Could there be a need for a horizontal finite-element discretization?

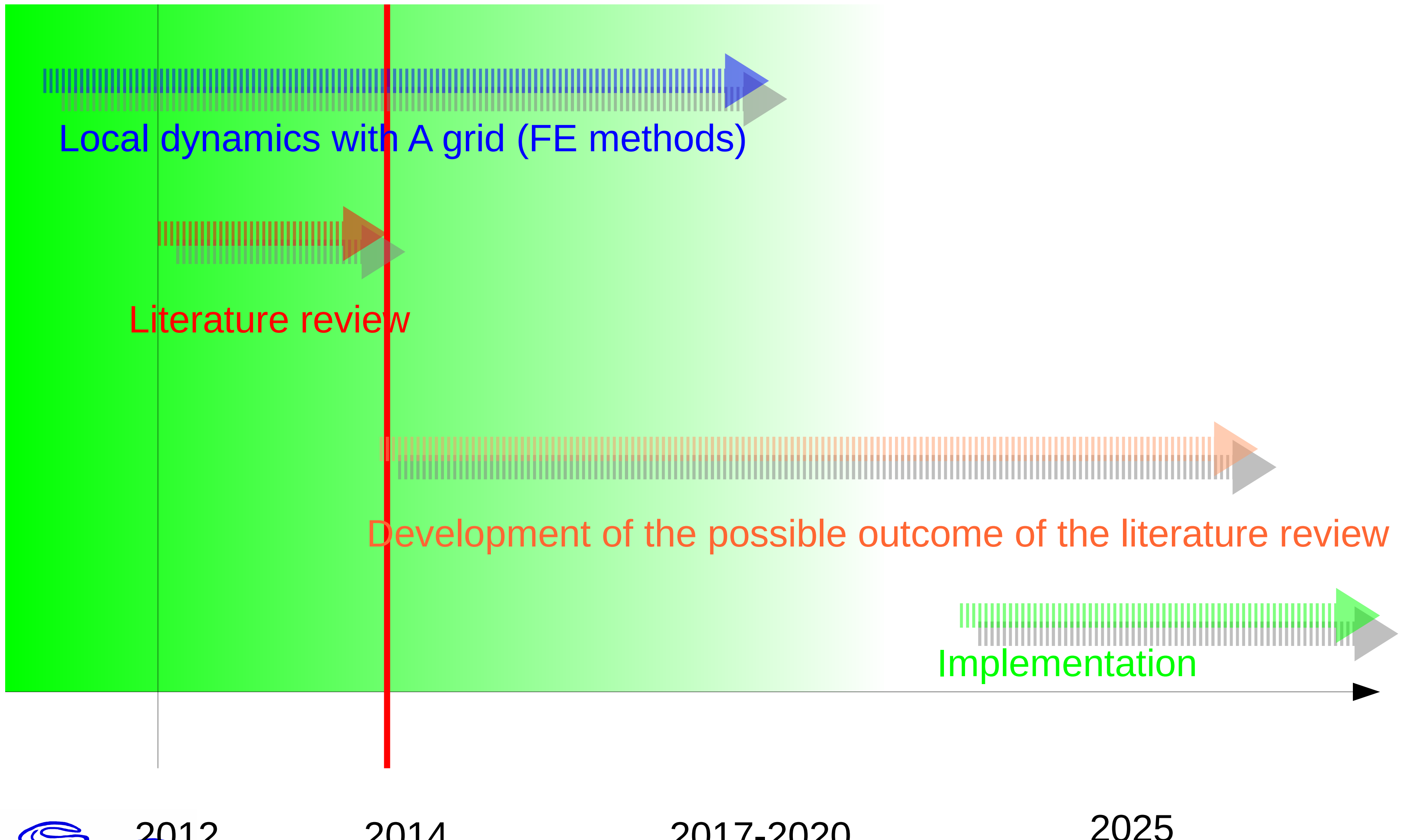
Piet Termonia

Caluwaerts, Degrauwe, Termonia, Voitus, Bénard, Geleyn, 2014: Importance of temporal symmetry in spatial discretization for geostrophic adjustment in semi-implicit Z-grid schemes, QJRMS, in press.

8/4/2014, Bucharest

Dynamics: road map

Eliminating the A grid means we have to overhaul the whole system.
We stay with the current system at least for the term of the current strategy plan (green area).



Some claims …, are they true?

- “A grid with `local' discretizations is bad for dispersion relations” (so we need a C grid)
- “Spectral methods will break down at high resolutions for flows over steep slopes.”
- “Spectral methods will not be suitable for the future massively parallel machines (scalability)”
- “so we need a dynamics with a `local' discretization” (local means finite differences, finite elements, in contrast to spectral)

