

Toulouse, 22th of March 2013

# On the use of moist entropy in moist turbulence.

*Pascal MARQUET*  
*(Météo-France. DPrévi / LABO)*



**METEO FRANCE**

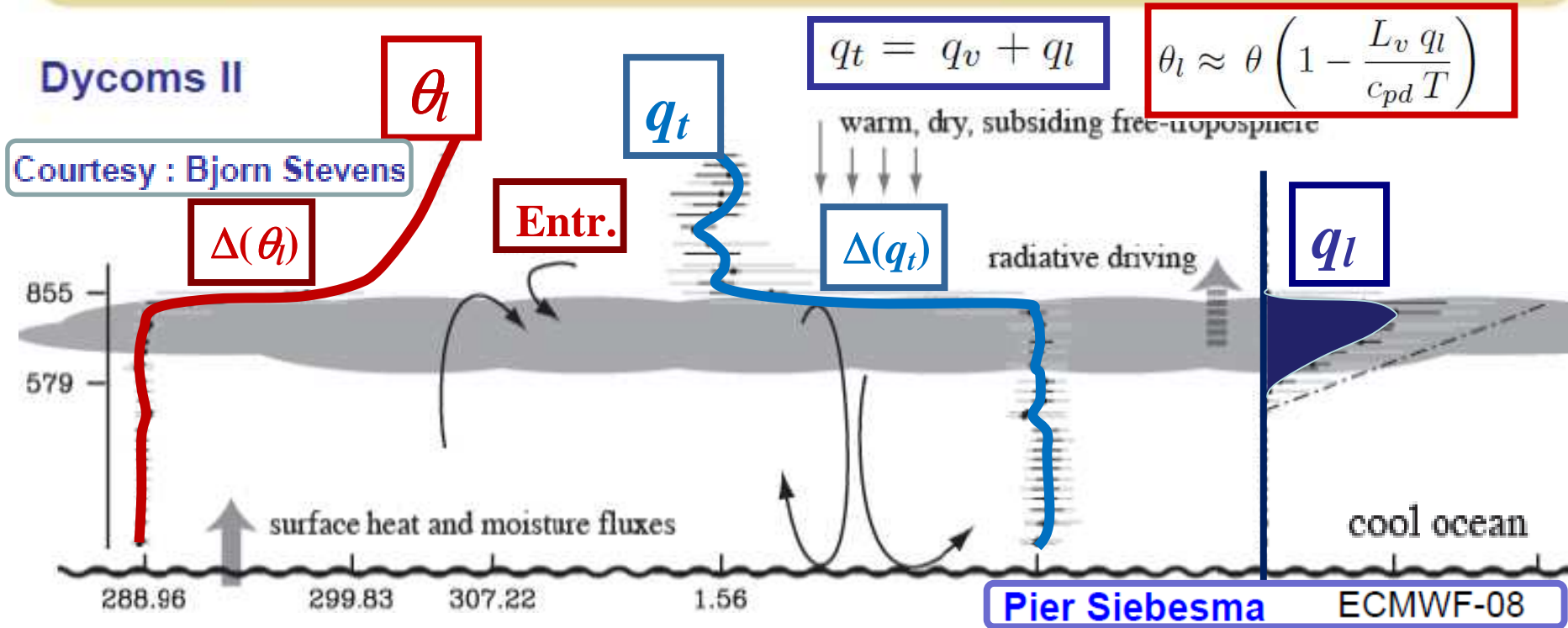
## Contents

- 1) Motivations: a wish to use the 2<sup>nd</sup> Law and Moist Entropy “ $s$ ”, because it is the quantity that the moist-turbulence want to see well-mixed ?
- 2) Opportunities: Marquet (2011) = a writing of “ $s$ ” as  $s_{ref} + c_{pd} \ln(\theta_s)$
- 3) The aim of this talk: EFB closure working days + the note of V. Masson  
→ a comparison of turbulent fluxes of “ $s$ ” either i) computed directly or ii) computed indirectly with the Betts variables ( $\theta_t, q_t$ ) ?

