from TOUCANS)
5. **Testing PC scheme in high resolutions** (see Czech national poster)

VFE in NH

- designed by Jozef Vivoda based on hydrostatic version of FE used in vertical (being developed by A.Untch, M.Hortal)
- **cooperation with HIRLAM colleagues** (J.Simarro, A.Subias)

Current status: there is a working implementation of the VFE method in the NH model since cycle CY40T1 **with remaining FD features** being tested in real simulations

Recent development:

1. Revised definition of boundary conditions
2. Fully FE vertical Laplacian
3. FE transformations d <-> w
4. Vertical operators satisfying the constraint C1

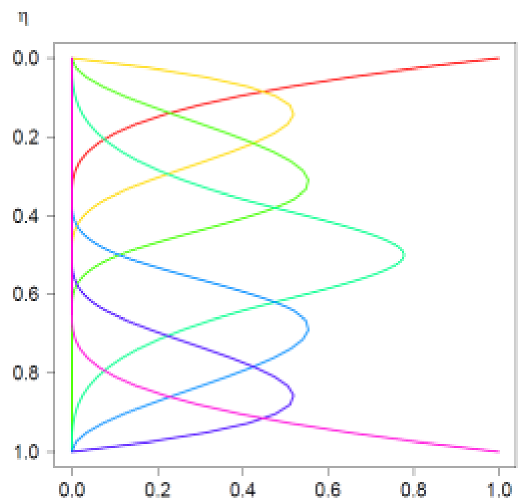
VFE in NH

Vertical operator $g\left(f(\eta)\right)$ sampled on L vertical levels without BC.

Projection from FD to FE space is LxL: $f(\eta_k) = \sum_{i=1}^{L} \hat{f}_i b_i(\eta_k)$

$$\begin{pmatrix} b_1(\eta_1) & \dots & b_L(\eta_1) \\ b_1(\eta_2) & \dots & b_L(\eta_2) \\ \dots & \dots & \dots \\ b_1(\eta_L) & \dots & b_L(\eta_L) \end{pmatrix} \begin{pmatrix} \widehat{f_1} \\ \widehat{f_2} \\ \dots \\ \widehat{f_L} \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \\ \dots \\ f_L \end{pmatrix}$$

L basis functions



VFE in NH

Vertical operator $g\left(f\left(\eta\right)\right)$ sampled on L vertical levels with top boundary condition $\frac{\partial f}{\partial \eta}(0) = c_0$ (possibly $f(0) = c_0$ or $\frac{\partial^k f(0)}{\partial \eta^k} = c_0$)

Discretized boundary condition: $\sum_{i=1}^{L} \hat{f}_i \frac{\partial b_i}{\partial \eta}(0) = c_0$

Explicit definition: extended projection matrix from FD to FE space, **added basis function b_0** with $\sum_{i=0}^{L} \hat{f}_i \frac{\partial b_i}{\partial \eta}(0) = c_0$

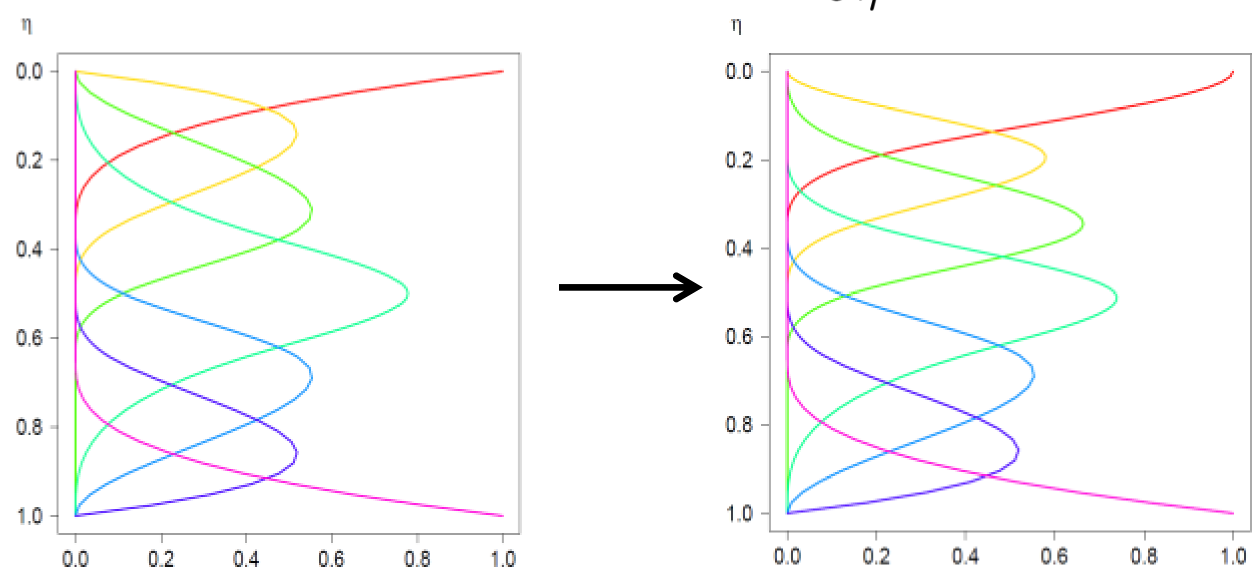
$$\begin{pmatrix} \frac{\partial b_0}{\partial \eta}(0) & \frac{\partial b_1}{\partial \eta}(0) & \dots & \frac{\partial b_L}{\partial \eta}(0) \\ b_0(\eta_1) & b_1(\eta_1) & \dots & b_L(\eta_1) \\ b_0(\eta_2) & b_1(\eta_2) & \dots & b_L(\eta_2) \\ \dots & \dots & \dots & \dots \\ b_0(\eta_L) & b_1(\eta_L) & \dots & b_L(\eta_L) \end{pmatrix} \begin{pmatrix} \widehat{f_0} \\ \widehat{f_1} \\ \widehat{f_2} \\ \dots \\ \widehat{f_L} \end{pmatrix} = \begin{pmatrix} c_0 \\ f_1 \\ f_2 \\ \dots \\ f_L \end{pmatrix}$$

VFE in NH

Vertical operator $g\left(f(\eta)\right)$ sampled on L vertical levels with top boundary condition $\frac{\partial f}{\partial \eta}(0) = c_0$ (possibly $f(0) = c_0$ or $\frac{\partial^k f(0)}{\partial \eta^k} = c_0$)

Discretized boundary condition: $\sum_{i=1}^{L} \hat{f}_i \frac{\partial b_i}{\partial \eta}(0) = c_0$

Implicit definition makes sense only for c_0=0: projection matrix not changed, basis functions changed to satisfy $\frac{\partial b_i}{\partial \eta}(0) = 0$ for any i = 1,…L



VFE in NH

Explicit definition of boundary conditions:

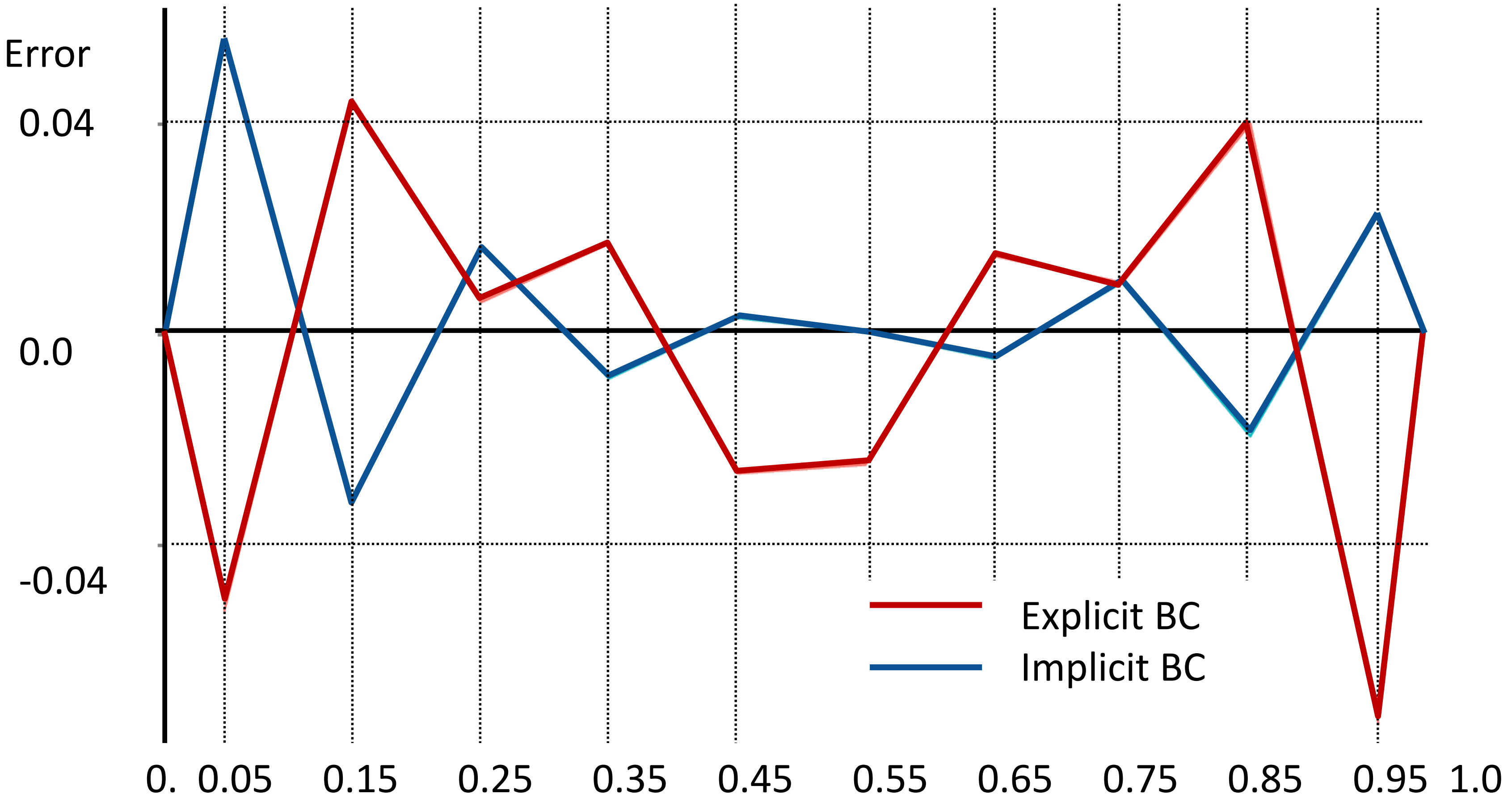
- c_0 arbitrary, possibly space dependent
- one basis function added for one boundary condition
- **projection operator extended** to (L+1) x (L+1)

Implicit definition of boundary conditions:

- $c_0 = 0$
- **basis functions changed** to satisfy BC
- projection operator remains LxL

VFE in NH

Derivative operator with boundary conditions: $\frac{\partial f(0)}{\partial \eta} = 0$ and $f(1) = 0$
applied on the testing function $f(\eta) = (sin(\pi\eta)^2 cos(\pi\eta))$ sampled at 10 levels



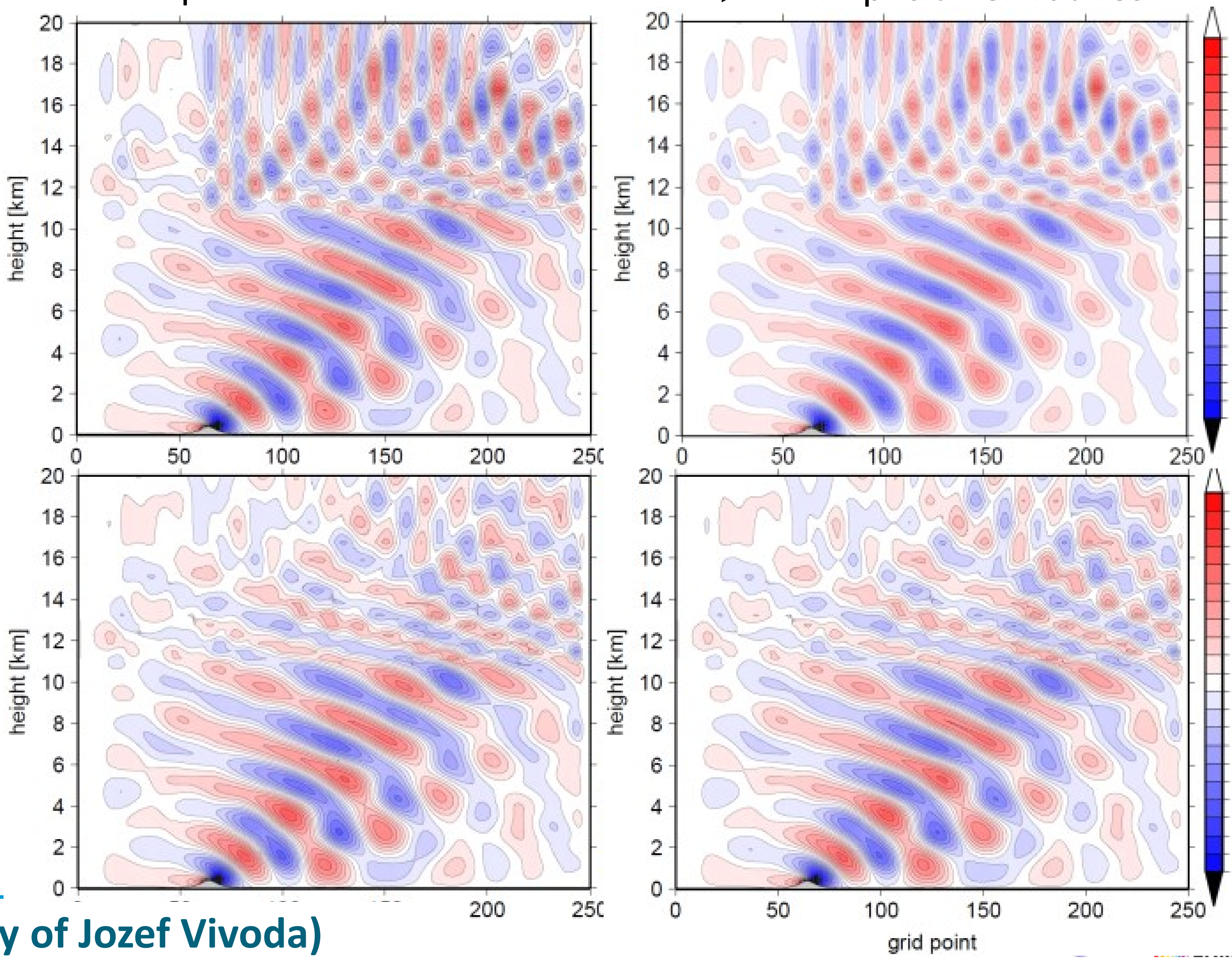
(courtesy of Jozef Vivoda)