Horizontal discretizations

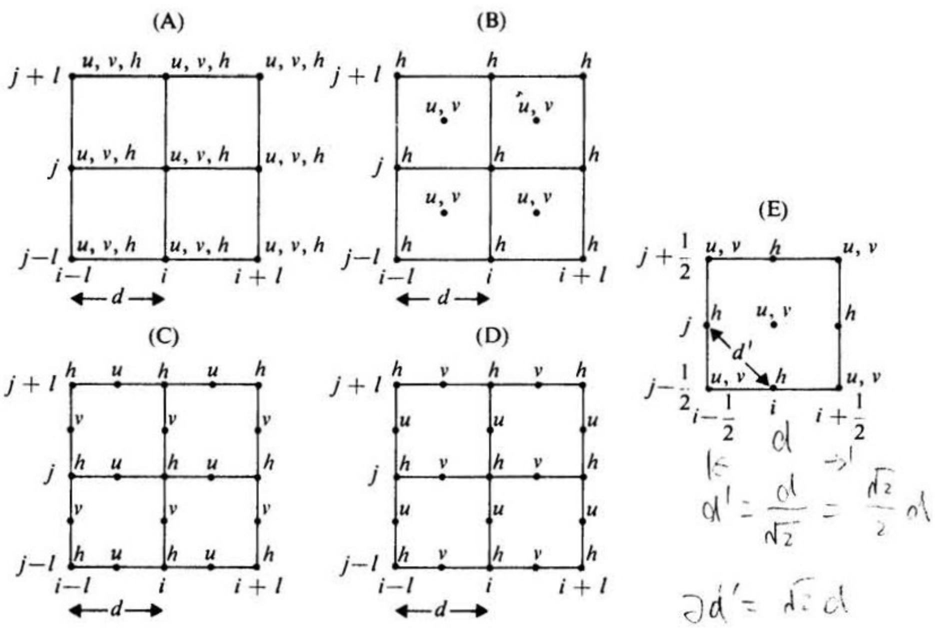
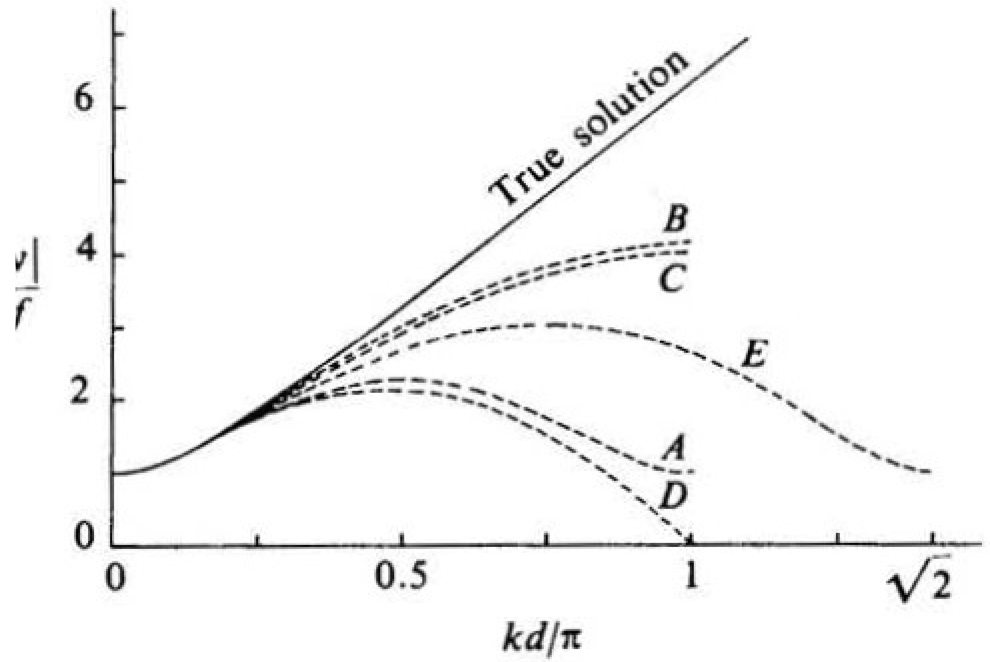


Figure 3.1 Five types of lattice considered for the finite difference solution of (3.1).



Mesinger and Arakawa, 1976

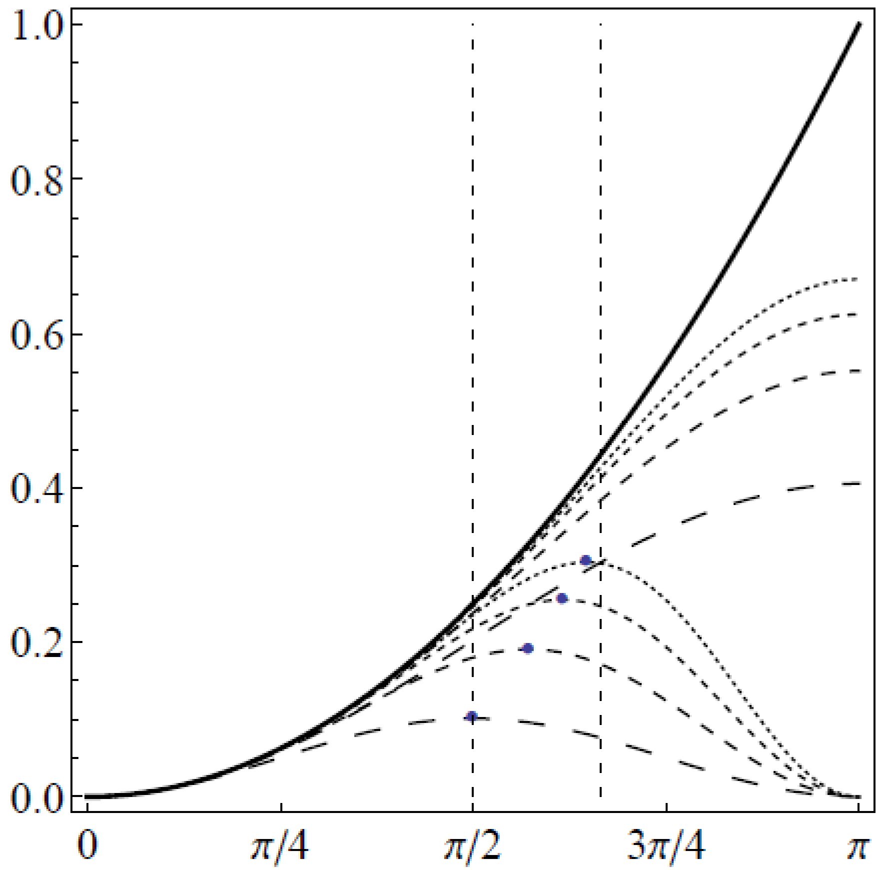
However, is A grid with finite differences/elements really so bad?

Pierre Bénard;

Summary and justification of a possible strategy for a local horizontal discretization of our future dynamical cores,