## Contents

- 1) Motivations: a wish to use the 2<sup>nd</sup> Law and Moist Entropy “*s*”, because it is the quantity that the moist-turbulence want to see well-mixed ?
- 2) Opportunities: Marquet (2011) = a writing of “*s*” as  $s_{ref} + c_{pd} \ln(\theta_s)$
- 3) The aim of this talk: EFB closure working days + the note of V. Masson  
→ a comparison of turbulent fluxes of “*s*” either i) computed directly or ii) computed indirectly with the Betts variables ( $\theta_t, q_t$ ) ?
- 4) Improvements: liquid water / ice? ... Cloud-Cover & subgrid / Sommeria-Deardorff / PDF ? ...  $N^2$  / M(C) / JFG ?
- 5) Improvements: more complicated : 3D-turbulent schemes? ... TOMS ?
- 6) Other ideas: “TPE” is in fact the APE of Lorenz  $\leftrightarrow$  EXERGY-Norm?

# On the use of moist entropy in moist turbulence ?



Marine Strato-cumulus are paradigm of the moist turbulence, with  $\theta_l$  and  $q_t$  often used as mixing variables (the so-called Betts' variables)

why  $\theta_l$  and  $q_t$ ? because  $(\theta_l, q_t)$  is more homogeneous than  $(\theta, q_v)$  (impact of  $q_l$ )

## *On the use of moist entropy in moist turbulence ?*

**Moist turbulence:** the so-called **Betts' (1973)** variables  $\theta_l$  and  $q_t$  / often used as mixing variables

1) *Total water content :*  $q_t = q_v + q_l + q_i$

2) *Liquid-water potential temperature:*

$$\theta_l = \theta \exp \left( - \frac{L_{vap} q_l + L_{sub} q_i}{c_{pd} T} \right)$$

3) *Optionally approximated by:*

$$\theta_l \approx \theta \left( 1 - \frac{L_{vap} q_l + L_{sub} q_i}{c_{pd} T} \right)$$
$$\exp(x) \approx 1 + x$$

## On the use of moist entropy in moist turbulence ?

1) **Specific moist-air entropy:** Marquet (QJ-2011),  
 + Marquet and Geleyn (QJ-2013)  
 + Marquet QJ-2013 (a,b, submitted)  $\rightarrow$  variable  $\theta_s$

$$s = s_{ref} + c_{pd} \ln(\theta_s)$$

$s_{ref}$  and  $c_{pd}$  constant

2) **Complete value:**

$$\theta_s = (\theta_s)_1 \left( \frac{T}{T_r} \right)^{\lambda q_t} \left( \frac{p}{p_r} \right)^{-\kappa \delta q_t} \left( \frac{r_r}{r_v} \right)^{\gamma q_t} \frac{(1 + \eta r_v)^{\kappa(1 + \delta q_t)}}{(1 + \eta r_r)^{\kappa \delta q_t}}$$

3) **Approximation:**

$$(\theta_s)_1 = \theta \exp \left( - \frac{L_{vap} q_l + L_{sub} q_i}{c_{pd} T} \right) \exp(\Lambda_r q_t)$$

4) **Impact of the Third law :**

$$\Lambda_r = (s_v^0 - s_d^0) / c_{pd} \approx 5.87$$

5) **Optionally approximated by :**

$$(\theta_s)_1 \approx \theta \left( 1 - \frac{L_{vap} q_l + L_{sub} q_i}{c_{pd} T} + \Lambda_r q_t \right)$$

**Betts' variable :**  $\theta_l \approx \theta \left( 1 - \frac{L_{vap} q_l + L_{sub} q_i}{c_{pd} T} \right)$

Comparisons  $(\theta_s)_1$   
 $\leftrightarrow [\theta_l ; \theta_v ; \theta_E ; \dots] ?$

$(\theta_s)_1$  different from  
 all other Pot. Temp.