VFE in NH

1. Boundary conditions: non-linear nh flow over an Agnesi shaped mountain

Explicit Derivatives → Implicit Derivatives

Explicit Integrals ↓ Implicit Integrals



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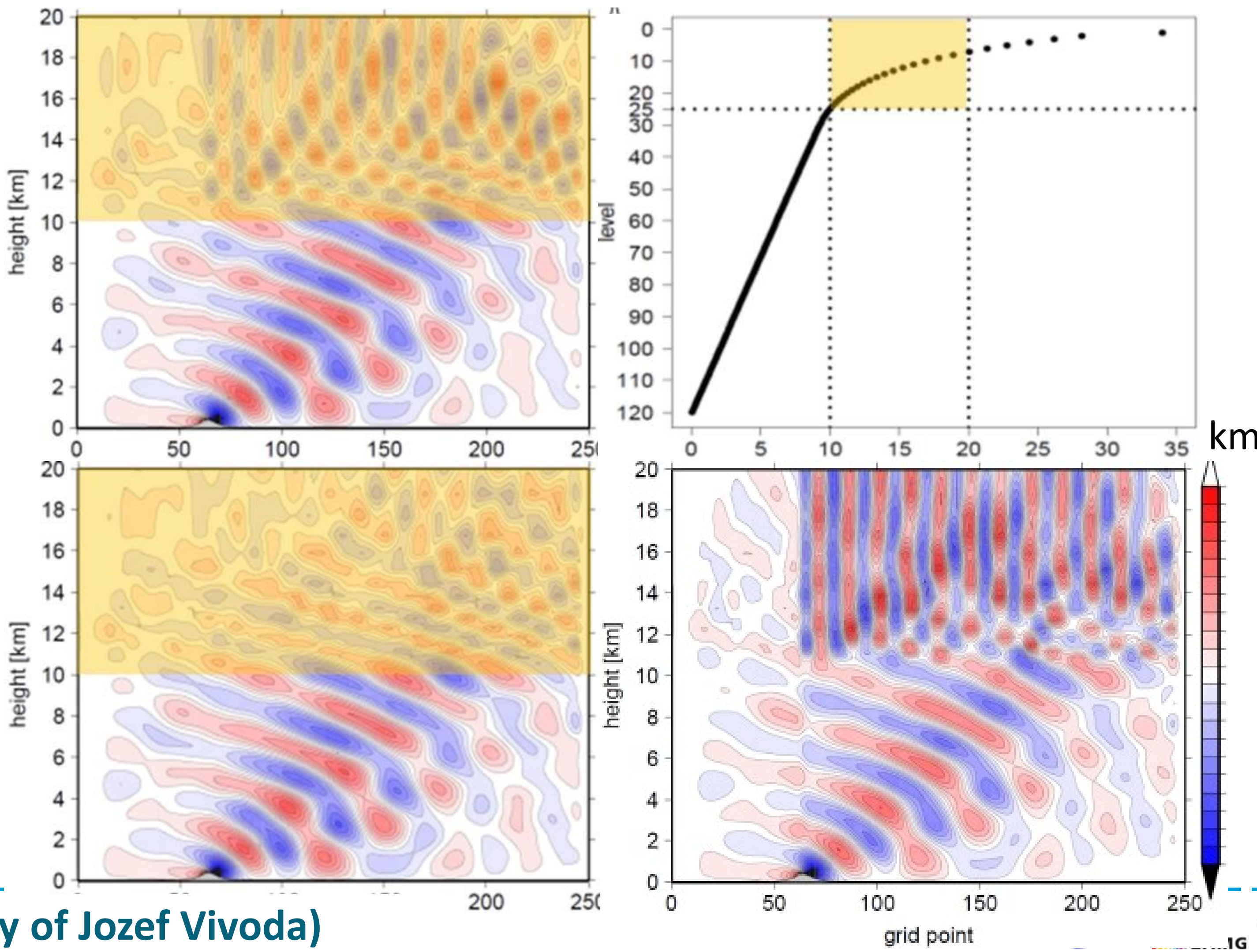
VFE in NH

1. Boundary conditions: non-linear nh flow over an Agnesi shaped mountain

Explicit Integrals

Implicit Integrals

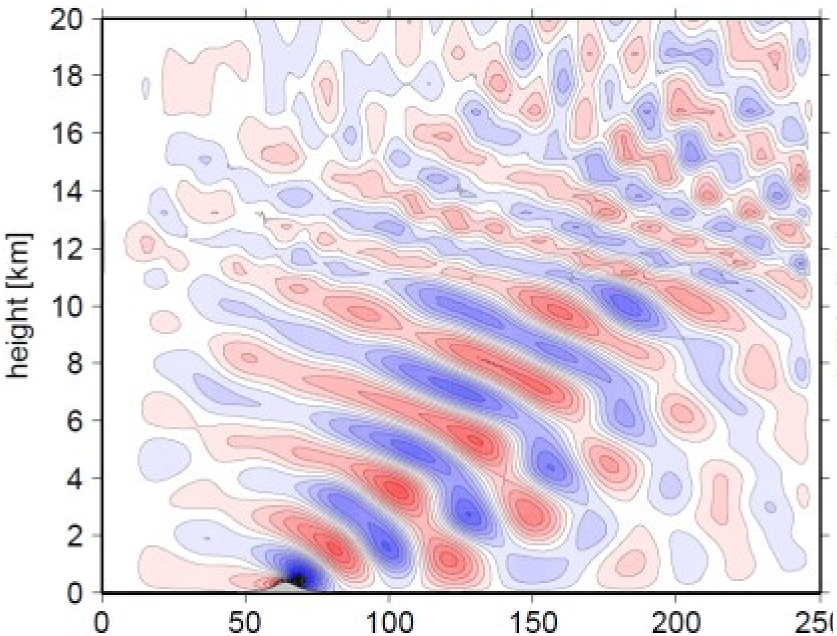


FE no BC

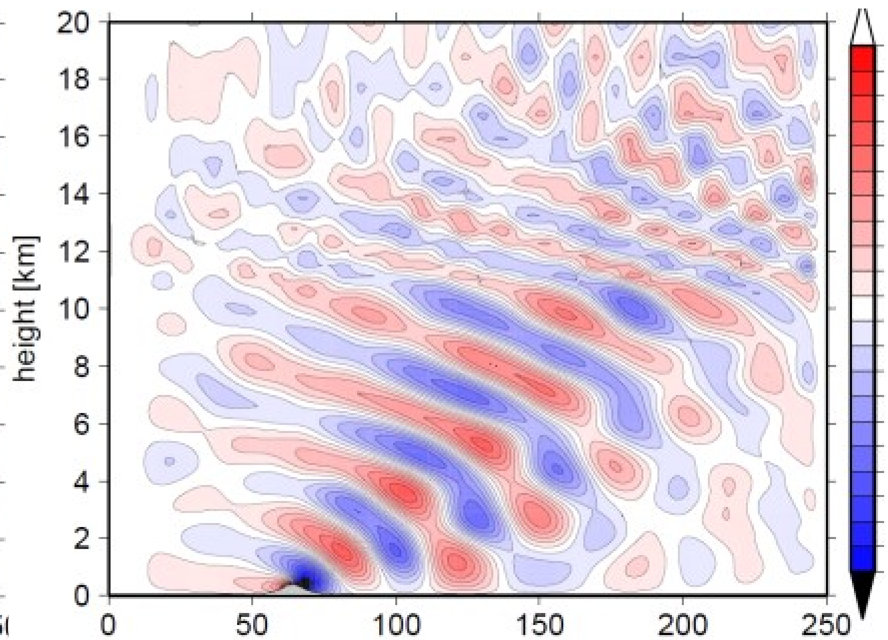
VFE in NH

2. FE vertical Laplacian

- with correct definition of vertical derivatives no need for FD boundary conditions as before



FD BC



fully FE

(courtesy of Jozef Vivoda)

3. FE transformations w <-> d (for w on full levels)

- vertical divergence d used for stability reasons in spectral space, while vertical velocity w is advected for accuracy reasons => transformations d -> w -> d every time step
- transformation d -> w -> d must be the identity to ensure correct steady state solution of an atmosphere in hydrostatic balance
- integral and derivative must be inverse of each other
- FD operators used up to now for transformations
- New solution: FE operators changing the spline order (increasing with integral and decreasing with derivative)

