Towards a common framework for (i) extensions of the Louis formalism, (ii) the RANS aspect of the QNSE theory & (iii) the class of 'No Ri(cr)' Reynolds-type prognostic TKE schemes ?

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13/5/09, Utrecht, HIRLAM ASM - ALADIN Workshop

Framework of the study

Goals
Main supporting evidence
Reference papers
Basic choices

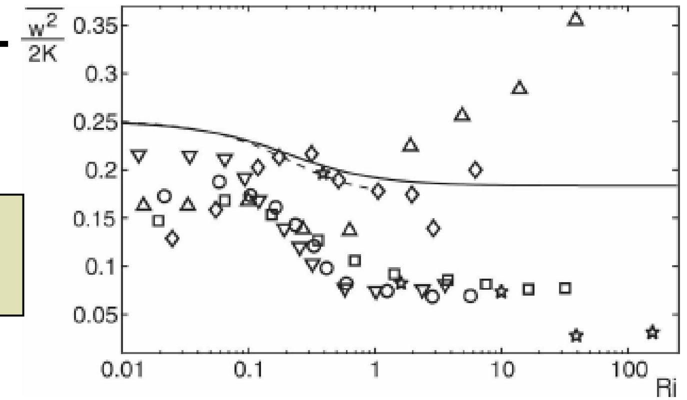
Aims of this study

- (i) Striping down to its most simple shape the problem of computing the shear production and buoyancy production-destruction terms of a prognostic TKE equation.
- (ii) Using the found framework to compare as fairly as possible three solutions for the specification of the remaining degrees of freedom:
  - The extension of a Louis-type 'static' computation towards memory from past time-steps, auto-diffusion and having a Newtonian-type formulation of the dissipation term;
  - The recently proposed spectral representation of turbulence (QNSE), when reduced to the sole specification of two stability dependency functions;
  - One recently proposed rather complete Reynolds-type scheme, in its version where (alike for both above cases) there is no critical value for the Richardson-number $\boldsymbol{R}_i$.

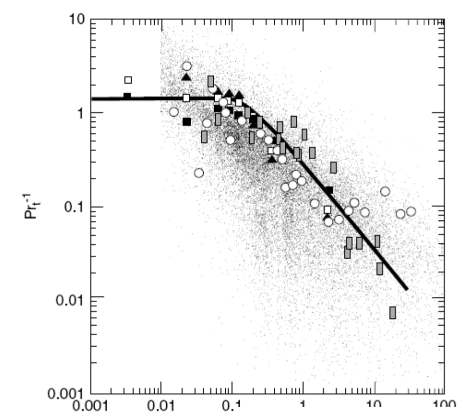
For the past 10 years, observations, LES and theoretical advances have shown that …

The anisotropy of turbulent flows should not be neglected

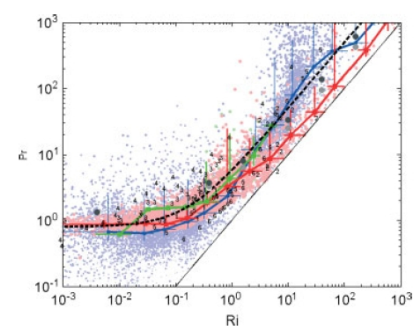
Unstable case asymptote



At very high stability there appears to be no limitation on the Richardson-number (but there exist a critical flux-Richardson-number R_{ifc})



The latter fact is the consequence of conversions between TKE and TPE happening even in very stable regimes



Papers used in the present study

- Redelsperger, Mahé & Carlotti, 2001. *Boundary Layer Meteorology*, **101**, pp. 375-408. (***RMC01***)
- Zilitinkevitch, Elperin, Kleeorin, Rogachevskii, Esau, Mauritsen & Miles, 2008. *Quart. J. Roy. Meteor. Soc.*, **134**, pp. 793-799. (***Z_et_al01***)
- Sukoriansky, Galperin & Staroselsky, 2005. *Phys. Fluids*, **17**, 085107, pp. 1-28. (***SGS05***)
- Galperin, Sukoriansky & Anderson, 2007. *Atmos. Sci. Lett.*, **8**, pp. 65-69. (***GSA07***)
- Cheng, Canuto & Howard, 2002. *J. Atmos. Sci.*, **59**, pp. 1550-1565. (***CCH02***)
- Canuto, Cheng, Howard & Esau, 2008. *J. Atmos. Sci.*, **65**, pp. 2437-2447. (***CCHE08***)

