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Consistent BBC treatment of HD in ALADIN-NH

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Contents

1 Introduction

In my previous report [4] there were described two residual problems which appear when ALADIN-NH dynamical core is used at very high resolution (typically 10–100 m). This report is devoted to the first problem, so called diffusive chimney.

In autumn 2003 it was recognized by P. Bénard (section 2.4 and appendix A of [4]) that diffusive chimney in ALADIN-NH is caused by applying horizontal diffusion (HD) on pseudovertical divergence d, but ignoring it in bottom boundary condition (BBC) for term $\frac{\partial \tilde{p}}{\partial \pi}$. Even though analysis was done in continuous framework using linear Long model, it immediately suggested cure for the problem. However, it soon became obvious that its full implementation in 3D ALADIN-NH will not be feasible. Some approximate treatments were proposed, taking into account only most significant terms. But there remained one important question unanswered: Will any of these treatments work also in non-linear case?

Looking for the answer constituted main task for this stay, implying following working plan:

- Propose consistent treatment of HD in BBC, which can be implemented at least in vertical plane 2D model.
- Implement proposed treatment and verify that it really removes HD chimney in linear regimes.
- Test whether implemented treatment cures also non-linear regimes.
- Test impact of further approximations, which would make 3D implementation more feasible.
- Propose implementation suitable for 3D model.

2 Theoretical analysis of BBC treatment

Analysis will be done assuming dry atmosphere, neglecting effects of earth curvature and rotation. Prognostic variable $d \equiv d_3$ (true vertical divergence in this case) will be used. Notations are taken from [2], where basic set of equations written in η-coordinate as well as discretization details can be found.

2.1 Continuous case

Momentum equations formulated in η coordinate together with free slip BBC have the form:

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\frac{RT}{p}\nabla p - \left(\frac{\partial \tilde{p}}{\partial \pi} + 1\right)\nabla\phi + \boldsymbol{\mathcal{V}} \tag{1}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(gw) = g^2\frac{\partial \tilde{p}}{\partial \pi} + g\mathcal{W} \tag{2}$$

$$gw_S = \mathbf{v}_S \cdot \nabla\phi_S \tag{3}$$

$$\tilde{p} \equiv p - \pi$$

In our case (no Coriolis force, no physics) source terms $\boldsymbol{\mathcal{V}}$ and $\mathcal{W}$ contain only HD tendencies.

Prognostic equation for vertical divergence d can be obtained now. It is sufficient to apply vertical derivative $\frac{\partial}{\partial\phi}$ on equation (2). After some manipulations this gives:

$$\frac{\mathrm{d}d}{\mathrm{d}t} = \frac{\partial}{\partial\phi}\left[g^2\frac{\partial\tilde{p}}{\partial\pi} + g\mathcal{W}\right] - d(d+X) + Z \tag{4}$$

$$X \equiv -\frac{\partial\mathbf{v}}{\partial\phi}\cdot\nabla\phi \qquad Z \equiv -\frac{\partial\mathbf{v}}{\partial\phi}\cdot\nabla(gw)$$

Appearance of extra terms on RHS of equation (4) is due to the fact that vertical derivative $\frac{\partial}{\partial\phi}$ does not commute with total time derivative $\frac{\mathrm{d}}{\mathrm{d}t}$.

Equation (4) still contains vertical velocity w hidden in term Z. It can be diagnosed by inverting definition of d:

$$d \equiv \frac{\partial}{\partial\phi}(gw) = -\frac{p}{mRT}\frac{\partial}{\partial\eta}(gw)$$

$$gw = gw_S + \int_\eta^1 \frac{mRT}{p} d\,\mathrm{d}\eta \tag{5}$$

Role of boundary conditions in the full system is to determine unique solution. When d is used as prognostic variable, it is not sufficient to impose free slip BBC (3) in diagnostic formula (5). BBC for w must also be transformed into BBC for $\frac{\partial\tilde{p}}{\partial\pi}$ in order to get well-posed system (this was nicely demonstrated by P. Bénard in linearized 1D vertical framework, see [1]). Elimination of time evolution from equations (1)–(3) gives (see section 2.1 of [3] for details):

$$\left[g^2 + (\nabla\phi_S)^2\right]\left(\frac{\partial\tilde{p}}{\partial\pi}\right)_S = \left[-\frac{RT}{p}\nabla p - \nabla\phi + \boldsymbol{\mathcal{V}}\right]_S \cdot \nabla\phi_S + J_S - g\mathcal{W}_S \tag{6}$$

$$J_S \equiv \frac{\partial^2\phi_S}{\partial x^2}{u_S}^2 + 2\frac{\partial^2\phi_S}{\partial x\partial y}u_S v_S + \frac{\partial^2\phi_S}{\partial y^2}{v_S}^2$$

It can be observed that in continuous system specifying of surface values $\mathbf{v}_S$, T_S, p_S, ϕ_S, $\boldsymbol{\mathcal{V}}_S$ and $\mathcal{W}_S$ influences 3D pressure field p via vertical derivative $\left(\frac{\partial\tilde{p}}{\partial\pi}\right)_S$. In other words, dynamical state of the system cannot be arbitrary, but it must respect condition (6). If on the other hand condition (6) is fulfilled at initial time, system evolving according to equations (1)–(3) will preserve it forever.