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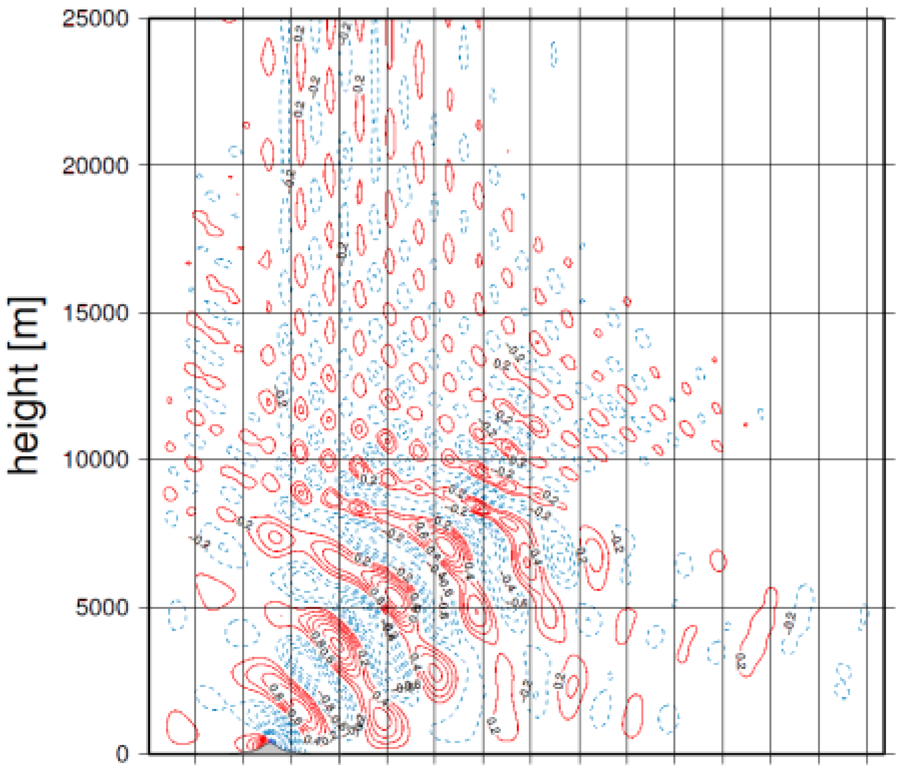
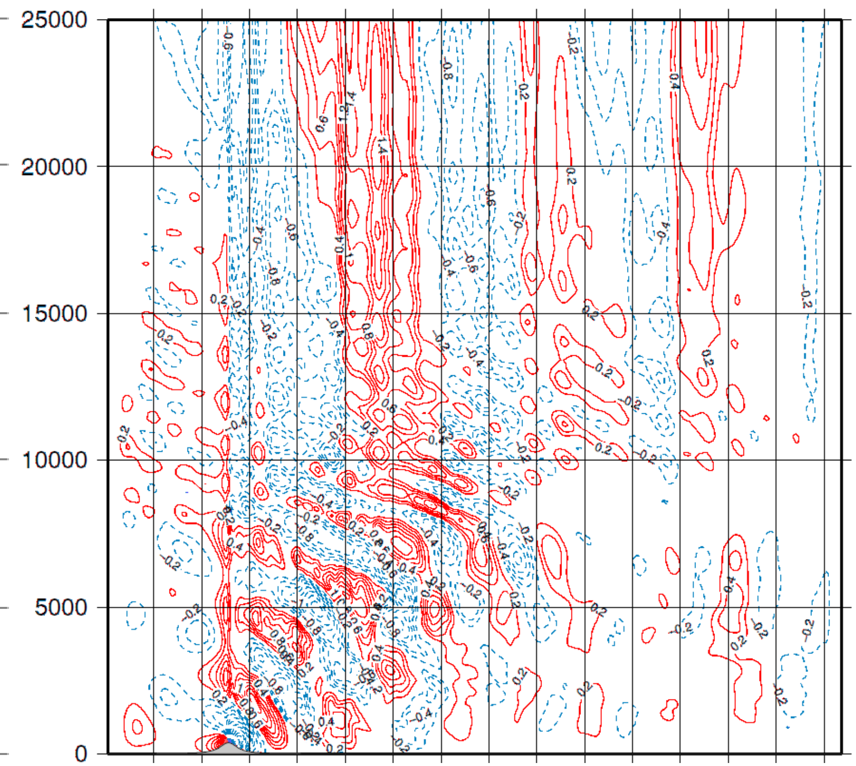
3. FE transformations w <-> d

FD, w on half levels



FE, w on full levels

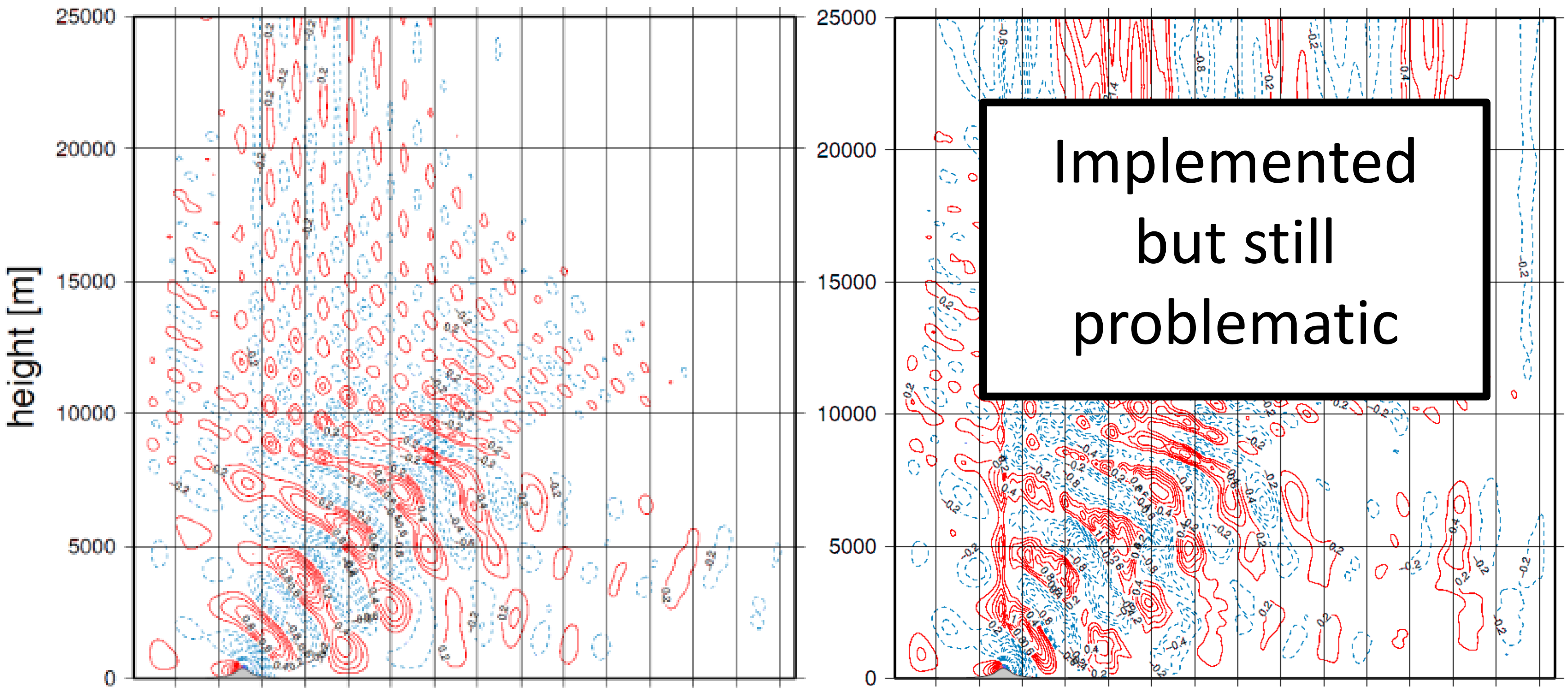


(courtesy of Jozef Vivoda)

VFE in NH

3. FE transformations w <-> d

FD, w on half levels | FE, w on full levels



(courtesy of Jozef Vivoda)