$(\theta_s)_1 \approx 2/3 - 1/3$  between  $\theta_l$  &  $\theta_E$

$$\theta_v = \theta (1 + \delta q_v - q_l - q_i)$$

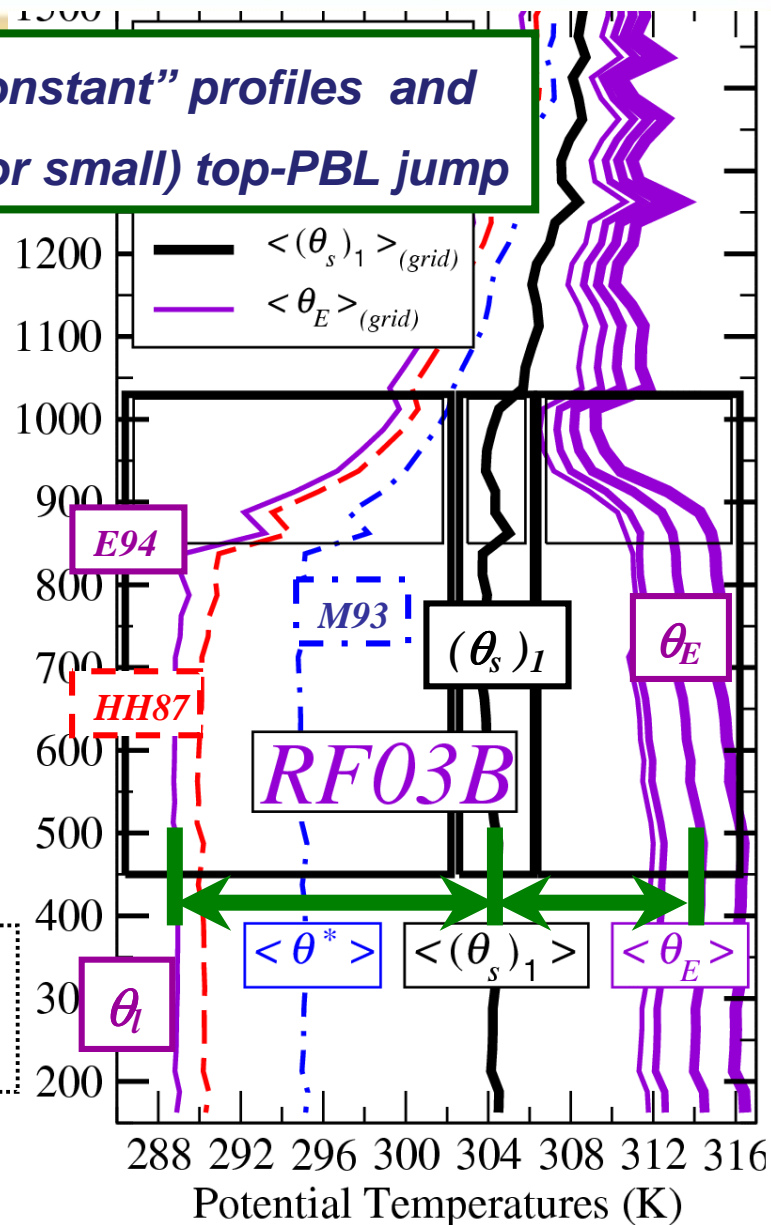
$$\theta_e \approx \theta \left( 1 + \frac{L_{vap} q_v}{c_{pd} T} \right) \quad \delta \approx 0.608$$

$$(\theta_s)_1 \approx \theta \left( 1 - \frac{L_{vap} q_l + L_{sub} q_i}{c_{pd} T} + \Lambda_r q_t \right)$$

