Computations of moist turbulent fluxes with the moist air entropy variable.

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1 Introduction.

Most (may be all?) present turbulent schemes are derived starting from systems of equations expressed with the flux of potential temperature $\overline{w'\theta'}$ and of water vapor $\overline{w'q'_v}$. It is then at the end of the computations that an important hypothesis is made: to replace (θ, q_v) by the couple of so-called Betts's (1973) variables (θ_l, q_t). The Betts (1973) variables are the potential temperature and the total water content defined by

$$\theta_l \;=\; \theta \, \exp\left(- \, \frac{L_{vap}\, q_l + L_{sub}\, q_i}{c_{pd}\, T} \right) \;\approx\; \theta \, \left(1 \; - \; \frac{L_{vap}\, q_l + L_{sub}\, q_i}{c_{pd}\, T} \right) \tag{1}$$

$$q_t \;=\; q_v \; + \; q_l \; + \; q_i \; . \tag{2}$$

The last part of (1) is obtained with the approximation $\exp(x) \approx 1 \, + \, x$, valid for small x.

The aim of this note is to analyze some of the consequences if θ_l were replaced by a quantity associated with the moist air entropy in a moist-air turbulent schemes. As an example of the turbulent scheme, we will consider the moist version of the turbulent scheme of Cuxart *et al.* (2000, i.e."CBR00") proposed in Masson (2013). As for the moist air entropy variable, it will be represented by the moist-air entropy potential temperature θ_s defined in Marquet (2011).

2 The specific moist air entropy.

The specific moist air entropy is defined in Marquet (2011) by

$$s \;=\; s_{ref} \; + \; c_{pd} \, \ln(\theta_s) \; , \tag{3}$$

where s_{ref} and c_{pd} are two constant terms and where the moist air entropy potential temperature writes

$$\theta_s \;=\; (\theta_s)_1 \, \left(\frac{T}{T_r} \right)^{\lambda \, q_t} \left(\frac{p}{p_r} \right)^{-\kappa \, \delta \, q_t} \left(\frac{r_r}{r_v} \right)^{\gamma \, q_t} \frac{(1 + \eta \, r_v)^{\,\kappa \, (1 + \delta \, q_t)}}{(1 + \eta \, r_r)^{\,\kappa \, \delta \, q_t}} \; . \tag{4}$$

The quantity $(\theta_s)_1$ is defined by

$$(\theta_s)_1 \;=\; \theta \, \exp\left(- \, \frac{L_{vap}\, q_l + L_{sub}\, q_i}{c_{pd}\, T} \right) \exp\left(\Lambda_r \, q_t \right) \tag{5}$$

$$(\theta_s)_1 \;=\; \theta \, \exp\left(- \, \frac{L_{vap}\, q_l + L_{sub}\, q_i}{c_{pd}\, T} \; + \; \Lambda_r \, q_t \right) \tag{6}$$

where $\Lambda_r = (s_v^0 - s_d^0)/c_{pd} \approx 5.87$ is a key quantity. It depends on the standard entropies of water vapor and dry air (s_v^0 and s_d^0) and it is computed in Marquet (2011) by using the Third law of thermodynamics (the Nernst's theorem).

It is shown in Marquet (2011), Marquet and Geleyn (2013) and Marquet (2013a, 2013b) that $(\theta_s)_1$ given by (5 is a good approximation of θ_s given by the full formula (4).

It is possible to further approximate $(\theta_s)_1$ by the formula

$$(\theta_s)_1 \;\approx\; \theta \, \left(1 \;-\; \frac{L_{vap}\, q_l + L_{sub}\, q_i}{c_{pd}\, T} \;+\; \Lambda_r \, q_t \right) \, , \tag{7}$$

where the exponential function is approximated by $\exp(x) \approx 1 \, + \, x$, like in the last part of (1).

For sake of simplicity, only the case of water vapour and without condensed water will be presented in this first version of the internal note.