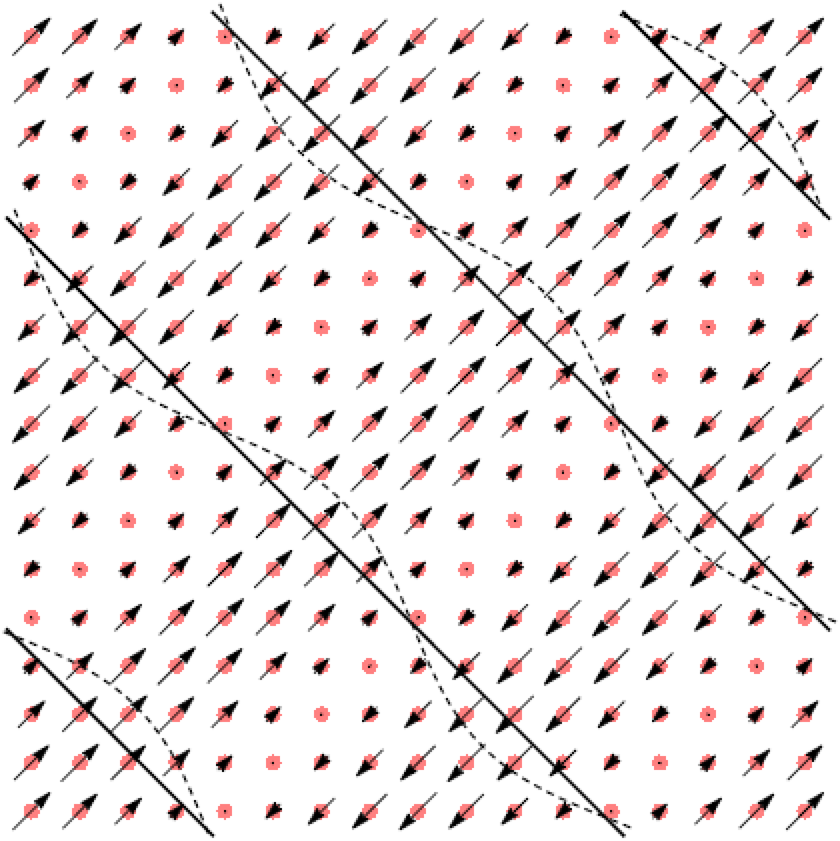
internal memorandum.

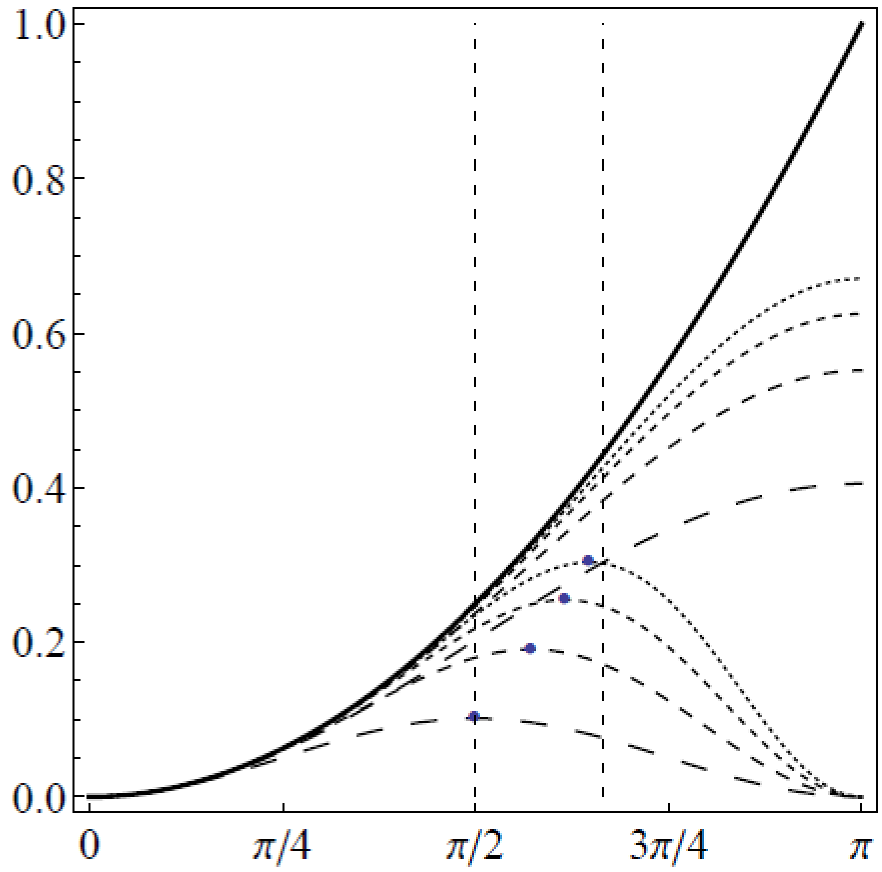
Square of the phase velocity of gravity waves



for A-grid (lowest group of curves), and for C-grid (middle group of curves) as a function of the normalized wave-number kDx (abcissa). Depicted accuracy orders are 2, 4, 6 and 8, from longest to shortest dashing patterns. Courtesy P. Bénard

Response of A Grid and C Grid for vortical mixing (Adv T)





Bottom curves: C-grid; Middle curves : A-grid. Top curve: exact response. The four curves for A and C grids are for accuracy orders 2, 4, 6, 8 in decreasing order of dashing length. Coutresy P. Bénard

So here C grid is bad and A grid is good

Z grid: best of both worlds?

1	inverse FFT, inverse Legendre transformation	
2	call physics in a parallel manner	$\mathcal{P}(X)$
3	update tendencies	
4	compute departure point **D** and interpolate to **D**	
5	explicit part dynamics	$(\mathcal{I} + \frac{\Delta t}{2}\mathcal{L}^*)X^0 + \Delta t(\mathcal{M} - \mathcal{L}^*)X^0$
6	add tendencies of adiabatic and diabatic processes	R
7	direct FFT, direct Legendre transformation	
8	Helmholtz, horizontal diffusion	$X^+ = (\mathcal{I} - \frac{\Delta t}{2}\mathcal{L}^*)^{-1}R$

$$R_u = u^0 - \frac{g\Delta t}{2}\left(\frac{\partial h}{\partial x}\right)^0 + \frac{f\Delta t}{2}v^0$$

$$R_v = v^0 - \frac{g\Delta t}{2}\left(\frac{\partial h}{\partial y}\right)^0 - \frac{f\Delta t}{2}u^0$$

$$R_h = h^0 - \frac{H_0\Delta t}{2}\left\{\left(\frac{\partial u}{\partial x}\right)^0 + \left(\frac{\partial v}{\partial y}\right)^0\right\}$$

Adv on A grid (u. v)

$$u^+ + \frac{g\Delta t}{2}\left(\frac{\partial h}{\partial x}\right)^+ - \frac{f\Delta t}{2}v^+ = R_u$$

$$v^+ + \frac{g\Delta t}{2}\left(\frac{\partial h}{\partial y}\right)^+ + \frac{f\Delta t}{2}u^+ = R_v$$

$$h^+ + \frac{H_0\Delta t}{2}\left\{\left(\frac{\partial u}{\partial x}\right)^+ + \left(\frac{\partial v}{\partial y}\right)^+\right\} = R_h$$

Z grid (vort, div)