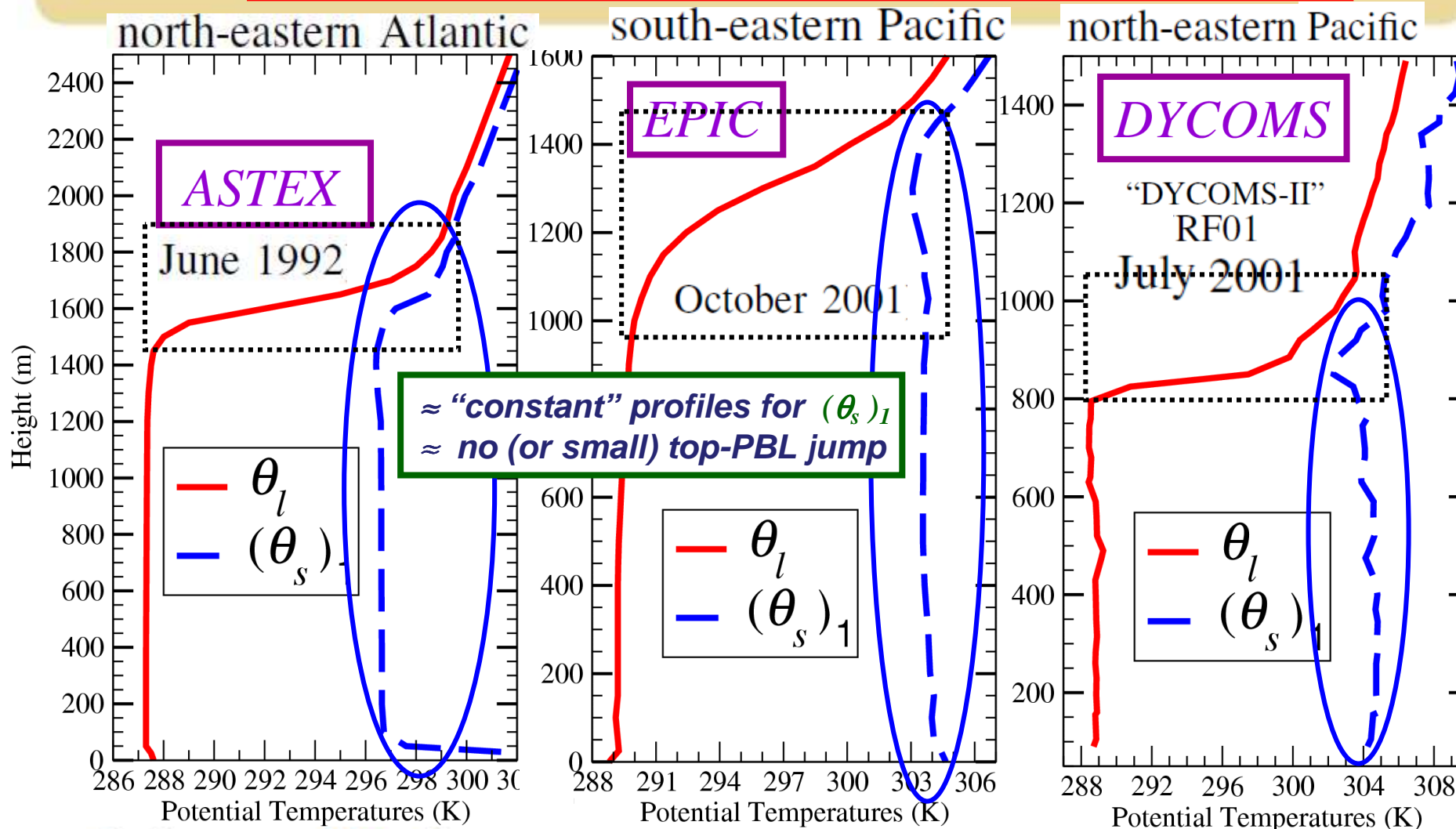
$$L_{vap}/(c_{pd} T) \approx 9$$

$$\Lambda_r \approx 6$$

$\approx$  "constant" profiles and  
 $\approx$  no (or small) top-PBL jump



***FIRE-I RF03B → 02B-04B-08B : OK ! ... and :***



Cuijpers and Bechtold (1995)

Bretherton et al. (2004)

Zhu et al. (2005)

P. Marquet - March. 2013



# On the use of moist entropy in moist turbulence ?

The 1D-scheme of Masson (2013):

$$\overline{w'\theta'} = -K_\theta \frac{\partial \Theta}{\partial z} + \Gamma_\theta E_\theta \overline{(\theta')^2} + \Gamma_\theta E_q \overline{\theta' q'}$$

$$\overline{w'q'} = -K_q \frac{\partial Q}{\partial z} + \Gamma_q E_\theta \overline{q' \theta'} + \Gamma_q E_q \overline{(q')^2}$$

$$\theta = \Theta + \theta'$$

$$q = Q + q'$$

Mean + eddies

First-order fluxes

Exchange-coeff. + Grad.