In current ALADIN-NH code source terms $\mathcal{W}_S$ and $\boldsymbol{\mathcal{V}}_S$ occuring in BBC (6) do not contain HD tendencies. In our case it means that they are assumed to be zero. This leads to incompatibility of equation (1) or (4) with BBC (6), as soon as HD is imposed on horizontal wind[1] $\mathbf{v}$ or vertical divergence d. As a result, HD chimney may occur.

Relative importance of the terms $\mathcal{W}_S$ and $\boldsymbol{\mathcal{V}}_S$ in BBC (6) can be estimated using linear Long model. It indicates that influence of term $\boldsymbol{\mathcal{V}}_S$ to HD chimney formation can be neglected in linear regimes.

2.2 Vertically discretized case

BBC in the form (6) cannot be used directly in ALADIN-NH, since some of the quantities are not available at surface half level $\tilde{L}$. It is therefore manipulated into different form, using assumption $\mathbf{v}_{\tilde{L}} = \mathbf{v}_L$:

$$g^2\left(\frac{\partial\tilde{p}}{\partial\pi}\right)_{\tilde{L}} = \left[-\frac{RT}{p}\nabla p - \left(\frac{\partial\tilde{p}}{\partial\pi}+1\right)\nabla\phi + \boldsymbol{\mathcal{V}}\right]_L \cdot \nabla\phi_{\tilde{L}} + J_L - g\mathcal{W}_{\tilde{L}} \tag{7}$$

Formula (7) contains problematic term $\left(\frac{\partial\tilde{p}}{\partial\pi}\right)_L$. There are several possibilities how to treat it, in current code it is evaluated employing additional hypothesis $\mathcal{P}_{\tilde{L}} = \mathcal{P}_L$, where $\mathcal{P} \equiv \frac{\tilde{p}}{\pi}$. This treatment might be assumed inconsistent, but more sophisticated approach tried in [3] had very weak impact on model results.

Now it can be examined where the surface source term $\mathcal{W}_{\tilde{L}}$ enters model dynamics. At the lowest full level L equation (4) can be discretized as:

$$\left(\frac{\mathrm{d}d}{\mathrm{d}t}\right)_L = \frac{1}{\delta\phi_L}\left[g^2\left(\frac{\partial\tilde{p}}{\partial\pi}\right)_{\tilde{L}} + g\mathcal{W}_{\tilde{L}} - g^2\left(\frac{\partial\tilde{p}}{\partial\pi}\right)_{\tilde{L}-1} - g\mathcal{W}_{\tilde{L}-1}\right] - d_L(d_L + X_L) + Z_L \tag{8}$$

Inserting BBC (7) into equation (8) gives:

$$\begin{aligned}\left(\frac{\mathrm{d}d}{\mathrm{d}t}\right)_L = &\frac{1}{\delta\phi_L}\left\{\left[-\frac{RT}{p}\nabla p - \left(\frac{\partial\tilde{p}}{\partial\pi}+1\right)\nabla\phi + \boldsymbol{\mathcal{V}}\right]_L \cdot \nabla\phi_{\tilde{L}} + J_L\right\} - \\ &- \frac{1}{\delta\phi_L}\left[g^2\left(\frac{\partial\tilde{p}}{\partial\pi}\right)_{\tilde{L}-1} + g\mathcal{W}_{\tilde{L}-1}\right] - d_L(d_L + X_L) + Z_L\end{aligned} \tag{9}$$

It can be observed that surface source term $\mathcal{W}_{\tilde{L}}$ does not appear in equation (9). It does not appear at higher levels as well.

Conclusion:

When source terms $\boldsymbol{\mathcal{V}}$ and $\mathcal{W}$ are treated consistently in vertically discretized system (using prognostic variable d and BBC (7) together with additional hypothesis $\mathcal{P}_{\tilde{L}} = \mathcal{P}_L$), surface term $\mathcal{W}_{\tilde{L}}$ cancels out from the equations. It has therefore, on the contrary to continuous case, no influence on model results.[2] This property was found desirable when proposing code implementation of HD chimney treatment.

[1] Indirectly, by diffusing vorticity ξ and divergence D.

[2] This would not be true if hypothesis $\mathcal{P}_{\tilde{L}} = \mathcal{P}_L$ was replaced by more consistent treatment proposed in [3]. In such case surface term $\mathcal{W}_{\tilde{L}}$ would not cancel in prognostic equation for d and it would appear also in pressure gradient term.