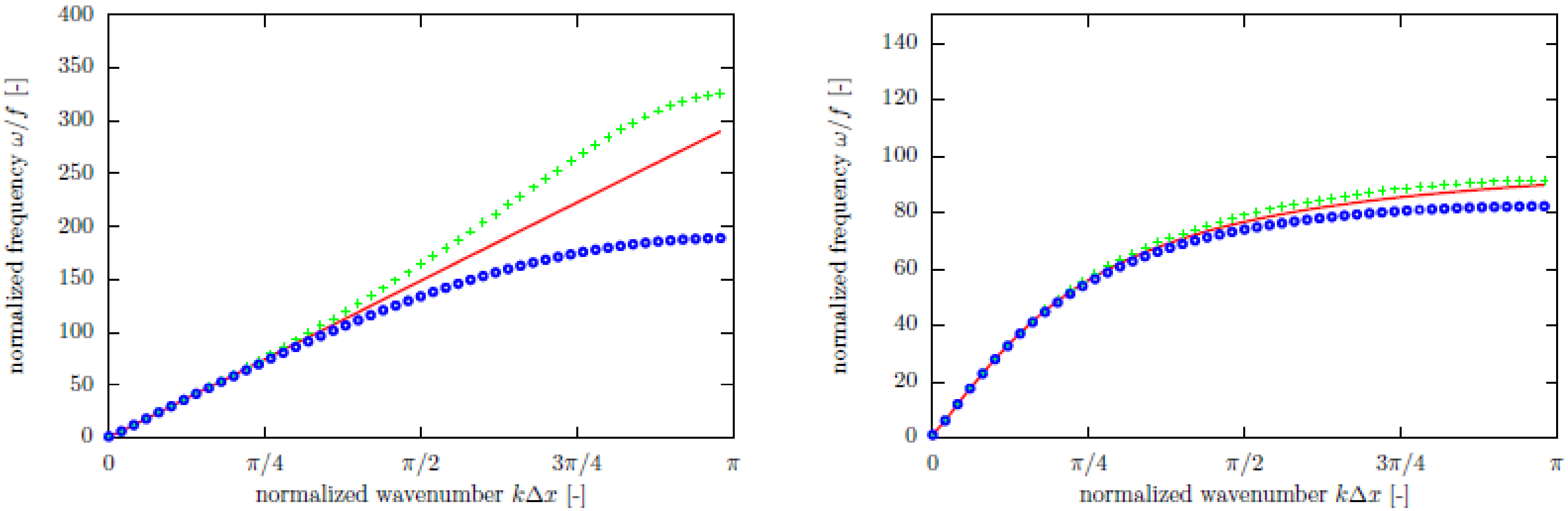
$$D^+ + \frac{g\Delta t}{2}\left\{\left(\frac{\partial^2 h}{\partial x^2}\right)^+ + \left(\frac{\partial^2 h}{\partial y^2}\right)^+\right\} - \frac{f\Delta t}{2}\zeta^+ = \frac{\partial R_u}{\partial x} + \frac{\partial R_v}{\partial y} \quad (14a)$$

$$\zeta^+ + \frac{f\Delta t}{2}D^+ = \frac{\partial R_v}{\partial x} - \frac{\partial R_u}{\partial y} \quad (14b)$$

$$h^+ + \frac{H_0\Delta t}{2}D^+ = R_h \quad (14c)$$

Here we need to solve a Poisson Eq.

Choosing the Z grid instead of an A grid leads also to correct dispersion relations, both for FE and FD

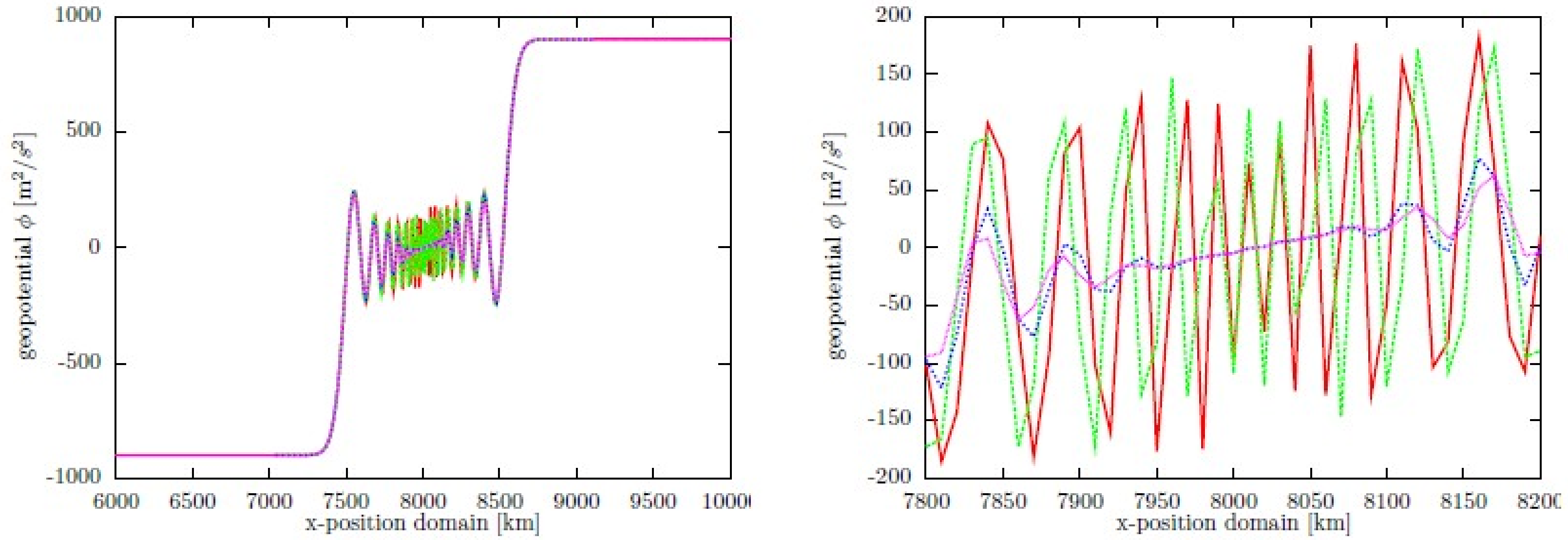


(a)

(b)

Figure 3. Normalized frequency of the IG wave of the symmetric 2TL SISL Z-grid scheme (27) combined with second order FD (blue circles) and linear FE (green crosses). The parameters are identical to the ones in Figure 2 with for the left figure $\Delta t = 10$s and for the right figure $\Delta t = 300$s. These plots are independent of the choice of $\mathcal{D}^*$. The spectral dispersion relation (red cont. line) is plotted as a reference.