This first non-saturated study is already an important step, because θ_s and $(\theta_s)_1$ defined by (4) to (7) are different from $\theta_l = \theta$ in case of water vapour, independently of existing cloud condensed water, or not. Moreover, the impact of q_v on $(\theta_s)_1$ is large. It is in particular much larger than the impact on the buoyancy potential temperature defined by

$$\theta_v \;=\; \theta \; (1 \, + \, \delta \, q_v \, - \, q_l \, - \, q_i) \, , \tag{8}$$

since $\Lambda_r \approx 5.87$ for θ_s is about ten time larger than $\delta \approx 0.608$ for θ_v. In fact, the impact is about 2/3 of the impact of q_v on the equivalent potential temperature defined by

$$\theta_e \;=\; \theta \, \exp\left(\frac{L_{vap}\, q_v}{c_{pd}\, T} \right) \;\approx\; \theta \, \left(1 \, + \, \frac{L_{vap}\, q_v}{c_{pd}\, T} \right) \, , \tag{9}$$

simply because $L_{vap}/(c_{pd}\, T) \approx 9$ is about 2/3 larger than $\Lambda_r \approx 6$.

Differently, there is no impact of q_v on θ_l. It is the reason why the use of the moist entropy potential temperature might lead to important differences even if no cloud exist, for instance in the moist (non-saturated) PBL.

3 The 1D CBR00-modified scheme - Masson (2013).

As explained before, only the case of water vapour will be presented in this first version of the internal note. Accordingly, the non-saturated (or just saturated / cloud-free) thermodynamic variables used in the next sections are equal to

$$\theta_l \;=\; \theta \, , \tag{10}$$

$$\theta_v \;=\; \theta \; (1 \, + \, \delta \, q_v) \, , \tag{11}$$

$$\theta_s \;\approx\; \theta \; (1 \, + \, \Lambda_r \, q_v) \, . \tag{12}$$

The purpose of this note is to compute the turbulent flux of the moist air entropy potential temperature by two methods :

- 1) by computing $\overline{w'\theta'_s}$ with $(\theta, \, q_v)$ replaced directly by $(\theta_s, \, q_v)$ in the non-saturated version of the 1D-scheme derived in section 0.3 of Masson (2013);

- 2) by computing $\overline{w'\theta'_s}$ as a function of the fluxes $\overline{w'\theta'_l}$ and $\overline{w'q_v}$, if $\overline{w'\theta'_l}$ and $\overline{w'q_v}$ are computed with the non-saturated version of the 1D-scheme system 0.3 of Masson (2013) where $(\theta,\, q_v)$ is replaced by $(\theta_l,\, q_v)$.

The first order fluxes $\overline{w'\theta'}$ and $\overline{w'q'}$ defined in section 0.3 of Masson (2013) can be rewritten as

$$\overline{w'\theta'} \;=\; -\,K_\theta\,\frac{\partial\Theta}{\partial z} \;+\; \Gamma_\theta\,E_\theta\,\overline{(\theta')^2} \;+\; \Gamma_\theta\,E_q\,\overline{\theta' q'}\;, \tag{13}$$

$$\overline{w'q'} \;=\; -\,K_q\,\frac{\partial Q}{\partial z} \;+\; \Gamma_q\,E_\theta\,\overline{q'\,\theta'} \;+\; \Gamma_q\,E_q\,\overline{(q')^2}\,. \tag{14}$$

The generic terms $(\theta,\, q)$ represent $(\theta_l,\, q_t)$, and so $(\theta,\, q_v)$ in non-saturation conditions. It is assumed that the eddies are defined by $\theta = \Theta + \theta'$ and $q = Q + q'$.

The exchange coefficients are defined by

$$K_\theta \;=\; \frac{L}{C_{p\theta}\,\sqrt{e}}\,\overline{(w')^2}\;, \tag{15}$$

$$K_q \;=\; \frac{L}{C_{pq}\,\sqrt{e}}\,\overline{(w')^2}\;, \tag{16}$$

where L represents the mixing length, e the turbulent kinetic energy and with $C_{p\theta}$ and C_{pq} two constants. These two constants are set to a common value in CBR00 and Masson (2013), and in (almost ?) all turbulent schemes.

The Gamma coefficients in front of the second order fluxes are equal to

$$\Gamma_\theta \;=\; \frac{2}{3}\,\beta\,\frac{L}{C_{p\theta}\,\sqrt{e}}\;, \tag{17}$$

$$\Gamma_q \;=\; \frac{2}{3}\,\beta\,\frac{L}{C_{pq}\,\sqrt{e}}\;, \tag{18}$$

where $\beta = g/\Theta$.