Choices for the analytical developments

- The common framework is that of a prognostic Turbulent Kinetic Energy (TKE) equation.
- There is no prognostic Turbulent Potential Energy (TPE) equation [***but*** the interplay of both forms of Turbulent Total Energy (TTE=TKE+TPE) is kept into account].
- In the case of the Reynolds decomposition, one uses the Mellor-Yamada assumption of neglecting the (small) influence of shear-turbulence interactions on the temperature-pressure correlation terms.
- Like proposed by SGS05 for such aims, the scope of the (wider) QNSE theory is limited here to the specification of stability dependency functions for the production-destruction terms of the TKE equation.
- In the stable case, the ALARO-0 (empirical) Louis-type functions have (as already since 2000) an asymptotic non-zero limit for momentum, no critical R_i and a finite R_{ifc}.

The ‘p-TKE’ starting point (‘p’ for pseudo) as used operationally in ALARO-0

Basic idea

Search of an extension [e-TKE, ‘e’ for emulation]

Some a-posteriori lessons

The 'p-TKE' extension of the Louis method (for more details see Ivan's poster)

- Basic Turbulent Kinetic Energy (***E***) prognostic scheme:

$$\frac{dE}{dt} = A_{dv}(E) + \frac{1}{\rho}\frac{\partial}{\partial z}\rho K_E \frac{\partial E}{\partial z} + Shear\_prod + Buoyancy\_prod / destr - \frac{C_\varepsilon E^{3/2}}{L_\varepsilon}$$

$$K_m = C_K L_K \sqrt{E}\chi_3(R_i) \quad K_h = C_H L_K \sqrt{E}\phi_3(R_i) \quad K_E = \alpha K_M$$

Louis-type scheme ⇔ this box ≡ 0

If we believe that we have a well-tuned (but too static) scheme for diagnostic values of K_m and K_h (via $F_m(R_i)$ and $F_h(R_i)$ in Louis' scheme), why not inverting the process?

From K_m (and its neutral equivalent K_n) one computes an equilibrium value $\tilde{E}$ for E towards which the prognostic variable will be relaxed with the time scale of the dissipation.

The 'e-TKE' extension of the 'p-TKE' method
(for more details see Ivan's poster)

- If we consider a 'full' TKE scheme ['f-TKE' in our 'slang'], 'emulating' it within the p-TKE framework just amounts to compute $\tilde{E}$ on the basis of the exact formulation of the production-destruction terms.
- In terms of dependency upon stability, it is equivalent to derive a formulation of $F_m(R_i)$ and $F_h(R_i)$ [Louis] starting from $\phi_3(R_i)$ and $\chi_3(R_i)$ ['f-TKE'].
- At first thought, one may believe that the problem is symmetric and that any Louis-type scheme can have a hidden 'e-TKE' equivalent.
- However the treatment of the length scales L_K and L_ε and their reduction to a single L one (RMC01) is such that it cannot be the case, except for rather particular conditions.

The lessons of the 'p-TKE' development

- In terms of adding 'memory', auto-diffusion and a Newtonian dissipation term to a well tuned pre-existing Louis scheme, it works perfectly well (operational in ALARO-0).
- As long as we stick to the 'e-TKE' data flow, 'p-TKE' can still take part in the intercomparison with QNSE and CCHE08.
- If this has to evolve (*for other considerations than those treated here*), 'p-TKE' is too restrictive a method to be setting the pace for future configurations.
- But the associated staggering, shape of the implicit 'solver' and time step algorithm can (& should) be preserved.

Without this 'intermediate step' we would anyhow not have been able to study the 'common framework problem' as below

The search for a single common framework

The ‘f’ function and its filtering role

The relation between the ϕ_3 & χ_3 stability functions

The QNSE case

The No-Ri(cr) Reynolds case

The resulting set of equations

The '*f*' function (RMC01) and its computation

- A bridge is needed between the shear- and buoyancy-terms of the TKE prognostic equation.
- The 'CBR' approach obtains it in a case where the only stability dependency is the one linked with the parameterisation of the TKE ⇔ TPE term, but this result can be shown to be absolutely general.

$$f = \frac{c_\varepsilon}{c_K} \frac{E}{L^2\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right]}$$

- There are two ways to compute '***f***' in practice:
  - Either explicitly while solving the TKE equation;
  - Or by solving a characteristic equation that expresses the stationnary solution **shear term + buoyancy term + dissipation = 0** . ***This delivers a second order equation for f(R_i) that admits a solution for R_i going from -∞ to +∞*** .