2.3 More code oriented explanation of chimney mechanism

Generalization of results from previous two sections to fully discretized model is not completely risk free, since some important aspects might not be captured by simplified analyses. Fortunately, there exists alternative explanation of chimney mechanism proposed by R. Brožková, which is directly applicable to the model. It is based on two rules:

1. Every particular evolution of $w_{\tilde{L}}$ (due to adiabatic dynamics, physics or HD) must respect kinematic rule $gw_{\tilde{L}} = \mathbf{v}_L \cdot \nabla\phi_{\tilde{L}}$. This means that evolution of $w_{\tilde{L}}$ is fully given by evolution of $\mathbf{v}_L$.

2. It must be remembered that $w_{\tilde{L}}$ is evolved implicitly whenever variable d is evolved, since $d_L \equiv \frac{gw_{\tilde{L}} - gw_{\tilde{L}-1}}{\delta\phi_L}$.

As long as rules 1 and 2 are respected in model code, chimney does not appear. This was confirmed already for semi-lagrangian (SL) chimney. Both successful treatments of SL chimney (`LGWADV` – advection of w and `LRDBBC` – diagnostic BBC) were proposed employing this simple theory.

Now the theory will be applied in order to explain HD chimney. Evolution of model field X during timestep without physics can be symbolically written as:

$$X^0 \xrightarrow{\text{dyn}} X^{(+)} \xrightarrow{\text{HD}} X^+$$

Symbol X^0 denotes value at time t, $X^{(+)}$ is preliminary $t + \Delta t$ value provided by dynamics and X^+ is final $t + \Delta t$ value after application of HD.

At the beginnig of timestep, vertical velocity $w^0_{\tilde{L}}$ is diagnosed using kinematic rule $gw^0_{\tilde{L}} = \mathbf{v}^0_L \cdot \nabla\phi_{\tilde{L}}$. If adiabatic dynamics itself does not suffer from chimney, provisional $t + \Delta t$ fields fulfil kinematic rule $gw^{(+)}_{\tilde{L}} = \mathbf{v}^{(+)}_L \cdot \nabla\phi_{\tilde{L}}$ with high accuracy.[3] Subsequent application of HD on d implicitly evolves $w^{(+)}_{\tilde{L}}$ into $w^+_{\tilde{L}}$, which becomes inconsistent with $\mathbf{v}^+_L$. Second source of inconsistency is evolution $\mathbf{v}^{(+)}_L \to \mathbf{v}^+_L$ due to HD acting on vorticity ξ_L and divergence D_L.

In the next timestep value X^+ becomes X^0. Velocity $w^0_{\tilde{L}}$ is correctly re-diagnosed from $\mathbf{v}^0_L$ using kinematic rule, but vertical divergence d^0_L is not changed accordingly. As a consequence, field w^0 diagnosed at half levels $\tilde{0}, \ldots, \tilde{L} - 1$ will be shifted by difference ($w^0_{\tilde{L}}$ re-diagnosed from $\mathbf{v}^0_L$) − ($w^0_{\tilde{L}}$ evolved via d_L). This shift is nothing else but HD chimney. It can be seen immediately that:

- HD chimney is zero at surface, but it evolves into its full strength within lowest model layer. It has no vertical structure above.

- HD chimney fully evolves during one timestep.

These conclusions should apply also to SL chimney.

[3]Value $w^{(+)}_{\tilde{L}}$ is hidden inside $d^{(+)}_L$.

3 Practical aspects

3.1 Proposed treatments

Starting from results of sections 2.1 and 2.2, natural solution for preventing diffusive chimney would be to incorporate HD tendency into the term $\mathcal{W}_{\tilde{L}}$ occuring in BBC (7). This was proposed by P. Bénard already one year ago. However, there are several complications connected to this approach:

- BBC is evaluated in gridpoint space, while HD is applied in spectral space. Evaluation of HD tendency for $w_{\tilde{L}}$ during gridpoint computations would therefore require extra transform of one 2D field into spectral space and back.

  For predictor-corrector (PC) scheme additional transforms would be needed at each iteration. This means $2\times(\texttt{NSITER}+1)$ extra spectral transforms per timestep.

- Implementation of this procedure cannot be achieved without significant changes in code design, since gridpoint computations are done using `NPROMA` slices (even if only single processor is used), while spectral transforms require global data.

- When HD is imposed on w as $-K\nabla^4 w$, corresponding term acting on d is not simply $-K\nabla^4 d$. There are two reasons for this:

  - diffusion coefficient K depends on vertical coordinate η
  - vertical derivative $\dfrac{\partial}{\partial\phi}$ does not commute with operator ∇^4

  This should be taken into account when converting HD tendencies between d and w. However, sometimes it is assumed that both expressions are equivalent at least for lowest layer, which can be denoted as $K_{\tilde{L}} = K_L = K_{\tilde{L}-1}$ and $\nabla\delta\phi_L = 0$ assumption.