The E-terms represent a way to express the flux of buoyancy potential temperature in terms of the basic first order fluxes (13) and (14). From (11), the non-saturated flux of $\theta_v^\star$ is equal to

$$\overline{w'\,\theta'_v} \;=\; (1\,+\,\delta\,Q\,)\;\overline{w'\,\theta'}\;+\;(\delta\,\Theta)\;\overline{w'\,q'_v}\;, \tag{19}$$

$$\overline{w'\,\theta'_v} \;=\; E_\theta\;\overline{w'\,\theta'}\;+\;E_q\;\overline{w'\,q'_v}\;, \tag{20}$$

leading to

$$E_\theta \;=\; 1\,+\,\delta\,Q\;, \tag{21}$$

$$E_q \;=\; \delta\,\Theta\,. \tag{22}$$

4 $E^{\star}$-terms for the moist entropy potential temperature.

Let us determine the $E^{\star}$-terms associated to the moist air entropy potential temperature. From (12), the non-saturated flux of θ_s is equal to

$$\overline{w'\,\theta'_s} \;=\; (1\,+\,\Lambda_r\,Q)\;\overline{w'\,\theta'}\;+\;(\Lambda_r\,\Theta)\;\overline{w'\,q'_v}\;. \tag{23}$$

If $\overline{w'\,\theta'}$ given by (23) is replaced in (19), the result is

$$\overline{w'\,\theta'_v} \;=\; \left(\frac{1\,+\,\delta\,Q}{1\,+\,\Lambda_r\,Q}\right)\,\overline{w'\,\theta'_s}\;+\;\Theta\,\left(\delta\;-\;\frac{\Lambda_r}{1\,+\,\Lambda_r\,Q}\right)\,\overline{w'\,q'_v}\;, \tag{24}$$

$$\overline{w'\,\theta'_v} \;=\; E^{\star}_{\theta}\;\overline{w'\,{\theta_s}'}\;+\;E^{\star}_{q}\;\overline{w'\,q'_v}\;, \tag{25}$$

leading to the $E^{\star}$-terms corresponding to the moist entropy potential temperature

$$E^{\star}_{\theta} \;=\; \frac{1\,+\,\delta\,Q}{1\,+\,\Lambda_r\,Q}\;, \tag{26}$$

$$E^{\star}_{q} \;=\; \Theta\;\left(\delta\;-\;\frac{\Lambda_r}{1\,+\,\Lambda_r\,Q}\right)\;. \tag{27}$$

It is easy to verify that the non-saturated CBR00 formulas (21) and (22) for $\theta_l = \theta$ are obtained from (26) and (27) with $\Lambda_r = 0$, which corresponds to $\theta_s = \theta$.

5 The first (non-saturated) method.

Let us compute $\overline{w'\theta'_s}$ with the generic variables $(\theta,\,q)$ replaced directly by $(\theta_s,\,q_v)$ in the non-saturated version of the 1D-scheme (13). The result is

$$\left(\overline{w'\,\theta'_s}\right)_1 \;=\; -\,K_s\,\frac{\partial\Theta_s}{\partial z}\;+\;\Gamma_s\;E^{\star}_{\theta}\;\overline{(\theta'_s)^2}\;+\;\Gamma_s\;E^{\star}_{q}\;\overline{\theta'_s\,q'_v}\;. \tag{28}$$

The terms $E^{\star}_{\theta}$ and $E^{\star}_{q}$ are given by (26) and (27). The moist air entropy exchange coefficient is equal to

$$K_s \;=\; \frac{L}{C_{ps}\,\sqrt{e}}\,\overline{(w')^2}\;, \tag{29}$$

where C_{ps} is a moist air entropy counterpart of the tow constants $C_{p\theta}$ and C_{pq}, to be determined.