With proper care, we can have a Z grid formulation with good dispersion relations/adjustment properties

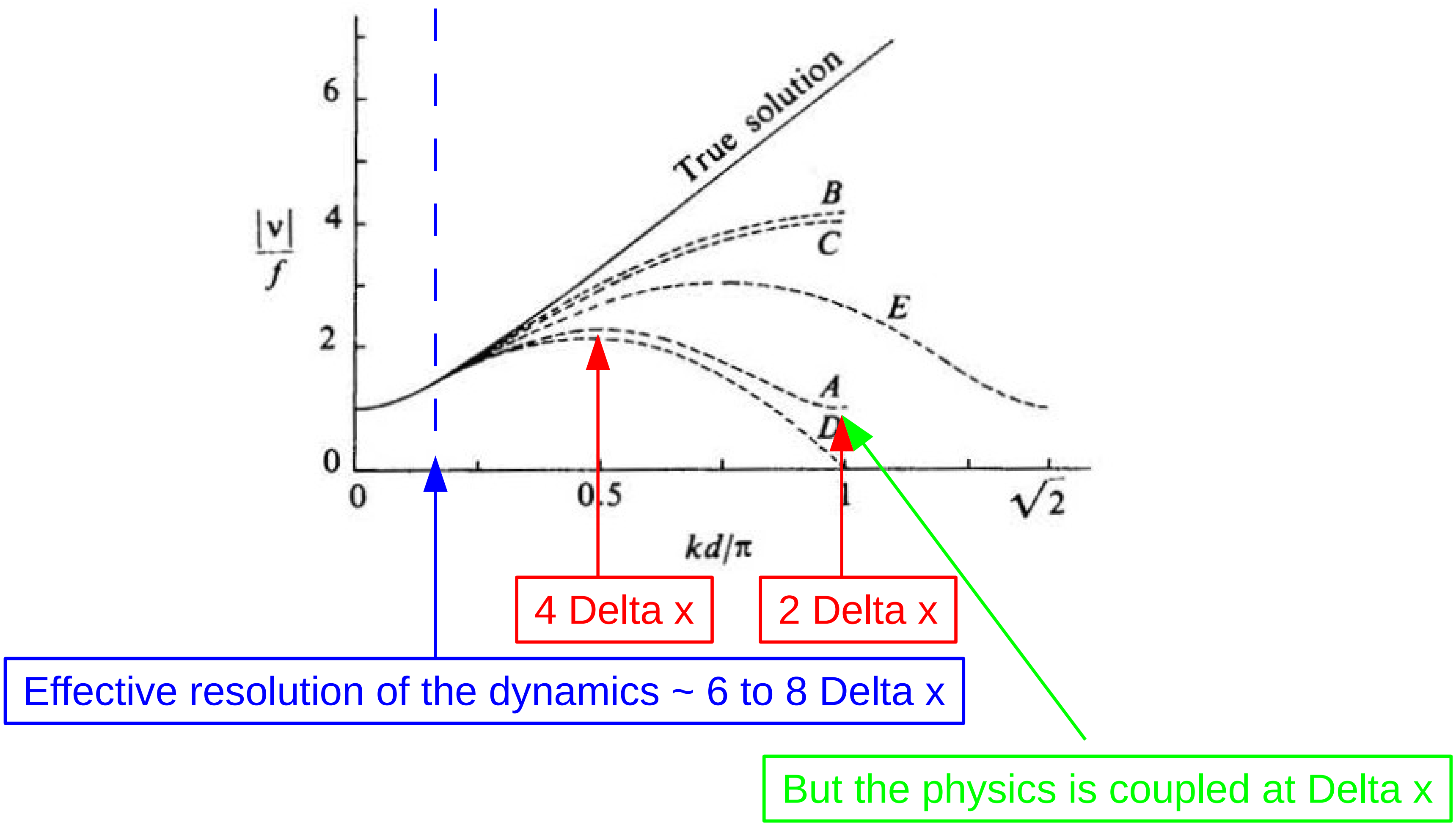


(a) Caluwaerts et al. (2014) (b)

Figure 5. The geopotential field after 20 timesteps in a 1D geostrophic adjustment test with parameters: $\Delta t = 300\mathrm{s}$, $\Delta x = 10\mathrm{km}$, $\Phi = 9000\mathrm{m}^2/\mathrm{s}^2$ and the height of the step is $1800\mathrm{m}^2/\mathrm{s}^2$. The right plot is a zoom in of the left plot around the location of the barrier. The spectral (red) and the linear FE symmetric scheme based on DZ reconstruction (green) show propagating IG waves whereas the TS87 scheme (magenta) and the linear FE TS12 scheme (blue) suppress the short scale IG waves.

Z grid: good dispersion, and advection is done in (u, v) set

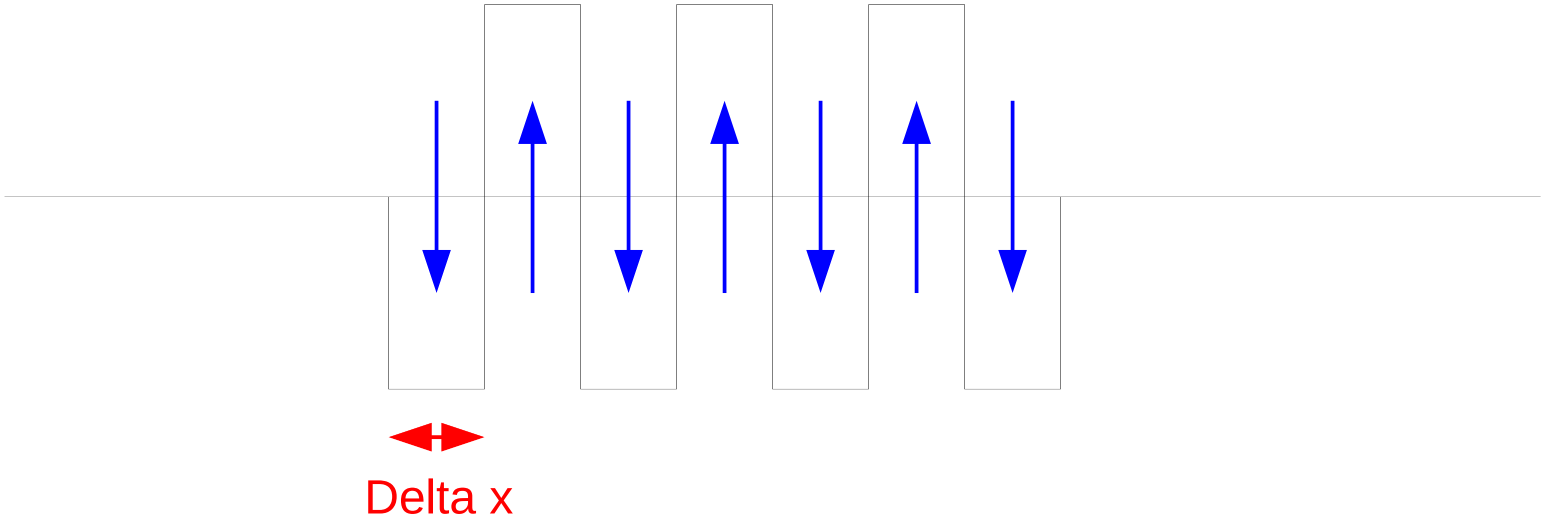
Should we care about the 2 Delta x mode? Physics people may ...