- Even if HD tendency for $w_{\tilde{L}}$ was correctly incorporated into BBC during gridpoint computations, temporal discretization might spoil its exact cancelation in prognostic equation for d. Cancelation should occur at the end of timestep, when HD is applied in spectral space. But since temporal discretization for dynamics and HD differs, it is likely that cancelation will be only approximate.

- Non-linear regimes may require also including the term $\boldsymbol{\mathcal{V}}_L$ in BBC (7). This would increase number of extra spectral transforms per timestep. Moreover, projection of HD tendencies onto velocity components u and v is not straightforward.

In order to avoid first two (purely technical) problems, approximate treatment of the term $\nabla^4(gw_{\tilde{L}})$ was proposed:

$$\nabla^4(gw_{\tilde{L}}) = \nabla^4(\mathbf{v}_L \cdot \nabla\phi_{\tilde{L}}) \approx \mathbf{v}_L \cdot \nabla(\nabla^4\phi_{\tilde{L}})$$

Approximation itself should work well at least in linear regimes. As for remaining problems, it is not easy to judge a priori how harmful they can be.

Using explanation described in section 2.3, R. Brožková proposed alternative approach recently: Do not modify BBC, but do not diffuse part of d_L containing $w_{\tilde{L}}$. Practical implementation would be to subtract term $\dfrac{gw_{\tilde{L}}}{\delta\phi_L}$ from d_L before HD and add it back after HD. In order to be consistent with kinematic rule, term to be subtracted must be diagnosed from undiffused quantitites while term to be added from diffused ones. There are of course some complications specific also for this approach:

- Horizontal diffusion is applied in spectral space, but the term $\frac{gw_{\tilde{L}}}{\delta\phi_L}$ can be diagnosed only in gridpoint space. This implies extra transforms of u_L, v_L, $\ln \pi_{\tilde{L}}$, $\tilde{p}_L$ and T_L (five 2D fields) into gridpoint space and then transformation of resulting term (one 2D field) back to spectral space. Because the term must be diagnosed both from undiffused and diffused quantities, there are 12 extra spectral transforms per timestep (this number does not depend on `NSITER` value).
- Implementation of this procedure is not straightforward, since on multiple processors spectral fields are splitted into parts, but again, transforms require global fields.

Considering advantages and disadvantages of both outlined treatments it was decided to try second approach. Extra argument supporting this choice was its easy generalization to HD applied purely on w. This was meant as a backup alternative for the case when proposed treatment would not work satisfactorily.

Remark:

First approach can be viewed as imposing "antidiffusion" on $w_{\tilde{L}}$ during adiabatic dynamics, which is afterwards compensated by HD applied on d_L. Since in this approach dynamics and HD are mixed, provisional value $w_{\tilde{L}}^{(+)}$ does not fulfil kinematic rule, which is restored only for final value $w_{\tilde{L}}^{+}$. Taking into account conclusion from section 2.2, this approach should give similar results as second one, i.e. not diffusing $w_{\tilde{L}}$.

3.2 Dirty implementation in vertical plane 2D model

Chosen approach will be analyzed in more detail first. Basic question to be answered is: What does it mean not to diffuse part of d_L containing $w_{\tilde{L}}$? Starting from $\mathcal{W}$, it is enough to realize that we do not want to have HD on $w_{\tilde{L}}$. For bottom layer this means:

$$\mathcal{W}_{\tilde{L}} = 0 \qquad \mathcal{W}_{\tilde{L}-1} = -K_{\tilde{L}-1}\nabla^4 w_{\tilde{L}-1}$$

Defining $K_L \equiv K_{\tilde{L}-1}$ and using assumption $\nabla\delta\phi_L = 0$, corresponding source term in prognostic equation for d evaluated at lowest full level becomes:

$$g\left(\frac{\partial\mathcal{W}}{\partial\phi}\right)_L = g\frac{\mathcal{W}_{\tilde{L}} - \mathcal{W}_{\tilde{L}-1}}{\delta\phi_L} = K_L\frac{\nabla^4(gw_{\tilde{L}-1})}{\delta\phi_L} \approx$$
$$\approx K_L\nabla^4\frac{gw_{\tilde{L}-1}}{\delta\phi_L} = -K_L\nabla^4\left(d_L - \frac{gw_{\tilde{L}}}{\delta\phi_L}\right)$$

In model, HD is applied at the end of timestep using fully implicit scheme. For prognostic field X it can be written as:

$$\frac{X^+ - X^{(+)}}{\Delta t} = -K\nabla^4 X^+$$
$$X^+ = \left[1 + \Delta t K\nabla^4\right]^{-1} X^{(+)} \tag{10}$$

Applying equation (10) to quantity $d_L - \frac{gw_{\tilde{L}}}{\delta\phi_L}$ gives:

$$d_L^+ = \left[1 + \Delta t K_L\nabla^4\right]^{-1}\left[d_L^{(+)} - \left(\frac{gw_{\tilde{L}}}{\delta\phi_L}\right)^{(+)}\right] + \left(\frac{gw_{\tilde{L}}}{\delta\phi_L}\right)^+ \tag{11}$$

This is searched formula for "diffusing d_L without part containing $w_{\tilde{L}}$".