The Gamma coefficients in front of the second order fluxes are then equal to

$$\Gamma_s \;=\; \frac{2}{3}\,\beta\,\frac{L}{C_{ps}\,\sqrt{e}}\;. \tag{30}$$

It is assumed that the definition (12) for θ_s corresponds to the following equation linking the vertical derivatives of the mean variables $\Theta_s = \overline{\theta_s}$, $\Theta = \overline{\theta}$ and $Q = \overline{q_v}$.

$$\frac{\partial\Theta_s}{\partial z} \;=\; (1\,+\,\Lambda_r\,Q)\;\frac{\partial\Theta}{\partial z}\;+\;(\Lambda_r\,\Theta)\;\frac{\partial Q}{\partial z}\;. \tag{31}$$

The second order flux $\overline{(\theta'_s)^2}$ can be written as

$$\overline{(\theta'_s)^2} \;=\; \overline{[\,\theta\,(1\,+\,\Lambda_r\,q_v)\,]'^{\,2}}\,, \tag{32}$$

$$\overline{(\theta'_s)^2} \;=\; \overline{\{\,[\,(1\,+\,\Lambda_r\,Q)\,\theta'\,]\,+\,[\,\Lambda_r\,\Theta\,q'_v\,]\,\}^2}\,, \tag{33}$$

leading to the following weighting sum of the three second order fluxes

$$\overline{(\theta'_s)^2} \;=\; (\,1\,+\,\Lambda_r\,Q\,)^2\;\overline{(\theta')^2}\;+\;(\,\Lambda_r\,\Theta\,)^2\;\overline{(q'_v)^2}\;+\;2\;(\,\Lambda_r\,\Theta\,)\,(\,1\,+\,\Lambda_r\,Q\,)\;\overline{\theta'\,q'_v}\,. \tag{34}$$

Similarly, the second order fluxes $\overline{\theta'_s\,q'_v}$ can be written as

$$\overline{\theta'_s\,q'_v} \;=\; \overline{\{\,[\,(1\,+\,\Lambda_r\,Q)\,\theta'\,]\,+\,[\,\Lambda_r\,\Theta\,q'_v\,]\,\}\,\{\,q'_v\,\}}\,, \tag{35}$$

leading to the following weighting sum of two of the second order fluxes

$$\overline{\theta'_s\,q'_v} \;=\; (\,1\,+\,\Lambda_r\,Q\,)\;\overline{\theta'\,q'_v}\;+\;(\,\Lambda_r\,\Theta\,)\;\overline{(q'_v)^2}\,. \tag{36}$$

According to all previous results, the flux of moist entropy potential temperature (28) can be rewritten as

$$\begin{aligned}
\left(\overline{w'\,\theta'_s}\right)_1 \;=\;& -\,K_s\;(1\,+\,\Lambda_r\,Q)\;\frac{\partial\Theta}{\partial z}\;-\;K_s\;(\Lambda_r\,\Theta)\;\frac{\partial Q}{\partial z}\\
&+\Gamma_s\;(\,1\,+\,\delta\,Q\,)\;(\,1\,+\,\Lambda_r\,Q\,)\;\overline{(\theta')^2}\\
&+\Gamma_s\;\left(\frac{1\,+\,\delta\,Q}{1\,+\,\Lambda_r\,Q}\right)\;(\,\Lambda_r\,\Theta\,)^2\;\overline{(q'_v)^2}\\
&+\Gamma_s\;(\,1\,+\,\delta\,Q\,)\;(\,2\,\Lambda_r\,\Theta\,)\,\overline{\theta'\,q'_v}\\
&+\Gamma_s\;\left(\delta\;-\;\frac{\Lambda_r}{1\,+\,\Lambda_r\,Q}\right)\Theta\;(\,1\,+\,\Lambda_r\,Q\,)\;\overline{\theta'\,q'_v}\\
&+\Gamma_s\;\left(\delta\;-\;\frac{\Lambda_r}{1\,+\,\Lambda_r\,Q}\right)(\,\Lambda_r\,\Theta^2\,)\;\overline{(q'_v)^2}\,.
\end{aligned} \tag{37}$$

