3-Fold Decomposition EFB Closure for Convective Turbulence and Organized Structures

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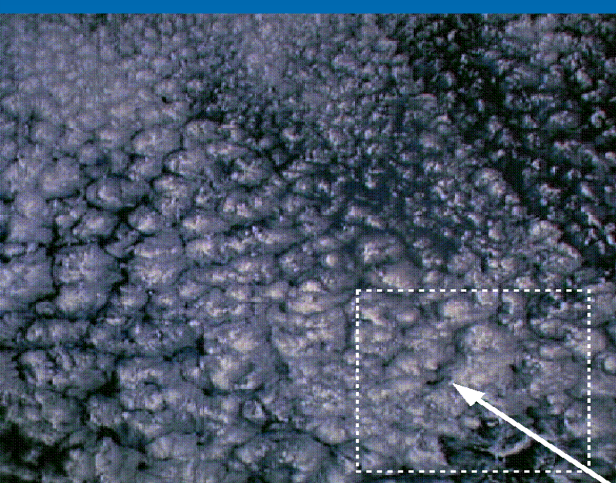
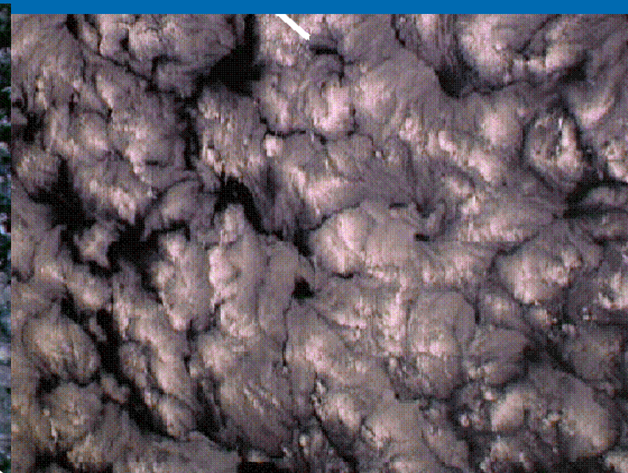




Outline

- Introduction
- Mechanism of formation of cloud cells in shear-free convection
- EFB closure for shear-free convection
- Mechanism of formation of cloud streets in sheared convection
- EFB closure for sheared convection
- Conclusions and future studies

Atmospheric Turbulent Convection

- **The atmospheric turbulent convection:**
  - the fully organized large-scale flow (the mean flow or mean wind)
  - the small-scale turbulent fluctuations,
  - long-lived large-scale semi-organized (coherent) structures.

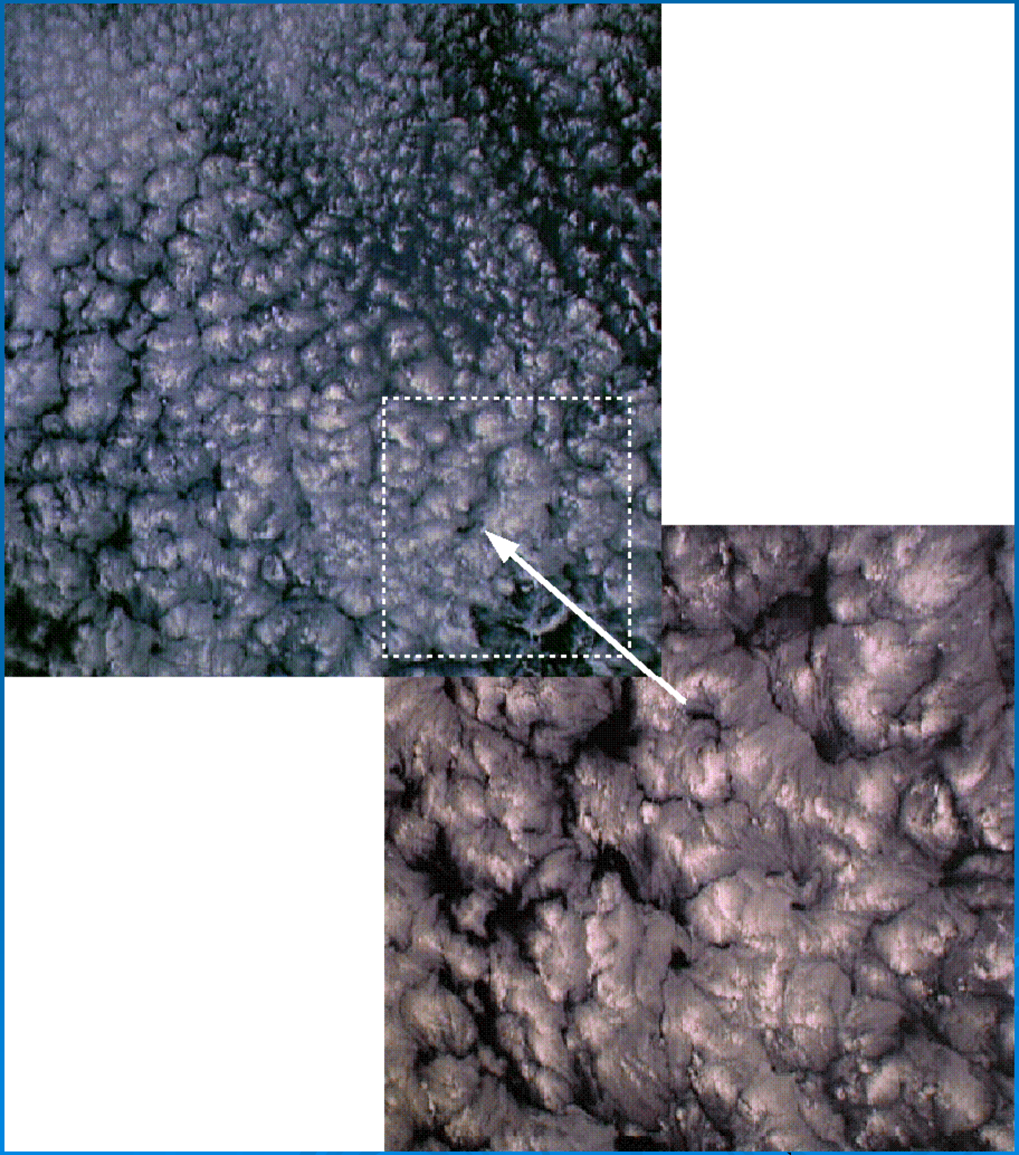
- **Two types of the semi-organized structures:**
  - cloud "streets"
  - cloud cells