Second order fluxes

$$K_\theta = C_{p\theta} \frac{L}{\sqrt{e}} \overline{(w')^2}$$

$$\Gamma_\theta = \frac{2}{3} \beta \frac{L}{C_{p\theta} \sqrt{e}}$$

$$K_q = C_{pq} \frac{L}{\sqrt{e}} \overline{(w')^2}$$

$$\Gamma_q = \frac{2}{3} \beta \frac{L}{C_{pq} \sqrt{e}}$$

$$\beta = g/\Theta$$

Constants:  $C_{p\theta}$  and  $C_{pq}$

## On the use of moist entropy in moist turbulence ?

$$\begin{aligned}\theta_l &= \theta, \\ \theta_v &= \theta (1 + \delta q_v), \\ \theta_s &\approx \theta (1 + \Lambda_r q_v).\end{aligned}$$

$$\delta \approx 0.608 \quad \Lambda_r \approx 5.87$$

**1) under-saturation approximation**

$$\overline{w' \theta'_v} = E_\theta \overline{w' \theta'} + E_q \overline{w' q'_v}$$

**2) Buoyancy fluxes? (Betts' variables)**

$$E_\theta = 1 + \delta Q$$

$$E_q = \delta \Theta$$

$$\overline{w' \theta'_s} = (1 + \Lambda_r Q) \overline{w' \theta'} + (\Lambda_r \Theta) \overline{w' q'_v}$$

$$\overline{w' \theta'_v} = E_\theta^* \overline{w' \theta'_s} + E_q^* \overline{w' q'_v}$$

**3) Buoyancy fluxes? (moist entropy)**

$$E_\theta^* = \frac{1 + \delta Q}{1 + \Lambda_r Q}$$

$$E_q^* = \Theta \left( \delta - \frac{\Lambda_r}{1 + \Lambda_r Q} \right)$$

## On the use of moist entropy in moist turbulence ?

1) A direct use of the Moist Entropy in 1D-turbulent scheme:  $\theta_l \rightarrow \theta_s$  ?

$$\overline{(w' \theta'_s)}_1 = -K_s \frac{\partial \Theta_s}{\partial z} + \Gamma_s E_\theta^* \overline{(\theta'_s)^2} + \Gamma_s E_q^* \overline{\theta'_s q'_v}$$

2) Flux of Moist Entropy indirectly computed from Betts' variables (what is presently done)

$$\theta_s \approx \theta (1 + \Lambda_r q_v)$$

$$\overline{(w' \theta'_s)}_2 = (1 + \Lambda_r Q) \overline{w' \theta'} + (\Lambda_r \Theta) \overline{w' q'_v}$$

$$\overline{w' \theta'} = -K_\theta \frac{\partial \Theta}{\partial z} + \Gamma_\theta E_\theta \overline{(\theta')^2} + \Gamma_\theta E_q \overline{\theta' q'}$$

$$\overline{w' q'_v} = -K_q \frac{\partial Q}{\partial z} + \Gamma_q E_\theta \overline{q' \theta'} + \Gamma_q E_q \overline{(q')^2}$$

## On the use of moist entropy in moist turbulence ?

1) The present method:  
Betts' variables ( $\theta_l, q_v$ )

$$\begin{aligned}
 (\overline{w' \theta'_s})_2 = & -K_\theta (1 + \Lambda_r Q) \frac{\partial \Theta}{\partial z} - K_q (\Lambda_r \Theta) \frac{\partial Q}{\partial z} \\
 & + \Gamma_\theta (1 + \delta Q) (1 + \Lambda_r Q) \overline{(\theta')^2} \\
 & + \Gamma_\theta (\delta \Theta) (1 + \Lambda_r Q) \overline{\theta' q'_v} \\
 & + \Gamma_q (1 + \delta Q) (\Lambda_r \Theta) \overline{q'_v \theta'} \\
 & + \Gamma_q (\delta \Lambda_r \Theta^2) \overline{(q'_v)^2}.
 \end{aligned}$$