The terms rearrange into

$$\begin{aligned}
\left(\overline{w'\,\theta'_s}\right)_1 \;=\;& -\,K_s\;(1\,+\,\Lambda_r\,Q)\;\frac{\partial\Theta}{\partial z}\;-\;K_s\;(\Lambda_r\,\Theta)\;\frac{\partial Q}{\partial z}\\
&+\Gamma_s\;(\,1\,+\,\delta\,Q\,)\;(\,1\,+\,\Lambda_r\,Q\,)\;\overline{(\theta')^2}\\
&+\Gamma_s\;\;(\,\delta\,\Theta\,)\,(\,1\,+\,\Lambda_r\,Q\,)\;\overline{\theta'\,q'_v}\\
&+\Gamma_s\;(\,1\,+\,\mathbf{2}\,\delta\,Q\,)\;(\,\Lambda_r\,\Theta\,)\;\overline{q'_v\,\theta'}\\
&+\Gamma_s\;\;(\,\delta\,\Lambda_r\,\Theta^2\,)\left[\,1+\left(\frac{\mathbf{\Lambda_r\,Q}}{\mathbf{1\,+\,\Lambda_r\,Q}}\right)\right]\overline{(q'_v)^2}\,.
\end{aligned} \tag{38}$$

6 The second (non-saturated) method.

Let us compute the same flux $\overline{w'\theta'_s}$ as in (28), but by using the fluxes $\overline{w'\theta'}$ and $\overline{w'q_v}$ expressed by the non-saturated version of the 1D-scheme (13) and (14), with the generic variables (θ, q) replaced as usual by ($\theta_l = \theta$, $q_t = q_v$).

This flux is already computed in (23), yielding

$$\left(\overline{w'\,\theta'_s}\right)_2 \;=\; (1 \,+\, \Lambda_r\, Q)\;\overline{w'\,\theta'} \,+\, (\Lambda_r\,\Theta)\;\overline{w'\,q'_v}\,. \tag{39}$$

The first order fluxes $\overline{w'\,\theta'}$ and $\overline{w'\,q'_v}$ are given by (13) and (14), with E_θ and E_q given by (21) and (22), leading to

$$\begin{aligned}
\left(\overline{w'\,\theta'_s}\right)_2 \;=\; & -K_\theta\;(1 \,+\, \Lambda_r\, Q)\;\frac{\partial\Theta}{\partial z} - \;K_q\;(\Lambda_r\,\Theta)\;\frac{\partial Q}{\partial z} \\
& +\Gamma_\theta\;(\,1 \,+\, \delta\,Q\,)\;(\,1 \,+\, \Lambda_r\,Q\,)\;\overline{(\theta')^2} \\
& +\Gamma_\theta\;\;(\,\delta\,\Theta\,)\,(\,1 \,+\, \Lambda_r\,Q\,)\;\overline{\theta'\,q'_v} \\
& +\Gamma_q\;(\,1 \,+\, \delta\,Q\,)\;(\,\Lambda_r\,\Theta\,)\;\overline{q'_v\,\theta'} \\
& +\Gamma_q\;\;\left(\,\delta\,\Lambda_r\,\Theta^2\,\right)\overline{(q'_v)^2}\,.
\end{aligned} \tag{40}$$

7 Comparison of the two (non-saturated) method.

Comparisons of (38) and (40) show that the two methods do not lead to the same results for the turbulent fluxes of moist-air entropy potential temperature.

The first result is that even for the flux-gradient case (i.e. if all the Γ's terms are equal to 0), the two formulations are equal to each others if and only if $K_s = K_\theta = K_q$, or equivalently in terms of the constants of the scheme: $C_{ps} = C_{p\theta} = C_{pq}$. This result is obtained by identifying the first lines of (38) and (40).

It seems that this assumption $C_{ps} = C_{p\theta} = C_{pq}$ is made in most of the turbulent schemes. However, the drag coefficient for water fluxes is sometimes set to a different value than the one for heat in some surface schemes over the ocean (like possibly $C_D \neq C_H \neq C_E$ in ECUME-SURFEX). This result is suggested by observations campains (POMME, FETCH, SEMAPHORE, CATCH, EQUALANT99). In that case, the flux of moist air entropy depends on the method chosen to compute it.

Moreover, results published in Siebesma et al. (2003, ATEX-S03) and Stevens et al. (2001, BOMEX-S01) show that K_θ might be different from K_q, as shown in Figs.(1) for ATEX and in Figs.(2) for BOMEX. Clearly $K_\theta < K_q$ within the in-cloud regions and within the PBL regions bellow the cloud, for both ATEX and BOMEX.