The 'f' function (RMC01) and its computation

- We follow here the second path, since:
  - We wish a solution without restriction of the range of possible Richardson-numbers;
  - We obtain this feature in a way very similar to the argument of Z_et_al_08: '***f***' acts as a 'filter' imposing that '***stationarity of the TKE equation + diagnostic TPE equation ⇔ conservation of TTE***'.
- Under these conditions it can be shown that the characteristic equation leading to '***f***' factorises as

$$f(R_i) = \chi_3(R_i)(1 - R_{if})$$

with $\boldsymbol{R_{if}}$ the flux-Richardson-number. With this, $\boldsymbol{\chi_3(R_i)}$ has the same range of validity as '***f***', i.e. from $-\infty$ to $+\infty$. Idem for $\boldsymbol{\phi_3(R_i)}$.

A key relationship

- We do not have yet the conditions for a full analytical solution of the problem.
- But, adding one constraint (too complex to be explicited here), that anyhow takes a different shape depending on which problem one wants to solve (CBR, CCH02, CCHE08), one can obtain a unique equation linking the two stability dependency functions:

$$C_3 R_i \phi_3{}^2 - \phi_3(\chi_3 + C_3 R_i / R_{ifc}) + \chi_3 = 0$$

with $\boldsymbol{C_3}$ the inverse Prandtl number at neutrality and $\boldsymbol{R_{ifc}}$ the critical flux-Richarson-number, i.e. two of the three ‘physical’ quantities relevant to our proposal.

A remaining degree of freedom (‘*R*’)

- On top of c_K, c_ε (Reynolds case only) and C_3, R_{ifc} (general case), a dependency analysis shows that we still have a degree of freedom to consider in our new system of equations.
- Let us define, for the time being as a function of stability (and by ‘eliminating’ the ‘*f*’ function),