4. Integral operators satisfying C1 constraint

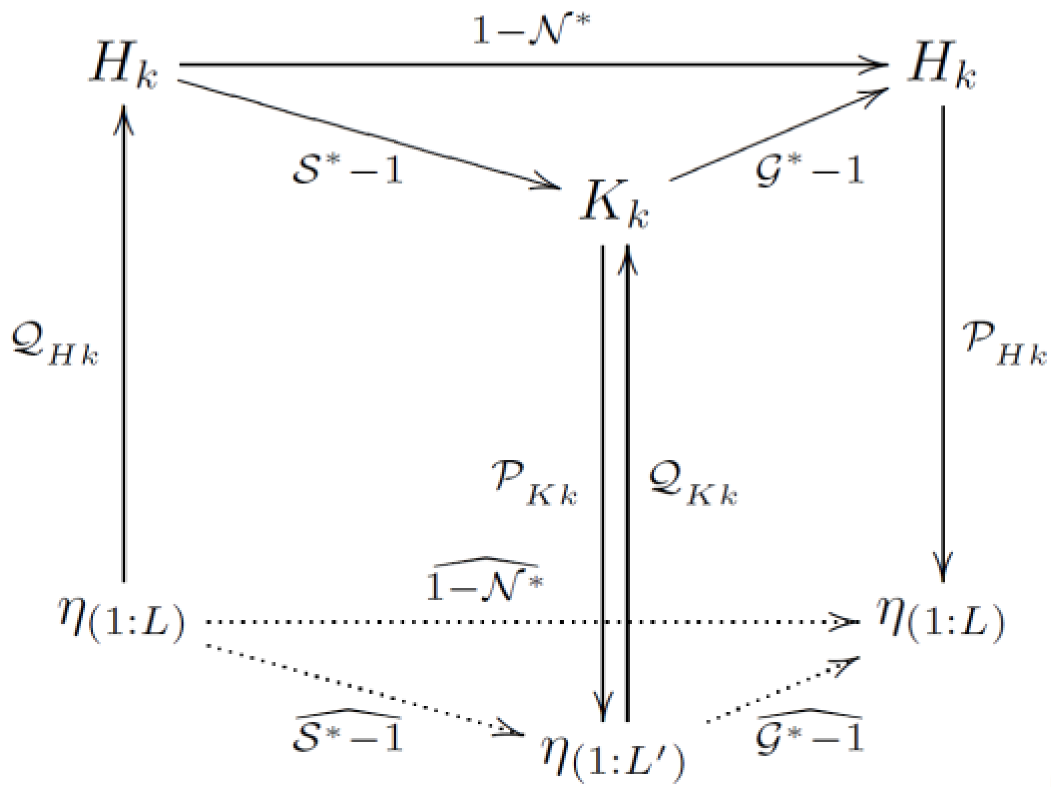
- only vertical operators (for integral, the first and the second derivative) are replaced by their FE versions
- in continuous case: vertical operators satisfy 2 conditions (C1,C2) which are not satisfied in discretized case
- VFD: designed to SATISFY (C1) & approximation to ALMOST SATISFY (C2)
- VFE: iterative stationary method used to satisfy (C1)

New idea in VFE (Alvaro Subías): to factorize (C1) to get

$$(\mathcal{G}^* - 1)(\mathcal{S}^* - 1) = (1 - \mathcal{N}^*)$$

and to focus on linear subspaces of normalised B-splines on which these vertical operators have „nice properties“

4. Integral operators satisfying C1 constraint



$$H_k := \langle\{\xi_{ik}\}_{i\in\mathbb{Z}}\rangle$$
$$K_k := \langle\{\sigma_{ik}\}_{i\in\mathbb{Z}}\rangle$$

functions defined on the base of normalized B-splines

$$(1-\mathcal{N}^*)\ \xi_{ik} = \xi_{ik} - \mu_{ik}$$
$$(\mathcal{G}^*-1)\ \sigma_{ik} = \xi_{ik} - \mu_{ik}$$
$$(\mathcal{S}^*-1)\ \xi_{ik} = \sigma_{ik}$$

C1: $(\mathcal{G}^*-1)(\mathcal{S}^*-1) = (1-\mathcal{N}^*)$

(courtesy of Alvaro Subías)

VFE in NH

Problems:

- higher spline orders and odd spline orders
- accuracy tests (Baldauf-Brdar test of the linear expansion of sound and gravity waves in a channel induced by a weak warm bubble)
- real simulations

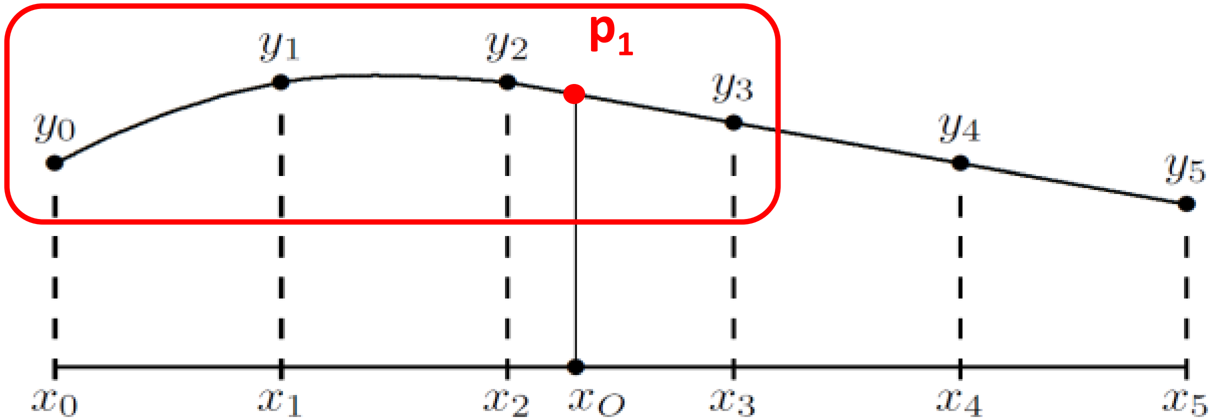
ENO technique in SL interpolations (Alexandra Craciun)

Previous work: already tested for quadratic interpolators in 2D and 3D, the results show that quadratic ENO/WENO is too smoothing

The aim: to implement cubic ENO interpolations in SL scheme – technically demanding, the stencil for SL 1D interpolation has to be extended from 4 to 6 points
(already partially prepared under NSTENCIL_WIDE=3)

ENO technique in SL interpolations

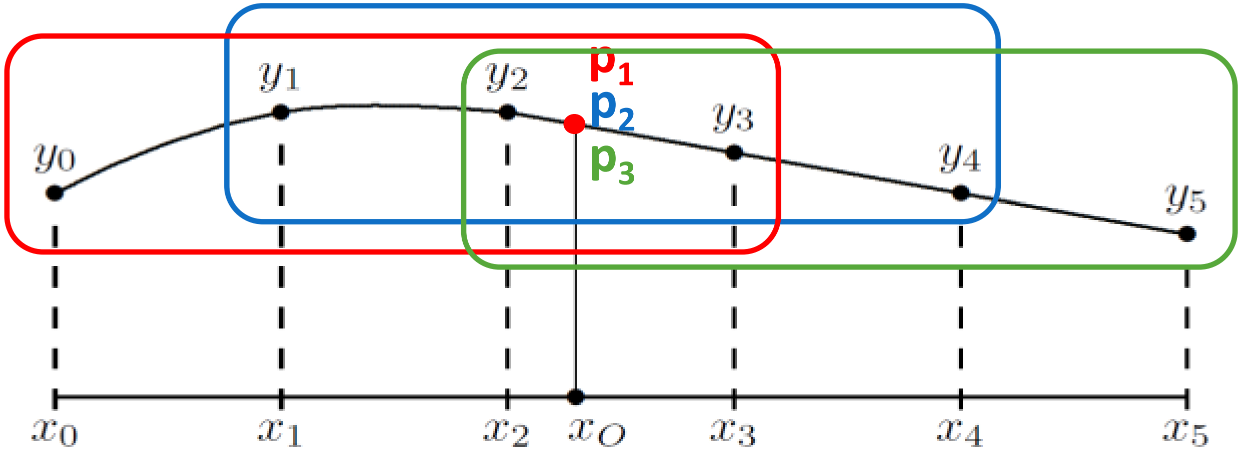
Third order interpolation scheme (cubic) needs 4 points to find the interpolated value:



Stencil 1 =>
interpolated value p_1

ENO technique in SL interpolations

With 6 points available, we can find the interpolated value 3 times on 4 points and choose the “best” solution from them:



Stencil 1 =>
interpolated value p_1

Stencil 2 =>
interpolated value p_2

Stencil 3 =>
interpolated value p_3

ENO technique in SL interpolations

In 3D :