$$K_s = K_\theta = K_q$$

$$C_{ps} = C_{p\theta} = C_{pq}$$

$$\Gamma_s = \Gamma_\theta = \Gamma_q$$

3) From the  
← first line

2) Direct use of ( $\theta_s, q_v$ )  
moist entropy variable

$$\begin{aligned}
 (\overline{w' \theta'_s})_1 = & -K_s (1 + \Lambda_r Q) \frac{\partial \Theta}{\partial z} - K_s (\Lambda_r \Theta) \frac{\partial Q}{\partial z} \\
 & + \Gamma_s (1 + \delta Q) (1 + \Lambda_r Q) \overline{(\theta')^2} \\
 & + \Gamma_s (\delta \Theta) (1 + \Lambda_r Q) \overline{\theta' q'_v} \\
 & + \Gamma_s (1 + 2\delta Q) (\Lambda_r \Theta) \overline{q'_v \theta'} \\
 & + \Gamma_s (\delta \Lambda_r \Theta^2) \left[ 1 + \left( \frac{\Lambda_r Q}{1 + \Lambda_r Q} \right) \right] \overline{(q'_v)^2}
 \end{aligned}$$

4) New terms with the use of  
( $\theta_s, q_v$ ) moist entropy variable

5) Conclusion: different formulations...

# On the use of moist entropy in moist turbulence ?

2) Not for these LES →

$$\overline{w'q'} = -K_q \frac{\partial Q}{\partial z}$$

$$\overline{w'\theta'} = -K_\theta \frac{\partial \Theta}{\partial z}$$

1) Is this relevant?

$$K_s = K_\theta = K_q$$

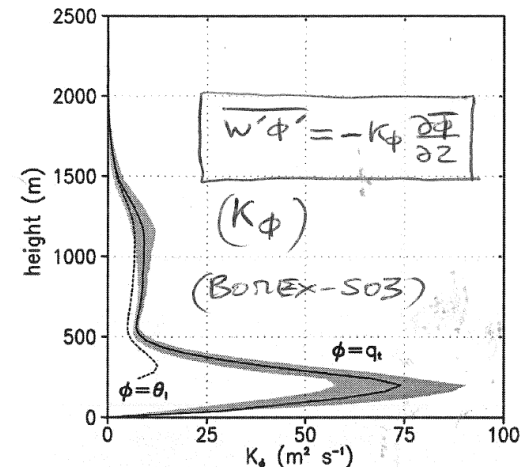
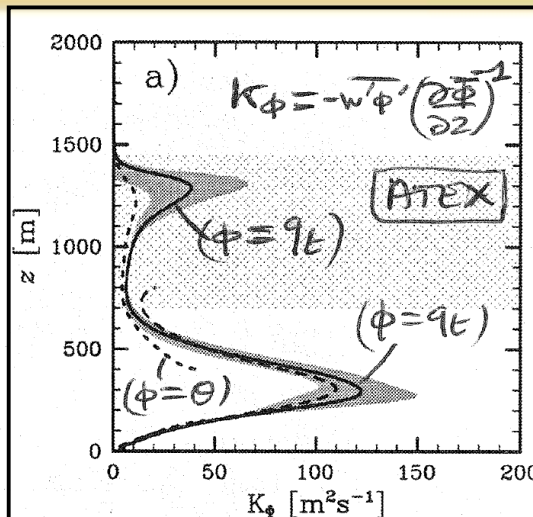
$$C_{ps} = C_{p\theta} = C_{pq}$$