So the physics may excite a 2 Delta x waves. Does it matter for adjustment?

ALADIN

A train of diabatic boxes (very idealistic)



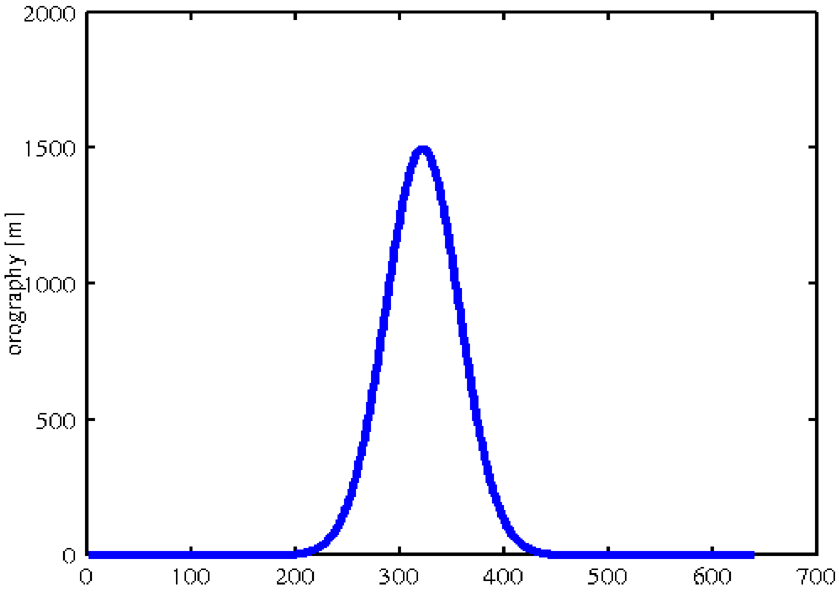
This is a 2 Delta x wave, but can be seen as a sequence of Heavisides on the previous slide (i.e. the adjustment of two slides ago), so a physics tendency like this will be suppressed by the dynamics.

Some claims …, are they true?

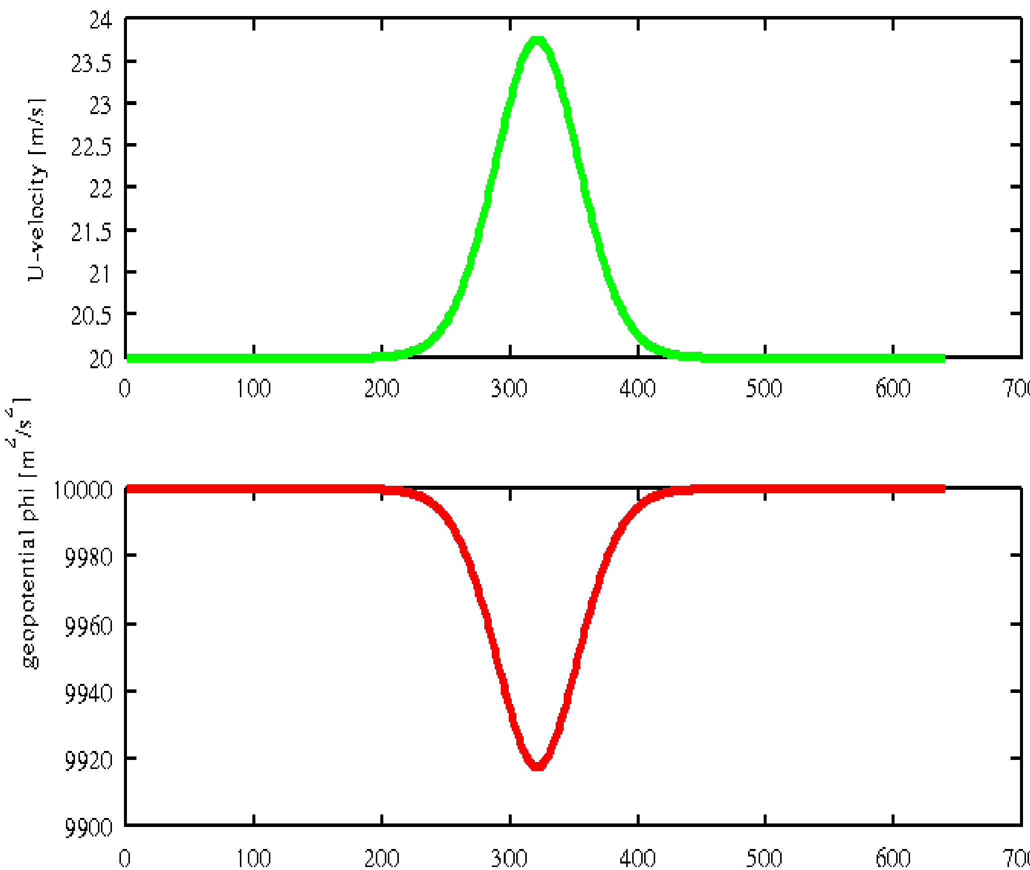
- “A grid with `local' discretizations is bad for dispersion relations” (so we need a C grid)
- “Spectral methods will break down at high resolutions for flows over steep slopes.”
- “Spectral methods will not be suitable for the future massively parallel machines (scalability)”
- “so we need a dynamics with a `local' discretization” (local means finite differences, finite elements, in contrast to spectral)

FE permits SISL treatment of some extra terms (e.g. orography), which is not possible if a spectral discretization is used.