6 vertical levels involved

3 linear interpolations in top and bottom level

+ 5 eno and 2 linear interpolations in 4 middle levels

+ 1 vertical eno interpolation

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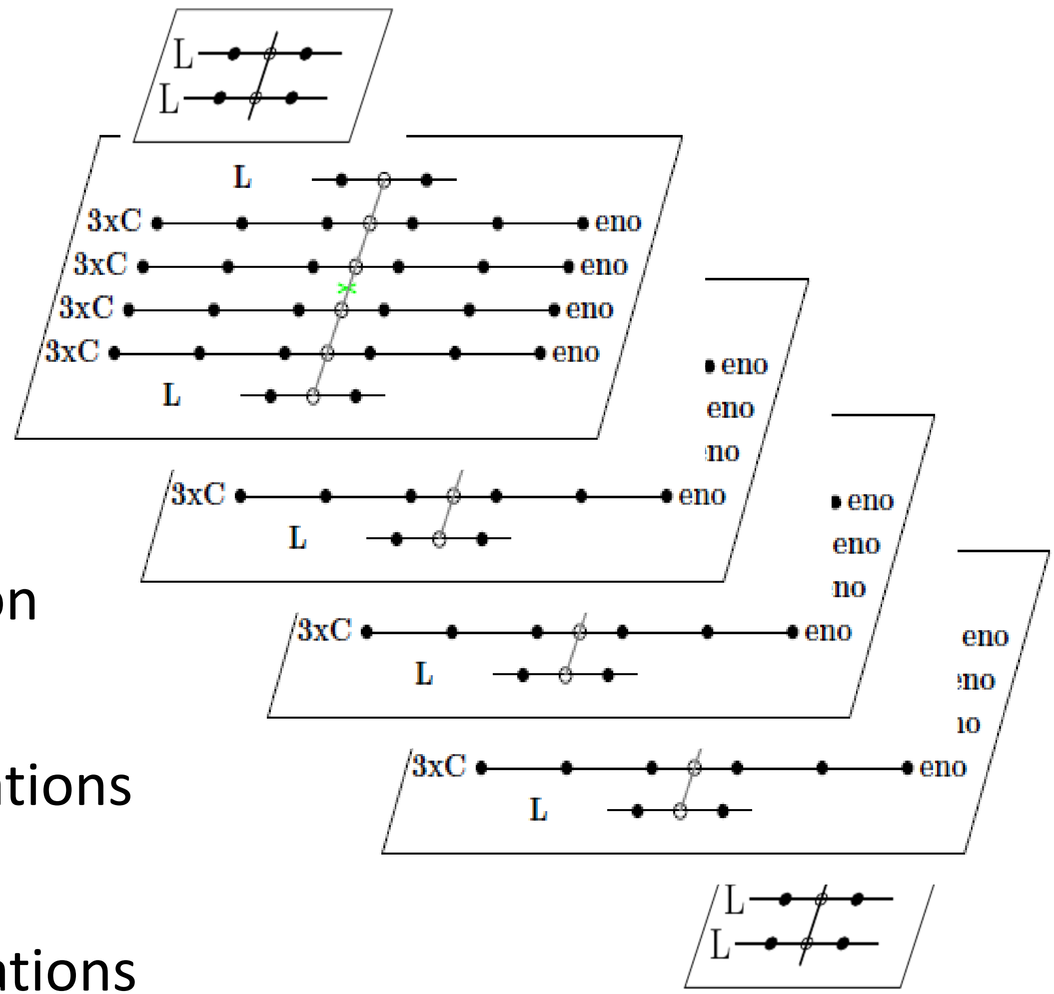
14 linear + 21 eno interpolations

instead of

10 linear + 7 cubic interpolations



ENO technique in SL interpolations

p_1, p_2, p_3 … interpolated values on 6-points stencils

Final interpolated value

$y = w_1 p_1 + w_2 p_2 + w_3 p_3$, where w_1, w_2, w_3 are weights with

$$w_1 + w_2 + w_3 = 1$$

$S_i = |y_{i+2} - 3y_{i+1} + 3y_{i-1} - y_{i-2}|$

smoothness on the given stencil (based on $\frac{\partial^3 y}{\partial x}$ approximation)

ENO chooses the smoothest solution ($S_i = min(S_1,S_2,S_3) \Rightarrow w_i = 1$)

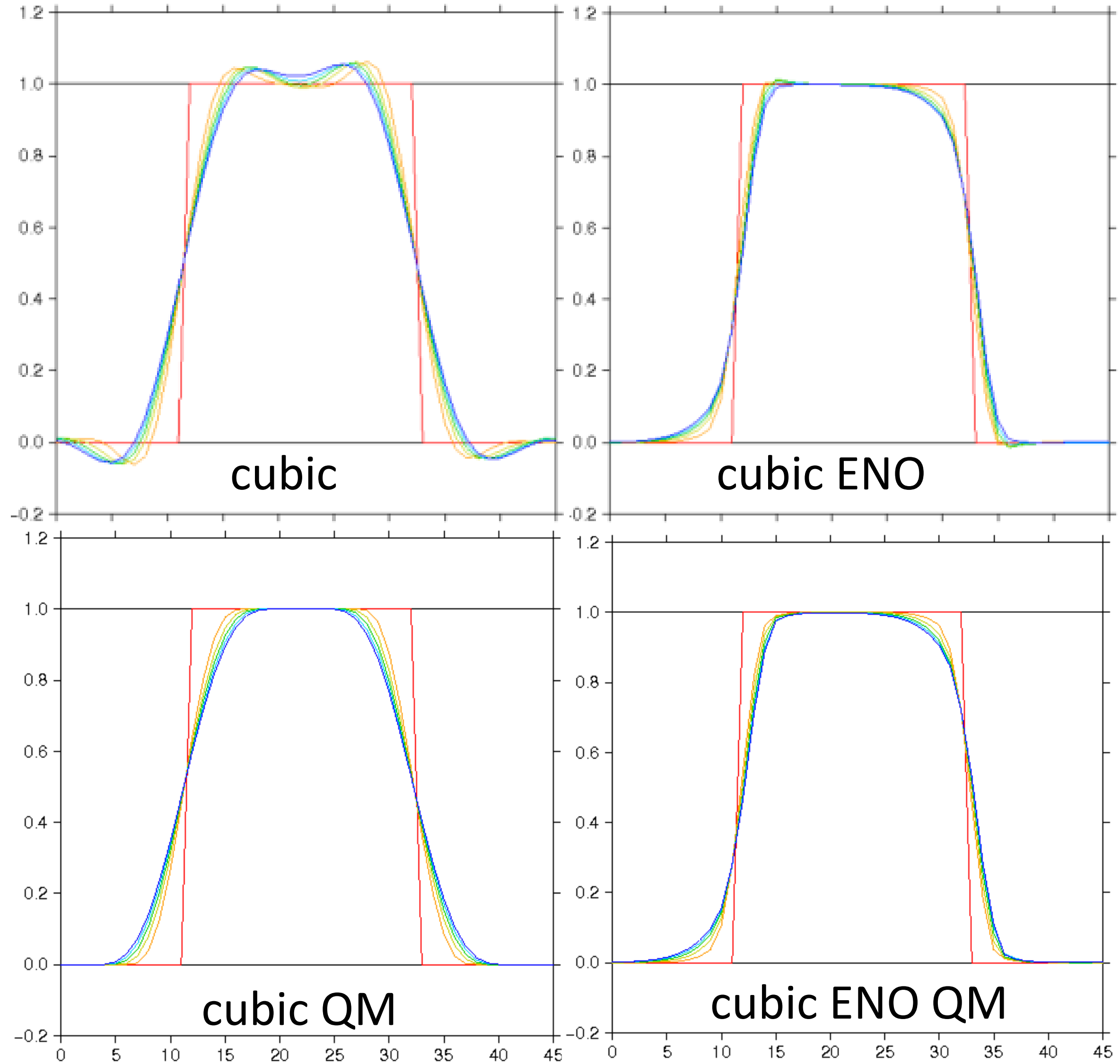
WENO weighted combination based on smoothness

ENO technique in SL interpolations

Toy model (courtesy of Jan Mašek):