$$\Gamma_s = \Gamma_\theta = \Gamma_q$$

3) Not for the Drag-coef.  
(ECUME / SURFEX) →

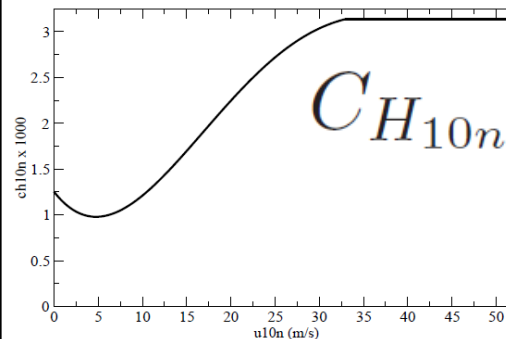
$$C_H = \frac{u_* \theta_*}{U(\theta_a - \theta_s)}$$

$$C_E = \frac{u_* q_*}{U(q_a - q_s)}$$

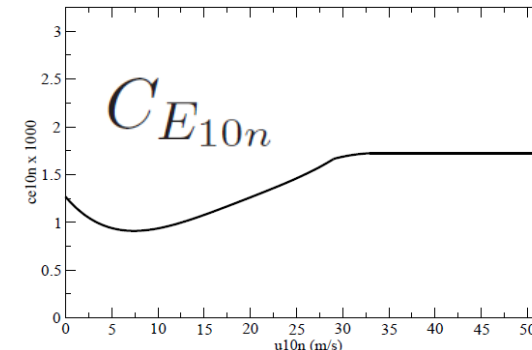


ATEX (Stevens et al, 2001) BOMEX (Siebesma et al, 2003)

SURFEX v7.2 - Issue n°2 - 2012



neutral heat coefficient



neutral evaporation coefficient

## Conclusions

1) Motivations: a wish to use the 2<sup>nd</sup> Law and Moist Entropy “ $s$ ”, because it is the quantity that the moist-turbulence want to see well-mixed ?

2) Opportunities: Marquet (2011) = a writing of “ $s$ ” as  $s_{ref} + c_{pd} \ln(\theta_s)$

3) The aim of this talk:

→ Turbulent fluxes of “moist entropy -  $\theta_s$ ” either computed directly or computed indirectly with the Betts variables ( $\theta_l, q_t$ ) ?

→ Equivalent only if ...  $C_{p\theta} = C_{pq}$  : not realistic? consequences? (rewind)

→ Other differences even if  $C_{p\theta} = C_{pq}$  : second-order fluxes / non-lin.

## Conclusion – Outlook

- 1) Motivations: a wish to use the 2<sup>nd</sup> Law and Moist Entropy “ $s$ ”, because it is the quantity that the moist-turbulence want to see well-mixed ?
- 2) Opportunities: Marquet (2011) = a writing of “ $s$ ” as  $s_{ref} + c_{pd} \ln(\theta_s)$
- 3) The aim of this talk:  
→ Turbulent fluxes of “moist entropy -  $\theta_s$ ” either computed directly or computed indirectly with the Betts variables ( $\theta_l, q_t$ ) ?
- 4) Improvements: liquid water / ice? ... Cloud-Cover & subgrid / Sommeria-Deardorff / PDF ? ...  $N^2$  / M(C) / JFG ?
- 5) Improvements: more complicated : 3D-turbulent schemes? ... TOMS ?

## Conclusion – Outlook

- 1) Motivations: a wish to use the 2<sup>nd</sup> Law and Moist Entropy “ $s$ ”, because it is the quantity that the moist-turbulence want to see well-mixed ?
- 2) Opportunities: Marquet (2011) = a writing of “ $s$ ” as  $s_{ref} + c_{pd} \ln(\theta_s)$
- 3) The aim of this talk:
  - Turbulent fluxes of “moist entropy -  $\theta_s$ ” either computed directly or computed indirectly with the Betts variables ( $\theta_l, q_t$ ) ?

- Other ideas: “TPE” is in fact the Available Potential Energy of Lorenz
- There is no “moist APE” ... (even MAE of Lorenz 1978-79 ...)
  - “Exergy” = an alternative for APE with a moist version of it ...
  - (total) Moist Available Enthalpy Norm ( $T, p_{surf}, q_v$ ): a new starting point?

<http://perso.numericable.fr/~pmarquet/>

Thank you ... Questions ?