Formula (11) confirms that the term $\frac{gw_{\tilde{L}}}{\delta\phi_L}$ should be diagnosed twice during spectral computations. In order to save some extra transforms it was also tried to diagnose it only once – either from undiffused quantities or from diffused ones.

Dirty implementation in vertical plane 2D version of model ALADIN-NH was straightforward, since work on single processor is not a problem for this configuration. As a result, spectral fields are not splitted, so that spectral transforms can be called directly from subroutine `ESPC`, which applies Helmholtz solver and HD. This enables to diagnose term $\frac{gw_{\tilde{L}}}{\delta\phi_L}$ directly in `ESPC`, without necessity to go deeper into the code.

Main target was to test whether exact treatment will work in non-linear regimes and then to propose approximations which will increase efficiency but not spoil results too much. Several implementations were prepared for this purpose. One exact, four approximate (requiring reduced number of extra transforms) and one purely academic (requiring significantly increased number of extra transforms). For each implementation, number of extra spectral transforms per timestep is given in square brackets:

1. Term $\frac{gw_{\tilde{L}}}{\delta\phi_L}$ diagnosed from undiffused quantities to be subtracted and from diffused quantities to be added back.
   `master_al25t2_37_sx6` [12]

2. Term $\frac{gw_{\tilde{L}}}{\delta\phi_L}$ diagnosed from undiffused quantities.
   `master_al25t2_35_sx6` [6]

3. Term $\frac{gw_{\tilde{L}}}{\delta\phi_L}$ diagnosed from diffused quantities.
   `master_al25t2_36_sx6` [6]

4. Term $\frac{gw_{\tilde{L}}}{\delta\phi_L}$ diagnosed as in 1, but using mean geopotential difference instead of $\delta\phi_L$.
   `master_al25t2_39_sx6` [6]

5. Term $\frac{gw_{\tilde{L}}}{\delta\phi_L}$ diagnosed using mean geopotential difference instead of $\delta\phi_L$ and mean wind instead of $\mathbf{v}_L$.[4]
   `master_al25t2_39b_sx6` [0]

6. HD applied on w, conversion $d \rightarrow w$ uses undiffused quantities, conversion $w \rightarrow d$ uses diffused ones.
   `master_al25t2_38_sx6` [$8 \times$ `NFLEVG` $+ 6$]

New logical key `LGWSHD` (meaning gw_S subject to HD) was introduced in namelist `NAMDYN`. It enables to switch between old (`.T.`) and new (`.F.`) HD treatment.

[4] Nobody believed that this one will work.

4 Experiments

All experiments were done in parallel both for eulerian and sl2tl advection scheme. In case of sl2tl scheme `LGWADV` treatment (advection of w) was used in order to prevent SL chimney. `LRDBBC` treatment (diagnostic BBC) was not tested.

Linear and quasi-linear potential flows were used as first test cases. They confirmed that exact treatment supresses HD chimney both in eulerian and sl2tl scheme. Aproximate treatments 2 and 3 behaved well in linear case, but there could be seen first indications of HD chimney in quasi-linear one. Results of these experiments are not shown here.[5] It was decided to concentrate on fully non-linear cases then: non-linear (NL) potential flow and non-linear non-hydrostatic (NLNH) regime with propagating waves.

4.1 Setup of experiments - NL potential flow

- Initial state:
  - isothermal with temperature 239 K
  - corresponding Brunt-Väisälä frequency $N = 0.02\,\mathrm{s}^{-1}$
  - constant wind profile with $V = 15\,\mathrm{ms}^{-1}$
  - sea level pressure 101 325 Pa
- Orography: Bell shaped mountain.

  height: $h = 100\,\mathrm{m}$
  half-width: $a = 100\,\mathrm{m}$
- Dimensionless flow parameters:

$$C_L = \frac{Nh}{V} = 0.13 \quad (C_L \ll 1 \Rightarrow \text{linear flow})$$

$$C_H = \frac{V}{Na} = 7.5 \quad (C_H \ll 1 \Rightarrow \text{hydrostatic flow})$$

- Geometry:

Δx	[m]	20	($a = 5\Delta x$)
Δz	[m]	≈20	(regular z-levels)
`NDGUX`		64	(C+I zone)
`NDGL`		64	(no E zone)
`NBZONG`		8	(I zone)
`NSMAX`		21	(quadratic grid)
`NFLEVG`		39	

- Vertical coordinate: σ
- Coupling files: Identical with initial file (time constant LBC).

[5] Setup was identical to non-linear potential flow, except for mountain height h which was 10 and 50 m respectively.