However, it is worth noticing that the exchange coefficients are determined from the crude formulas $\overline{w'\theta'} = -\,K_\theta\,\partial\,\Theta/\partial z$ and $\overline{wq'_v} = -\,K_q\,\partial\,Q/\partial z$, with the second order fluxes missing. It is difficult to know the impact of the missing terms in the computed values of K_θ and K_q.

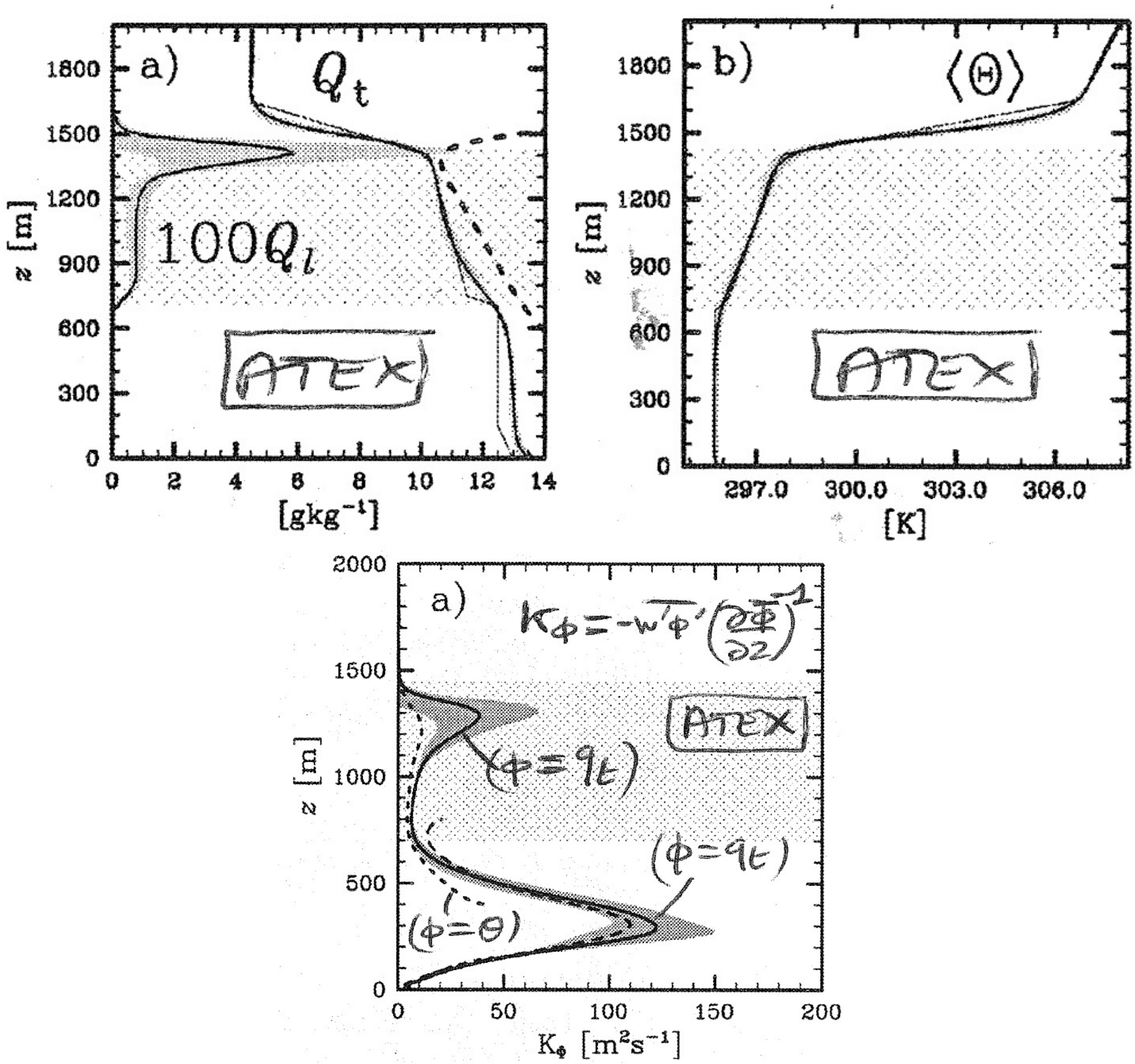

Figure 1: *The vertical profiles for ATEX (Stevens et al, 2001).*

The second result is obtained if the Γ's terms are different from 0, with all the second order fluxes acting in (38) and (40). The equality of the three constants (C_{ps}, $C_{p\theta}$, C_{pq}) implies that $\Gamma_s = \Gamma_\theta = \Gamma_q$. But even for this simplified case, differences exist between (38) and (40). These differences are highlighted in red in (38). First, the factor $(1 + \delta Q)$ in front of the flux $\overline{q'_v\,\theta'}$ in (40) is replaced by the factor $(1 + 2\,\delta Q)$ in (38). Second, there is a new factor $[1 + (\Lambda_r\,Q)\,/(1+\Lambda_r\,Q)]$ in front of the flux $\overline{(q'_v)^2}$ in (38).

8 Conclusion.

This is a first preliminary study, limited to the non-saturated (or the just-saturated) case. But this preliminary study already shows that the numerical value of the moist air-entropy flux may depend on the way this flux is computed.