To test potential advantages we compare the spectral SISL scheme with FE SISL schemes. A 1D stationary state over orography is used as initial state. For this test: f=0 (no Coriolis force). A 2TL predictor-corrector scheme is used for the integrations.



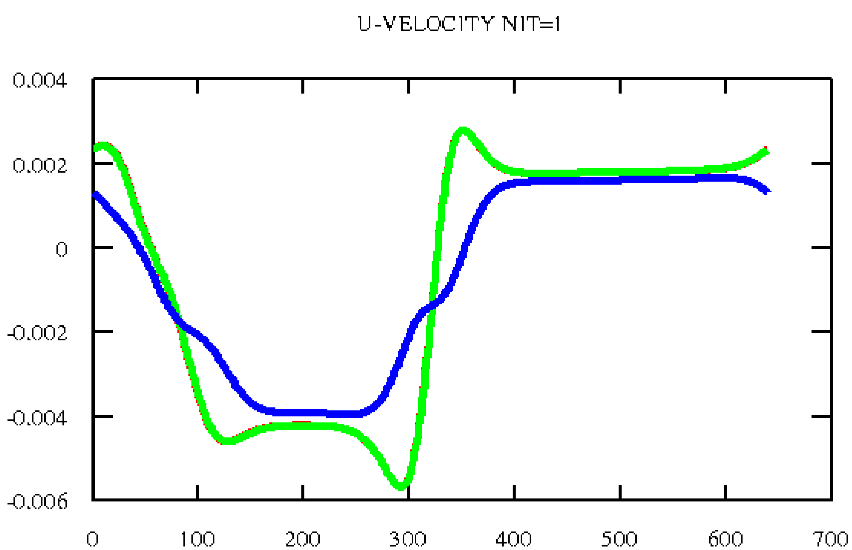
orography

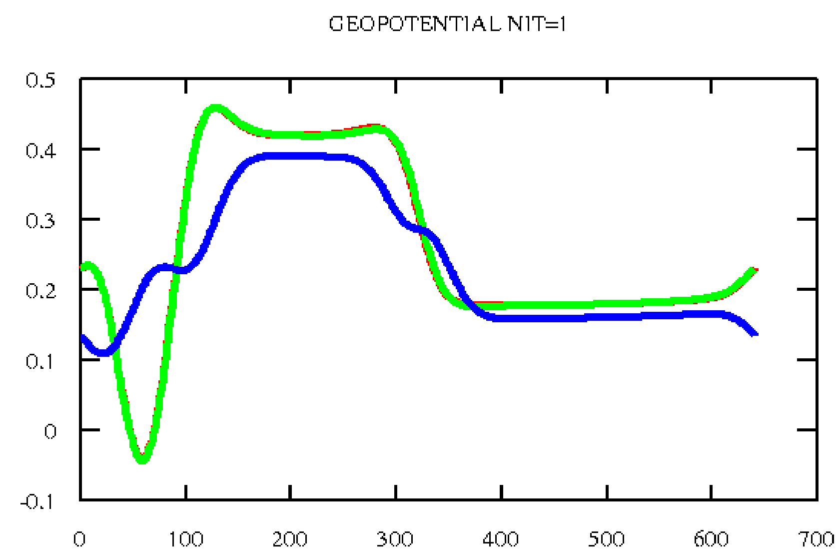


corresponding stationary state

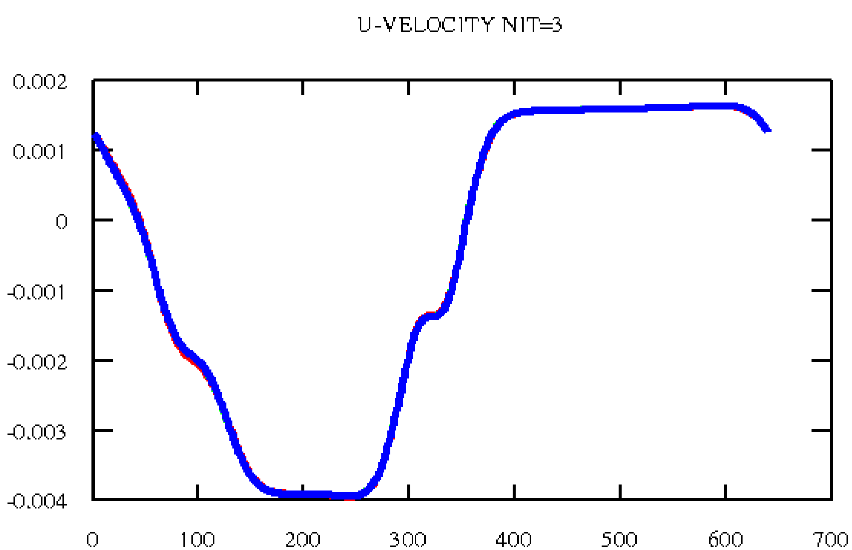
Differences with stationary state after 50 timesteps.
PRELIMINARY RESULTS

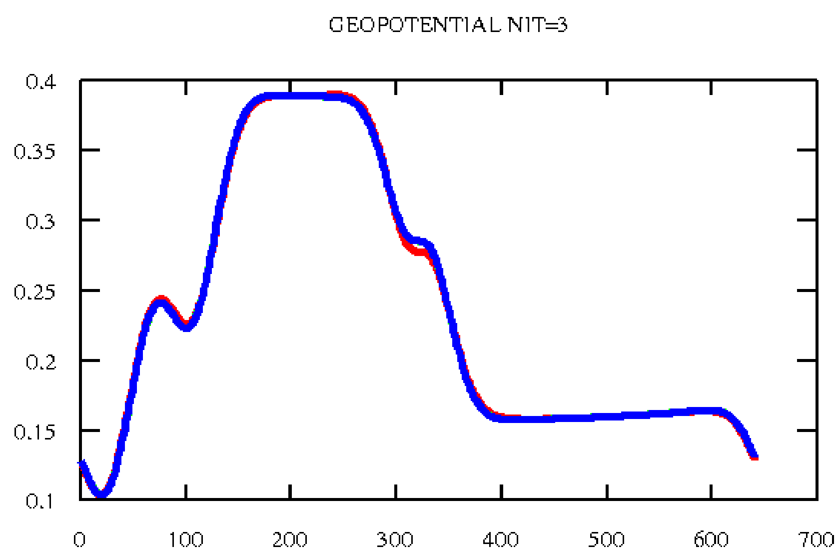
Green = FE SISL / Red = spectral SISL / Blue = FE SISL (orography included in SI)