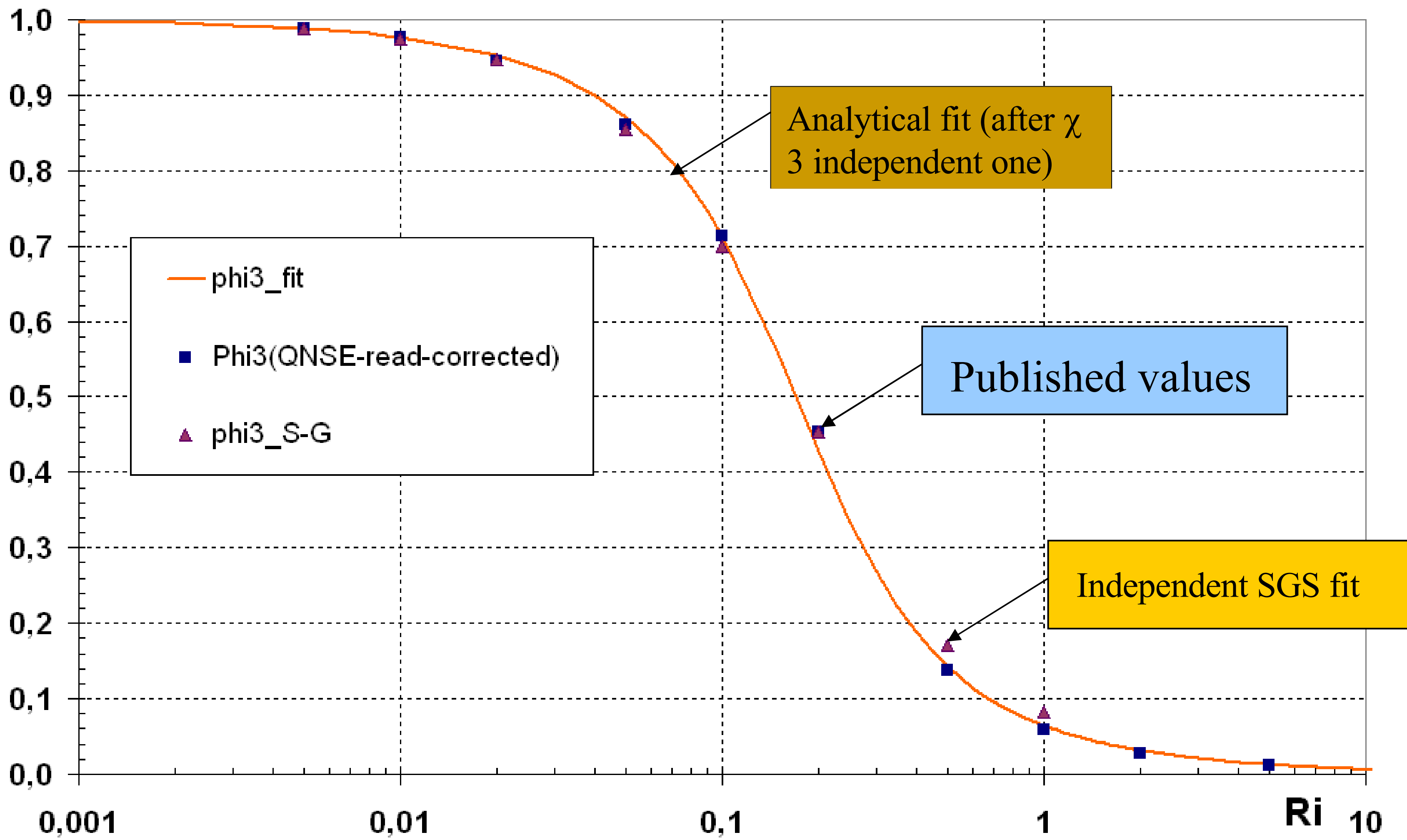
$$R(R_i) = R_{if} /(1 - f(R_i))$$

- ***R*** can be seen as a measure of the anisotropy. For an isotropic flow one shall have ***R≡1*** (CBR case for instance); lower and lower ***R*** values will indicate more and more anisotropy.
- Our system can now be solved analytically once the 3(+2) degrees of freedom are specified and using the ‘filtering’ characteristic equation as well as the link between the stability functions.

Quasi Normal Scale Elimination (QNSE) case

- QNSE is a ‘spectral’ alternative to the Reynolds-averaging technique for describing the detailed properties of turbulence.
- The ‘working hypotheses’ are imbedded in the derivation method => no a-posteriori tuning possible.
- Anisotropy of the flow is central to the algorithm. But nothing distinguishes its impact from the impact of TPE ⇔ TKE => one prognostic equation only.
- The resulting data are valid only for stable and slightly unstable case => we need a strategy for extrapolation to the full unstable regime.
- The basic theory delivers wave-number dependency that has to be converted to R_i-type one (see SGS05).

Fits of the function $\phi_3(R_i)$ for QNSE



What about the handling of anisotropy?

- After doing the analytical fit of $\chi_3(R_i)$ one may look at what are the implicit values of R associated with the resulting function (fitted exclusively from published values)
  - For $R_i \rightarrow -\infty$, we get $R=0.404$ *(through extrapolation)*
  - For $R_i = 0$, we get $R=0.359$
  - For $R_i \rightarrow +\infty$, we get $R=0.440$
- So a fit with $R=0.4$ (rather than 'reading' $\chi_3(R_i)$) would not be as good, but still quite acceptable.
- The other constants corresponding to the QNSE fit are $C_3=1.39$ (given by the authors) and $R_{ifc}=0.377$ (vs. 0.4 suggested by the authors).

The case of Reynolds averaging models

- Contrary to QNSE, we have here complete control of all relevant parameters.
- The CBR case is not interesting (***R*=*1***, no χ_3 function).
- The CCH02 formulation does not match the searched generality (***either*** limitation of the range of possible R_i values [with strong associated ***R*** variations] ***or*** need to artificially decouple momentum and heat equations).
- The modification for 'No Ri(cr)' [CCHE08] on the contrary leads to interesting perspectives: not only do we have the full range of R_i but we get far more homogeneous ***R*** values (a bit alike QNSE).
- **Hope to soon justify the use of a constant *R* parameter changing value only with the set-up ?**

Resulting set of equations (for *R* constant)

$$R_{if} = C_3 R_i \frac{\phi_3(R_i)}{\chi_3(R_i)}$$

$$\chi_3(R_i) = \frac{1 - R_{if} / R}{1 - R_{if}}$$

$$\phi_3(R_i) = \frac{1 - R_{if} / R_{ifc}}{1 - R_{if}}$$

$$R_i \in [-\infty, +\infty]$$

C_3 : inverse Prandtl number at neutrality

R : parameter characterising the flow's anisotropy

R_{ifc} : critical flux-Richardson number (***R_{if}*** at $+\infty$)

Plus the 'developed' prognostic TKE equation, of course

Conclusions

- The development is rather complex, but the result is synthetic [as aimed at] and simple [a nice surprise].
- It also seems to be quite general and compatible with recent basic findings (beyond CBR, so to say).
- As a by-product, it gives a consistent QNSE extension for unstable cases.
- We have yet to justify it in more details.
- Anyhow, it can already play its role for a perfectly fair intercomparison of formulations.