- **The life-times and spatial scales of the semi-organized structures are much larger than the turbulent scales.**

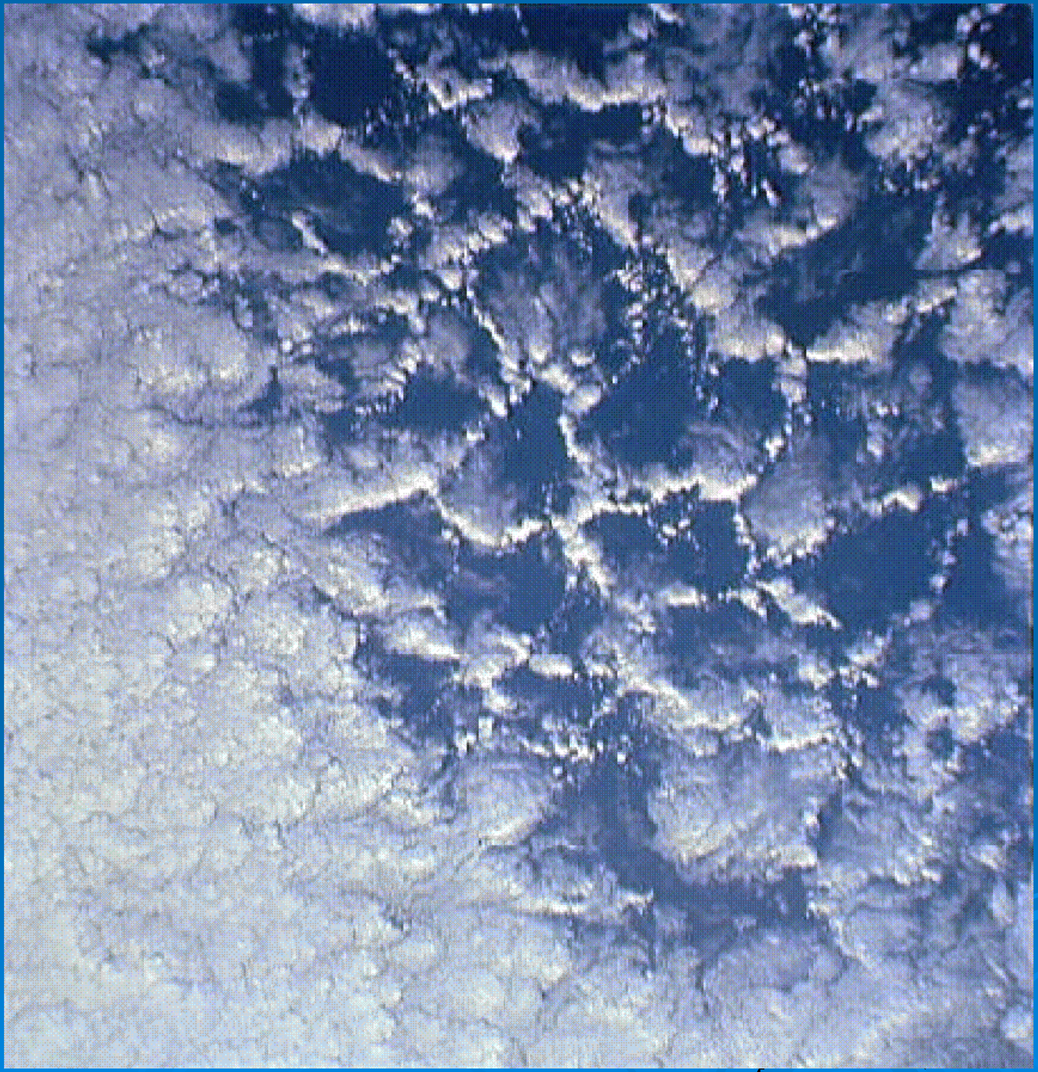
Etling, D. and **Brown**, R. A., 1993. *Boundary-Layer Meteorol.*, **65**, 215—248.