1D linear advection of rectangular pulse in a periodic domain

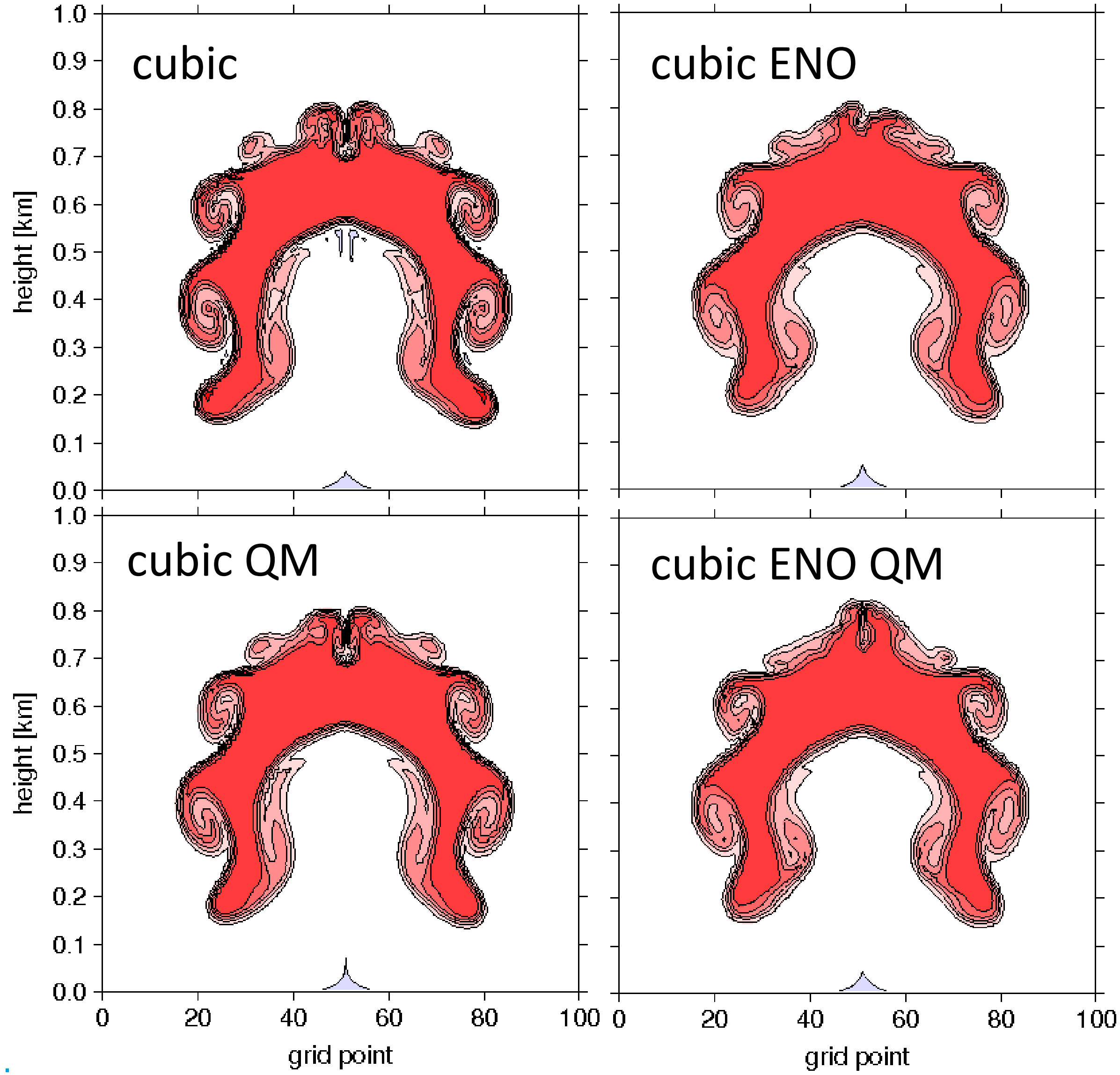
QM for ENO is closer to ENO than QM cubic to cubic interpolation => less overshooting with ENO



ENO technique in SL interpolations

Robert's test in 2D model: warm bubble in the field of constant potential temperature without advection

Difference between S and S QM is smaller for ENO => less overshooting with ENO



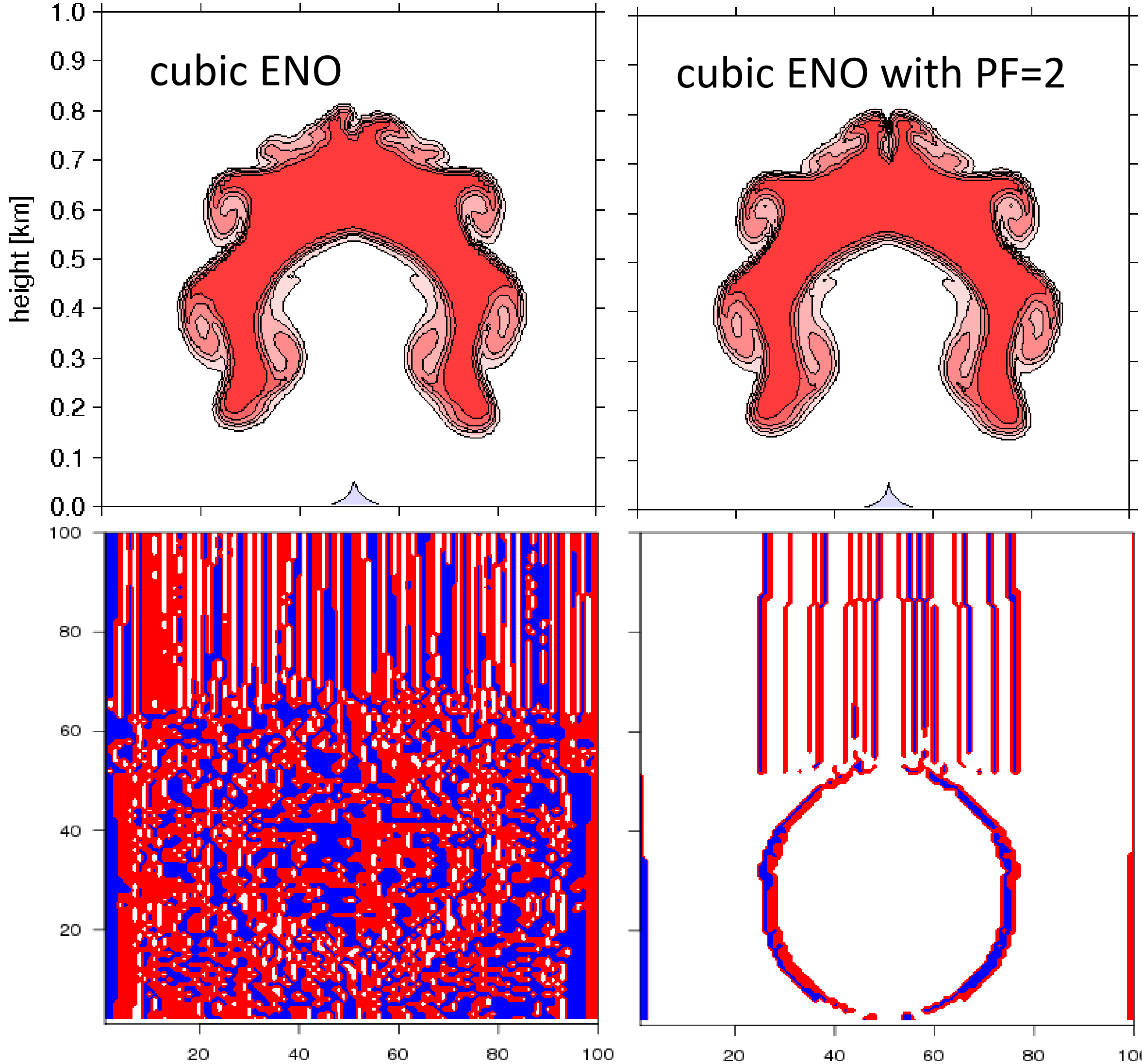
ENO technique in SL interpolations

Which stencil is used ?
left, **central**, **right**

in areas with modest gradients the stencils are chosen “randomly” because smoothness is almost constant

⇒ the central stencil is preferred by preference factor PF

⇒ outer stencils may be chosen only if their smoothness is PF times bigger then for central stencil



ENO technique in SL interpolations

Conclusions and questions:

- Cubic ENO interpolator seems to be too smoothing, but less overshooting than the cubic Lagrange operator.
- What about cubic WENO ? It is promising and suitable for linearization (TL code).
- The choice of preference factor need to be studied.
- What is the cost of ENO in 3D ?

Physical tendency of w

Turbulent tendency of vertical velocity w was implemented in CY38t1 with ALARO-1, where TOUCANS are already mature and tuned; the turbulent diffusion flux of w calculated in TOUCANS from TKE is used to calculate the turbulent tendency of w

Physical tendency of w

Tests in 2D vertical plane: **turbulent wave behind a hill**

Δx=1km, Δz=50m, Δt=20s

initial profile

- two isothermal layers divided by a strong inversion
- uniform horizontal wind
- several surface parameters (water reservoir, roughness, albedo, emissivity etc.)