- Common integration settings:

t_{STOP}	[s]	200
NPDVAR		2
NVDVAR		3
SIPR	[Pa]	90000.
REPONBT	[m]	450.
REPONTP	[m]	750.
REPONTAU	[s]	1.0
HDIRDIV*	[s]	0.2
HDIRVD*	[s]	0.2
HDIRVOR*	[s]	1.0
HDIRT*	[s]	1.0
VESL		0.0
XIDT		0.0

(*) When HD was turned on, zero otherwise.

- Scheme dependent integration settings:

		euler	sl2tl
TSTEP	[s]	0.2	1.0
RCMSLP0		0.0	1.0
SITR	[K]	239.	300.
SITRA	[K]	239.	100.
LGWADV		.F.	.T.
LPC		OLD	FULL NESC
NSITER		1	3

- Experiment dependent settings:

figure		HD treatment
euler	sl2tl	
1	2	no HD
3	4	0 – old
5	6	6 – HD on w without $w_{\tilde{L}}$
7	8	1 – exact
9	10	2 – term $\frac{gw_{\tilde{L}}}{\delta\phi_L}$ diagnosed from undiffused fields
11	12	3 – term $\frac{gw_{\tilde{L}}}{\delta\phi_L}$ diagnosed from diffused fields
13	14	4 – approximate $\delta\phi_L$
15	16	5 – approximate $\delta\phi_L$ and $\mathbf{v}_L$

4.2 Setup of experiments - NLNH regime

- Initial state:
  - temperature profile with constant Brunt-Väisälä frequency $N = 0.01\,\mathrm{s}^{-1}$ up to tropopause at height 21 km, isothermal above tropopause
  - sea level temperature 293 K
  - tropopause temperature 133 K
  - constant wind profile with $V = 10\,\mathrm{ms}^{-1}$
  - sea level pressure 101 325 Pa
- Orography: Bell shaped mountain.

  height: $h = 1000\,\mathrm{m}$
  half-width: $a = 1000\,\mathrm{m}$
- Dimensionless flow parameters:

  $$C_L = \frac{Nh}{V} = 1.0 \quad (C_L \ll 1 \Rightarrow \text{linear flow})$$

  $$C_H = \frac{V}{Na} = 1.0 \quad (C_H \ll 1 \Rightarrow \text{hydrostatic flow})$$
- Geometry:

Δx	[m]	200	($a = 5\Delta x$)
Δz	[m]	≈300	(regular z-levels)
NDGUX		128	(C+I zone)
NDGL		128	(no E zone)
NBZONG		14	(I zone)
NSMAX		42	(quadratic grid)
NFLEVG		100	(30 levels above tropopause)

- Vertical coordinate: η
- Coupling files: Identical with initial file (time constant LBC).
- Common integration settings:

t_{STOP}	[s]	5000
NPDVAR		2
NVDVAR		3
SIPR	[Pa]	90000.
REPONBT	[m]	20000.
REPONTP	[m]	29500.
REPONTAU	[s]	100.
HDIRDIV*	[s]	1.0
HDIRVD*	[s]	1.0
HDIRVOR	[s]	0.0
HDIRT	[s]	0.0
VESL		0.0
XIDT		0.0

(∗) When HD was turned on, zero otherwise.

- Scheme dependent integration settings:

	euler	sl2tl
`TSTEP` [s]	`2.5`	`10.0`
`RCMSLP0`	`0.0`	`1.0`
`SITR` [K]	`220.`	`300.`
`SITRA` [K]	`220.`	`50.`
`LGWADV`	`.F.`	`.T.`
`LPC`	`OLD`	`FULL NESC`
`NSITER`	`1`	`3`

- Experiment dependent settings:

figure		HD treatment
euler	sl2tl	
17	18	no HD
19	20	6 – HD on w without $w_{\tilde{L}}$
21	22	1 – exact
23	24	4 – approximate $\delta\phi_L$
25	26	0 – old

4.3 Experimental results

Field of vertical velocity w is shown on all figures, since the chimney problem (either SL or HD) always demonstrates itself in this field. Background flow in all experiments is from left to right.

Figures 1–16 show results for NL potential flow. Only bottom half of integration domain (up to 400 m) is displayed. Sponge region is not shown. It can be observed that:

- Solutions without HD do not contain chimney, but they are noisy (fig. 1, 2). For sl2tl scheme there is some problem behind the mountain (fig. 2). It might be related to the use of non-extrapolating scheme, but it was not further examined.
- Turning on HD using old treatment smoothes the fields, but chimney appears (fig. 3, 4). It is almost 2 times stronger for eulerian scheme. Problem of sl2tl scheme behind the mountain disappeared (fig. 4).
- HD applied on w without $w_{\tilde{L}}$ suppresses the chimney completely (fig. 5, 6). There can be seen some non-chimney like distortion in eulerian scheme (fig. 5 versus 1). Applied HD was not strong enough to create smooth fields.
- Exact HD treatment gives almost identical results to HD applied on w without $w_{\tilde{L}}$ (fig. 7, 8).
- Approximate HD treatment 2 removes big part of the chimney, but its remnants are still clearly visible (fig. 9, 10). Some noise reappears compared to old HD treatment.
- Approximate HD treatment 3 is only slightly better than 2 (fig. 11, 12).
- Approximate HD treatment 4 gives almost identical results to the exact treatment (fig. 13, 14).