0 iterations





1 iterations

Interpretation of the previous experiment

- differences with stationary state are very small for all schemes
- both FE and spectral discretization give very similar differences (compare red and green lines)
- iterations are not needed if the orography terms (orography * divergence) is treated in a SI-way (blue line). The other two schemes give similar differences after some iterations.
- we suspect that the remaining difference is due to the SL interpolations (cubic)

This strongly simplified test (1D without Coriolis terms) hints that by **adding the orography terms in the SI treatment one can avoid iterations** in the predictor-corrector scheme.

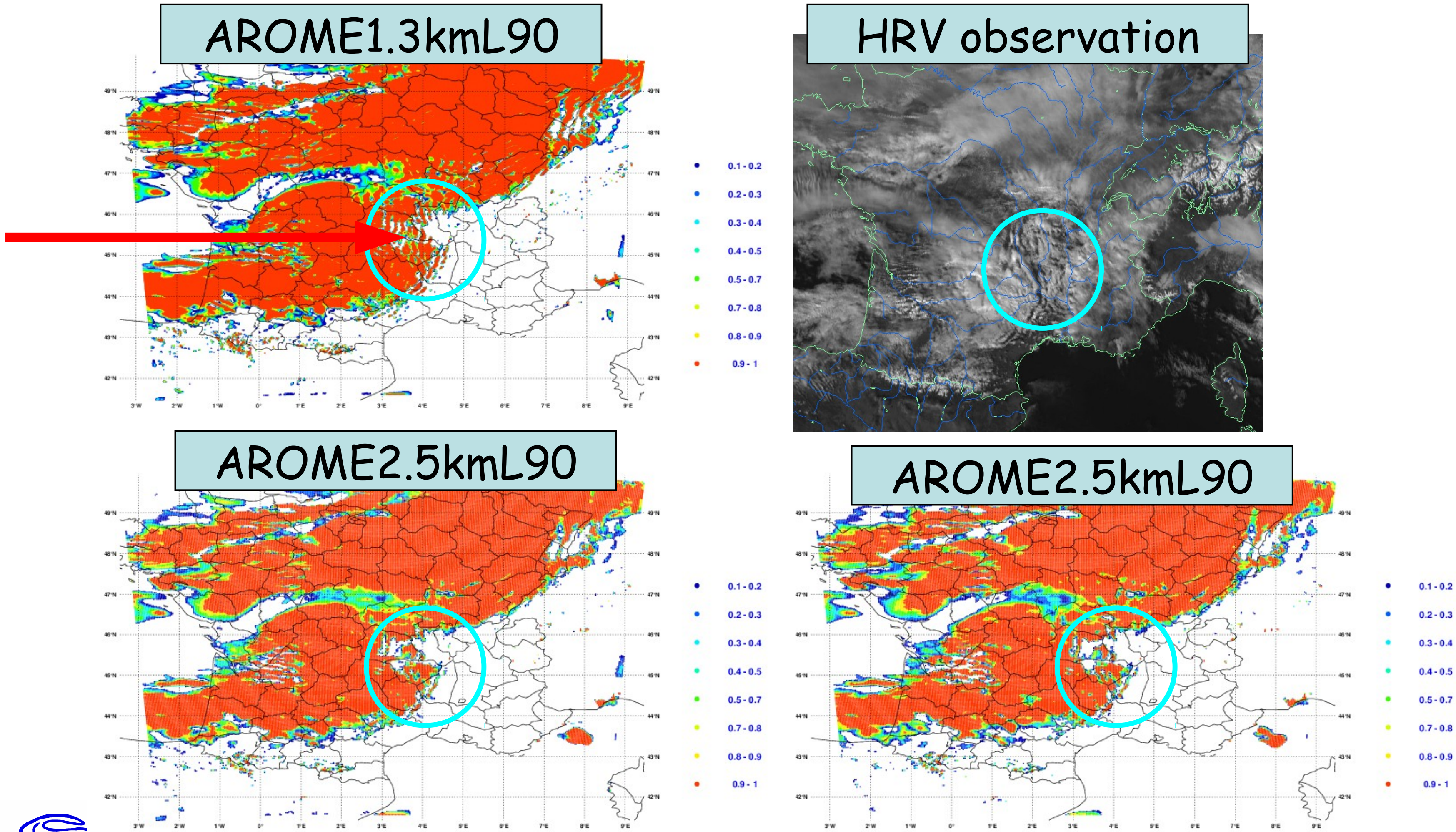
Further testing needed with different U_0 and longer/shorter time steps and the varying slopes of the mountain

- NDLON = 640
- DELTA_X = 1000 m
- PHI_0 = 10000 m^2/s^2
- U_0 = 20m/s
- DT = 30 s

+ the slope of the mountain

Increasing the resolution : Prototype AROME 1.3km

- Runs OK with dt=45s PC_CHEAP (NSITER=1), LGWADV
- Stronger NH impact at 1.3 km (orographic waves): 31st January 2013 +14TU



Courtesy Yann Seity

Additional advantages of A/Z grids

- No problems due to staggering: no corresponding computational modes.
- No impact on our operational chains/fields in file formats ...

disadvantages the Z grid

- Extra Poisson equation to solve. But this may be compensated by not having to use an iteration.

Conclusions

- Rather than making educated guesses about what could be the best option, we like to test them, first in a highly idealized set up.
- We propose(d) to implement an FE solver **next to** the spectral one. **We do NOT want to get rid of the spectral model!** At least such a testbed would allow to test,
  - Whether the 2 Delta x mode really plays a role, testing a good dispersion (spectral Zgrid) relation w.r.t. To a “wrong” one (A grid) with highly fragmented physics fields.
  - Whether an implicit treatment of the orography may decrease the need for an iteration in the solver (in terms of accuracy).
- The goal is to carry out *clean* tests (keeping all else equal) to see how long (in terms of resolution) spectral methods will be safe
- Ideally we would find that we will be safe with spectral methods for a long time...