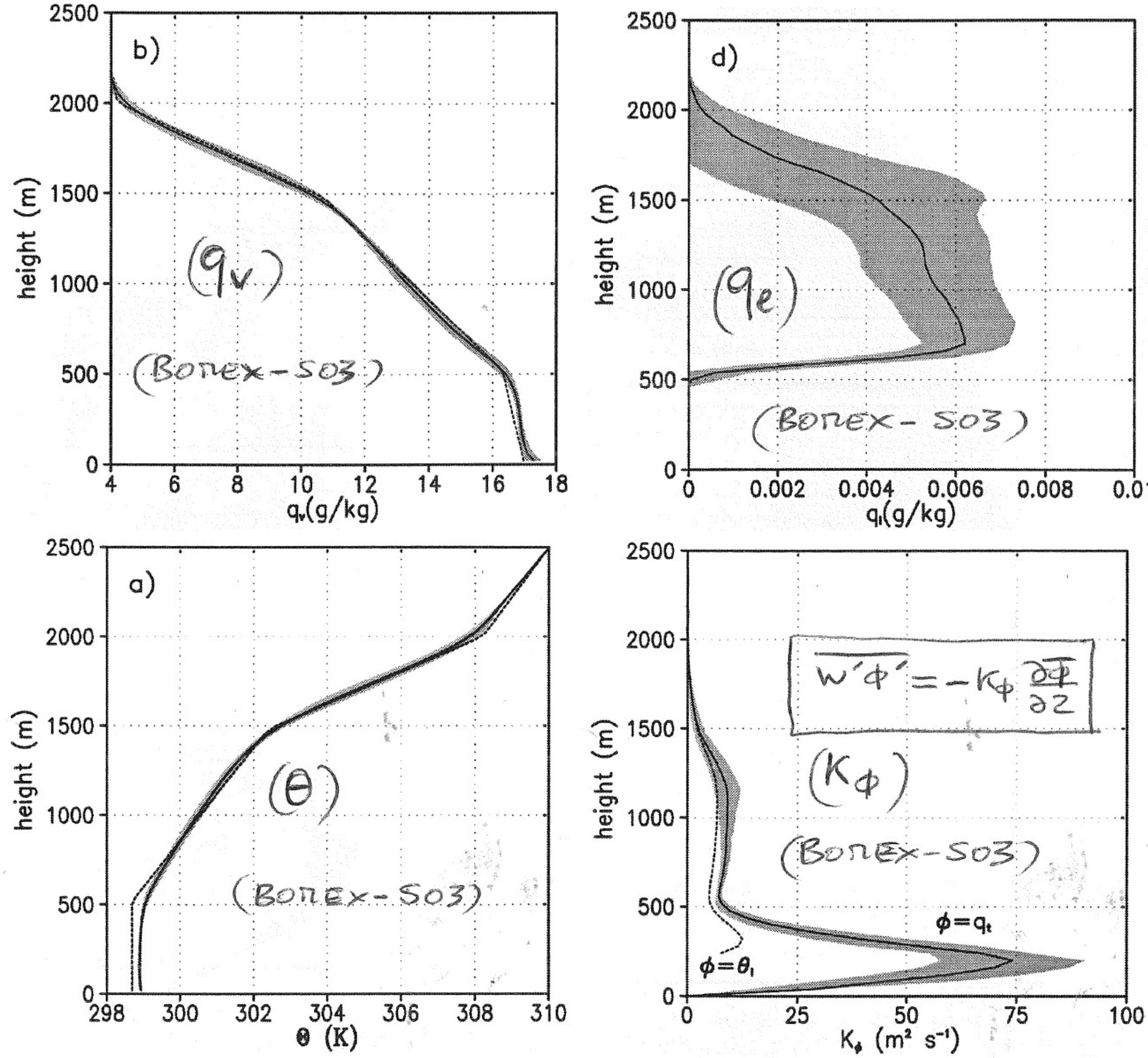

Figure 2: *The vertical profiles for BOMEX (Siebesma et al, 2003).*

If the moist air entropy is indeed a key (thermal) variable to be used in the mixing processes (turbulence and convection), it is not equivalent to use (θ_l, q_t) or (θ_s, q_t), except in the simplified case where $C_{ps} = C_{p\theta} = C_{pq}$ and $\Gamma_s = \Gamma_\theta = \Gamma_q = 0$.

Differences are generated by the non-linearities and by the second order fluxes in the turbulent scheme equations. Larger differences may appear as soon as the more realistic choice for $C_{ps} \neq C_{p\theta} \neq C_{pq}$ will be managed in the future.

9 References.

• Betts AK. (1973). Non-precipitating cumulus convection and its parameterization. *Q. J. R. Meteorol. Soc.* **99** (419): 178–196.

• Cuxart J, Bougeault Ph, Redelsperger J-L. (2000) A turbulence scheme allowing for mesoscale *and large-eddy simulations. Quart. J. R. Met. Soc.*, **126**, 1–30.

• Marquet P. (2011). Definition of a moist entropic potential temperature. Application to FIRE-I data flights. *Q. J. R. Meteorol. Soc.* **137**, (656): 768-791. Published paper is available on `http://perso.numericable.fr/~pascath31/marquet_2011_QJ787.pdf`)

• Marquet P, Geleyn J-F. (2013). On a general definition of the squared Brunt-Väisälä frequency associated with the specific moist entropy potential temperature. *Q. J. R. Meteorol. Soc.* **139** (670) : 85–100. Last revised version of the paper available on `http://perso.numericable.fr/~pascath31/QJ4_N2m_V2_20120131b_sp.pdf`)