- Approximate HD treatment 5 has disastrous consequences. Chimney appears having $\frac{2}{3}$ of its original strength (fig. 15, 16).

Taking into account results for NL potential flow, HD treatments 1, 4 and 6 were tested further in NLNH regime. Strong HD was used in order to get smooth fields. Results of these tests are shown on figures 17–26:

- Solutions without HD do not contain chimney, but they are noisy (fig. 17, 18). There is some difference between eulerian and sl2tl response, especially for the first tilted maximum behind the mountain.
- HD applied on w without $w_{\tilde{L}}$ suppresses the chimney completely (fig. 19, 20). Fields are smooth and the difference between eulerian and sl2tl response is smaller.
- Exact HD treatment gives results very close to HD applied on w without $w_{\tilde{L}}$ (fig. 21, 22).
- Approximate HD treatment 4 gives results similar to the exact treatment (fig. 23, 24). Some difference can be seen above the mountain top, especially for sl2tl scheme (fig. 24 versus 22).
- Old HD treatment is completely unusable (fig. 25, 26). Chimney strength is comparable to the amplitude of wave response. Moreover, wave response itself is amplified.

5 Conclusions

- HD chimney can be prevented by "not diffusing part of d_L containing $w_{\tilde{L}}$", even in strongly non-linear regimes.

- Exact implementation of this treatment requires 12 extra spectral transforms per timestep.

- Approximate treatment using mean value of $\delta\phi_L$ reduces number of extra spectral transforms per timestep to 6. It works satisfactorily in academic 2D tests. Some deterioration must be expected in real 3D cases, since there will be greater horizontal variation of fields determining $\delta\phi_L$. Anyway, this treatment is proposed as the best candidate for 3D implementation.

- Precise diagnostics of $w_{\tilde{L}}$ in the term $\frac{g w_{\tilde{L}}}{\delta\phi_L}$ is crucial. This means that expression $\mathbf{v}_L \cdot \nabla\phi_{\tilde{L}}$ must be evaluated in spectral space twice per timestep. No transforms can be saved here.

- Exact treatment gives very similar results to HD applied on w without $w_{\tilde{L}}$. This justifies use of $\nabla\delta\phi_L = 0$ assumption when converting HD tendencies between d and w, at least in academic 2D framework. But again, in real 3D cases difference between the two treatments might become more significant.

- For 3D implementation, call of spectral transforms from the level of `ESPC` seems to be unavoidable. Communications between processors will have to be solved in order to enable this.

- To prevent extra spectral transforms completely, there is an alternative to evaluate product $\mathbf{v}_L \cdot \nabla\phi_{\tilde{L}}$ directly in double Fourier space (for products this is possible, see appendix A). However, this approach does not remove the problem with inter-processor communications. Moreover, advantage of higher efficiency would be compensated by increased demands for maintaining such piece of code.

6 Info section

6.1 Unfinished work

- There were no tests performed using `LRDBBC` (diagnostic BBC) instead of `LGWADV` (advection of w). But there is no reason to suppose that proposed HD treatment will not work also for this case.
- 3D implementation was not solved.

6.2 Code info

All work was based on cycle `25t2`. Several versions of the code were used:

`34` = reference version + modifications from J. Vívoda:
- bugfix
- `SITRA`
- `LGWADV` + `LPC_FULL` + `LPC_NESC` for all d variables
- `HDIR[X]` $\leq$ `1.0` enabled

`35` = `34` + HD treatment 2

`36` = `34` + HD treatment 3

`37` = `34` + HD treatment 1

`38` = `34` + HD treatment 6

`39` = `34` + HD treatment 4

`39b` = `34` + HD treatment 5

CVSTUC branches (`kappa`):

```
Ald_mma157_AL25t2_34
Arp_mma157_CY25t2_34
Ald_mma157_AL25t2_35
Arp_mma157_CY25t2_35
```

Modified sources (`kappa`):

```
~mma157/cycle_25t2/mod_34_ald/
                   mod_34_arp/
                   mod_35d34_ald/
                   mod_35d34_arp/
                   mod_36_ald/
                   mod_36_arp/
                   mod_37d36_ald/
                   mod_37d36_arp/
                   mod_38d36_ald/
                   mod_38d36_arp/
                   mod_39d36_ald/
                   mod_39d36_arp/
                   mod_39bd36_ald/
                   mod_39bd36_arp/
```

Sources + dependencies for compilation (`kappa`):

```
~mma157/cycle_25t2/dep_<ver>_ald/
                   dep_<ver>_arp/

  <ver> = 34, 35, 36, 37, 38, 39, 39b
```

Loading scripts (`kappa`):

```
~mma157/cycle_25t2/load/load_<ver>_sx6

  <ver> = 34, 35, 36, 37, 38, 39, 39b
```

Executables (`archiv`):

```
~mma157/bin/master_al25t2_<ver>_sx6

  <ver> = 34, 35, 36, 37, 38, 39, 39b
```

Integration scripts (`voodoo`):

```
~mma157/m2d/exp/script_05/
```

7 Final remark

"The Devil is really in the detail!!"