Physical tendency of w

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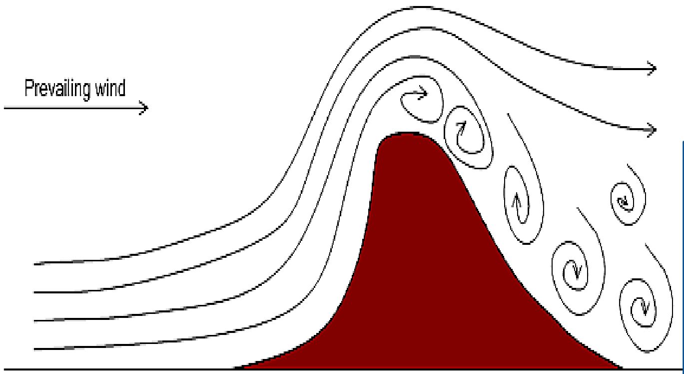
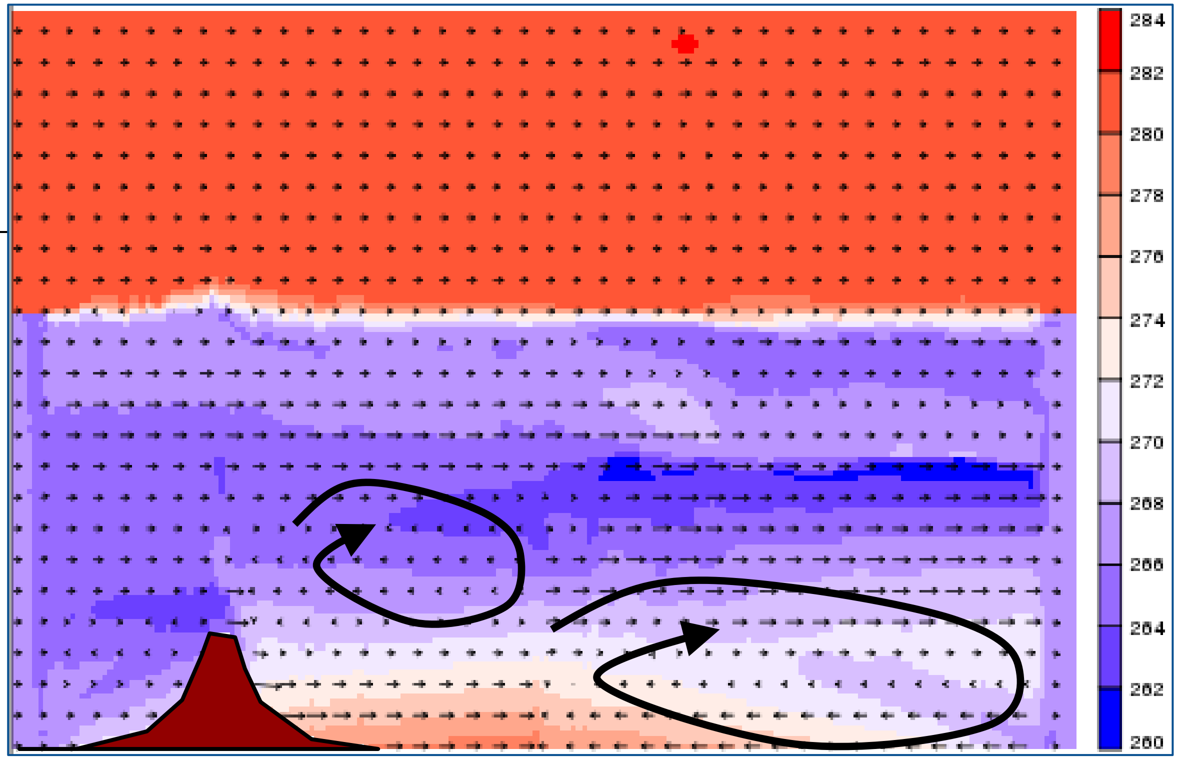
Tests in 2D vertical plane: **turbulent wave behind a hill** is influenced by turbulent tendency of w

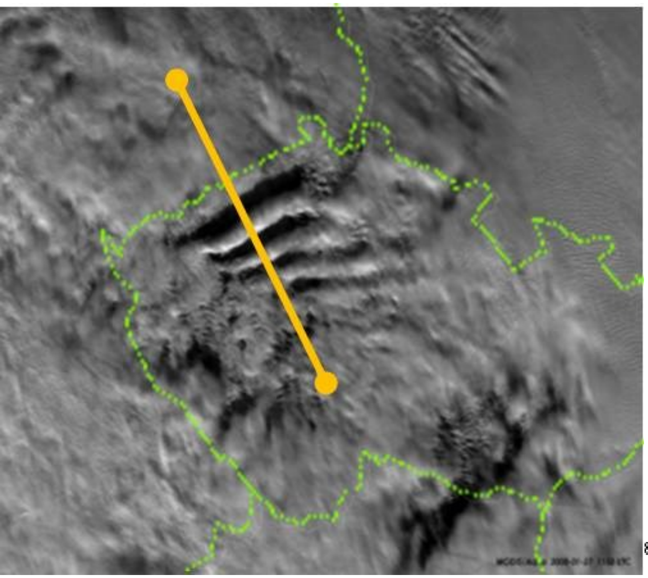


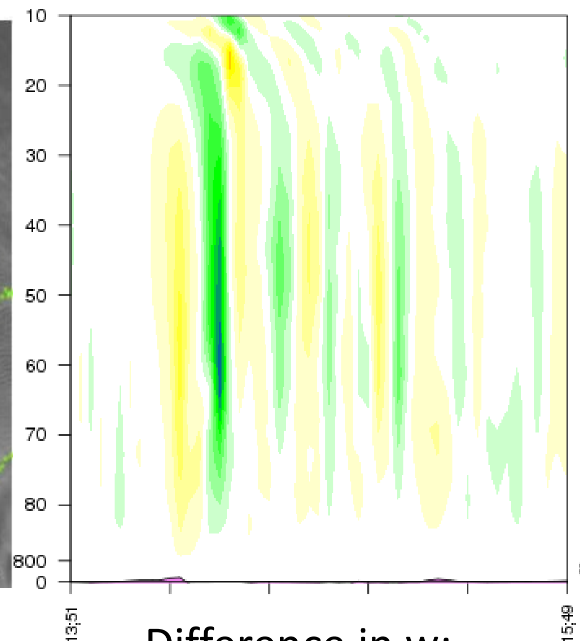


(courtesy of David Lancz)

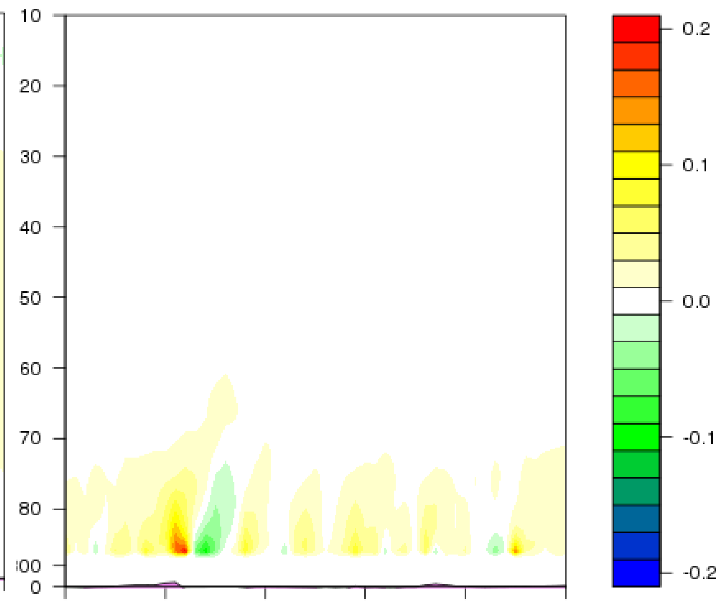
Physical tendency of w

Real simulations: an orographic wave in the flow over north-western Czech boundary





Difference in w: tend-notend



Turbulent tendency of w

(courtesy of David Lancz)

Physical tendency of w

Conclusions:

- vertical plane tests for turbulent flow achieved
- the impact of the vertical diffusion on vertical velocity corresponds to the expected trend (the damping of an orographic wave as an example)
- the observed effect on vertical velocity field is modest
- an original solution for this process has been proposed which can be further developed, used and tested

Dynamics of Lisbon