• Marquet P (2013a). Computations of moist air thermal enthalpy. Plotting of Mollier, Bernoulli and available enthalpy diagrams. *Q. J. R. Meteorol. Soc.* Revised version sent on 17th of March 2013. Available on `http://perso.numericable.fr/~pascath31/QJ4R1_Hm_20130317a_large.pdf`)

• Marquet P (2013b). On the definition of a moist-air potential vorticity. *Q. J. R. Meteorol. Soc.* Revised version sent on 17th of March 2013. Available on `http://perso.numericable.fr/~pascath31/QJ4R1_PVs1_20130317a_COL.pdf`)

• Masson V. (2013). Modification of the turbulent scheme to fit the TPE approach. *Internal work document for the Working days on EFB closure for turbulence schemes.* 18-22 March, 2013. CNRM. Toulouse.

• Siebesma AP, Bretherton CS, Brown A, Chlond A, Cuxart J, Duynkerke PG,Jiang H, Khairoutdinov M, Lawellen D, Moeng C-H, Sanchez E, Stevens B, Stevens DE. (2003). A large eddy simulation intercomparison study of shallow cumulus convection. *J. Atmos. Sci.* **60** (10): 1201–1219.

• Stevens B, Ackerman AS, Albrecht BA, Brown AR, Chlond A, Cuxart J, Duynkerke PG, Lewellen DC, Macvean MK, Neggers AJ, Sánchez E, Siebesma AP, Stevens DE. (2001). Simulation of trade wind cumuli under a strong inversion. *J. Atmos. Sci.* **58** (14): 1870–1891.

• Additional bibliography on Exergy is available on `http://perso.numericable.fr/~pmarquet/`),