JFG

"And only God knows, in which one," I add.

Appendix

A Evaluation of products in double Fourier space

Assume real biperiodic function $f(x, y)$ with periods L_x, L_y, which can be expressed as elliptically truncated double Fourier serie with corresponding truncations M and N:

$$f(x, y) = \sum_{m',n'} \hat{f}_{m',n'} \exp\left[im'\frac{2\pi x}{L_x}\right] \exp\left[in'\frac{2\pi y}{L_y}\right] \tag{12}$$

$$\frac{m'^2}{M^2} + \frac{n'^2}{N^2} \leq 1 \tag{13}$$

This means that f is spectrally fitted ($f = \lfloor f \rfloor$, where $\lfloor\ \rfloor$ denotes truncation operator).

Because the function f is real, spectral coefficient $\hat{f}_{-m',-n'}$ is a complex conjugate of $\hat{f}_{m',n'}$. This means that only one half of spectral coefficients is independent. In case of rectangular truncation there would be $(2M+1) \times (2N+1)$ independent real values. For elliptical truncation this number is reduced by factor $\frac{\pi}{4}$.

Assume another function g with analogical properites as f. Then $g(x, y)$ can be expressed as:

$$g(x, y) = \sum_{m'',n''} \hat{g}_{m'',n''} \exp\left[im''\frac{2\pi x}{L_x}\right] \exp\left[in''\frac{2\pi y}{L_y}\right] \tag{14}$$

$$\frac{m''^2}{M^2} + \frac{n''^2}{N^2} \leq 1 \tag{15}$$

Now the Fourier serie for spectrally fitted product $h \equiv \lfloor f \cdot g \rfloor$ will be derived. Unfitted product $f \cdot g$ can be obtained directly by combining expressions (12) and (14):

$$f(x, y) \cdot g(x, y) = \sum_{m',n'} \sum_{m'',n''} \hat{f}_{m',n'} \hat{g}_{m'',n''} \exp\left[i(m' + m'')\frac{2\pi x}{L_x}\right] \exp\left[i(n' + n'')\frac{2\pi y}{L_y}\right] \tag{16}$$

Summation region is given by constraints (13) and (15). If new indexes $m \equiv m' + m''$ and $n \equiv n' + n''$ are introduced, equation (16) can be rearranged into the form:

$$f(x, y) \cdot g(x, y) = \sum_{m',n'} \sum_{m,n} \hat{f}_{m',n'} \hat{g}_{m-m',n-n'} \exp\left[im\frac{2\pi x}{L_x}\right] \exp\left[in\frac{2\pi y}{L_y}\right] \tag{17}$$

When spectrally fitted product $\lfloor f \cdot g \rfloor$ is searched, there must be additional constraint imposed on (17):

$$\frac{m^2}{M^2} + \frac{n^2}{N^2} \leq 1 \tag{18}$$

Taking into account constraint (18), summation order can be changed in (17):

$$h(x, y) = \sum_{m,n} \left[\sum_{m',n'} \hat{f}_{m',n'} \hat{g}_{m-m',n-n'}\right] \exp\left[im\frac{2\pi x}{L_x}\right] \exp\left[in\frac{2\pi y}{L_y}\right] \tag{19}$$

Finally, expression for spectral coefficients $\hat{h}_{m,n}$ follows directly from (19). It must be remembered that constraints (13) and (15) still apply to it, but the second one must be re-expressed in terms of m, m', n and n':

$$\hat{h}_{m,n} = \sum_{m',n'} \hat{f}_{m',n'} \hat{g}_{m-m',n-n'} \tag{20}$$

$$\frac{m'^2}{M^2} + \frac{n'^2}{N^2} \leq 1 \qquad \frac{(m'-m)^2}{M^2} + \frac{(n'-n)^2}{N^2} \leq 1 \tag{21}$$

Geometrical interpretation of constraints (21) is very simple. For fixed m and n, summation in (m', n') space is done through the intersection of two ellipses: first centered at $(0, 0)$ and second centered at (m, n). Thanks to the constraint (18) this intersection is always non-empty.

References

[1] Bénard, P., 2003: On the ill-posedness of d-type variables, *internal document*

[2] Bénard, P., 2003: Scientific Documentation for ALADIN-NH Dynamical Kernel, *ALADIN documentation, version 2.0*

[3] Mašek, J., 2003: Reformulation of Bottom Boundary Condition for Term $\frac{\partial \tilde{p}}{\partial \pi}$, *report*

[4] Mašek, J., 2003: Residual Problems in ALADIN-NH Dynamical Core, *report, revised 24.6.2004*