Atkinson, B. W. and **Wu Zhang**, J., 1996. *Reviews of Geophysics*, **34**, 403—431.

Closed cloud cells over the Atlantic Ocean

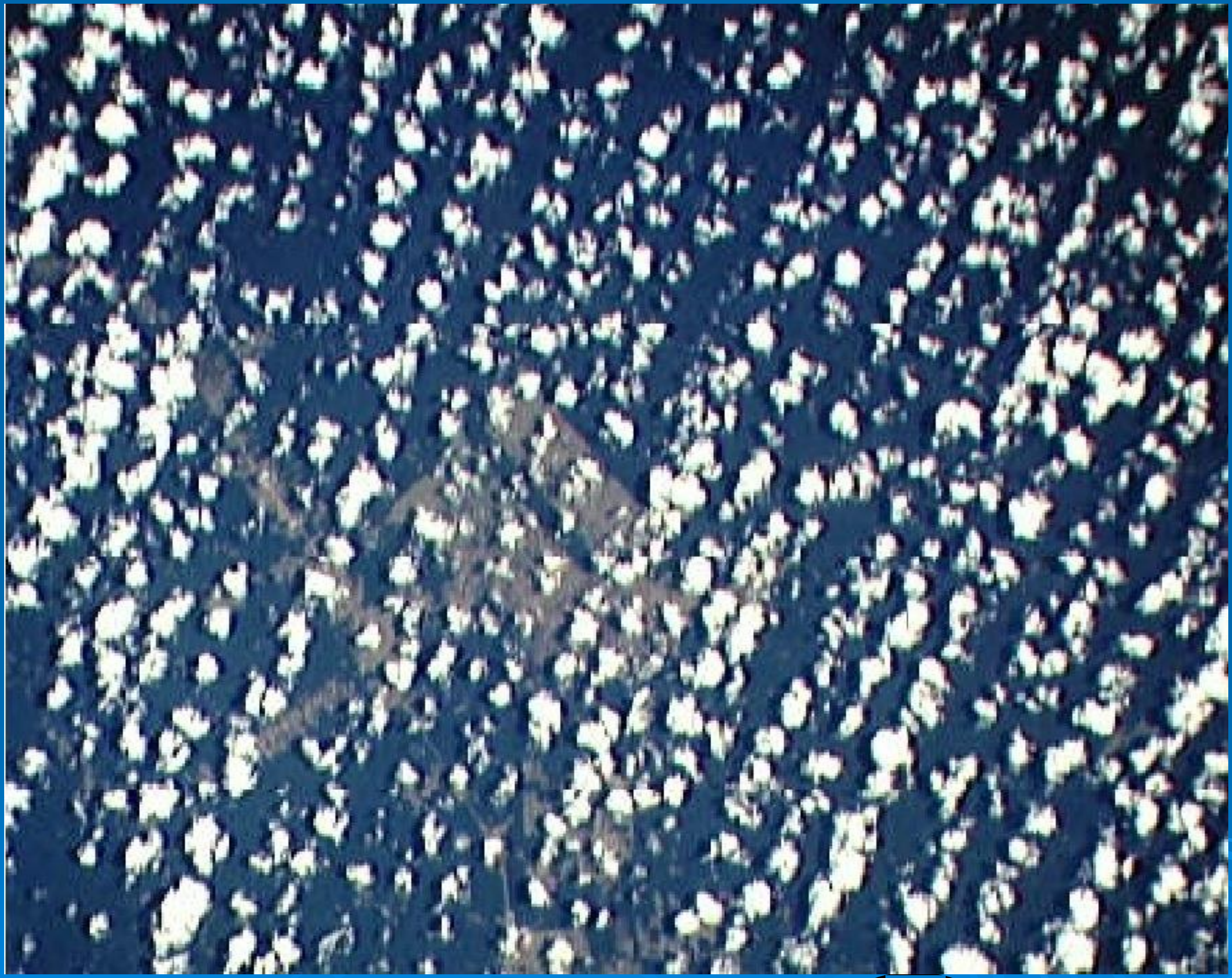


Open cloud cells over the Pacific Ocean



Cloud “streets” over Indian ocean

Cloud “streets” over the Amazon River



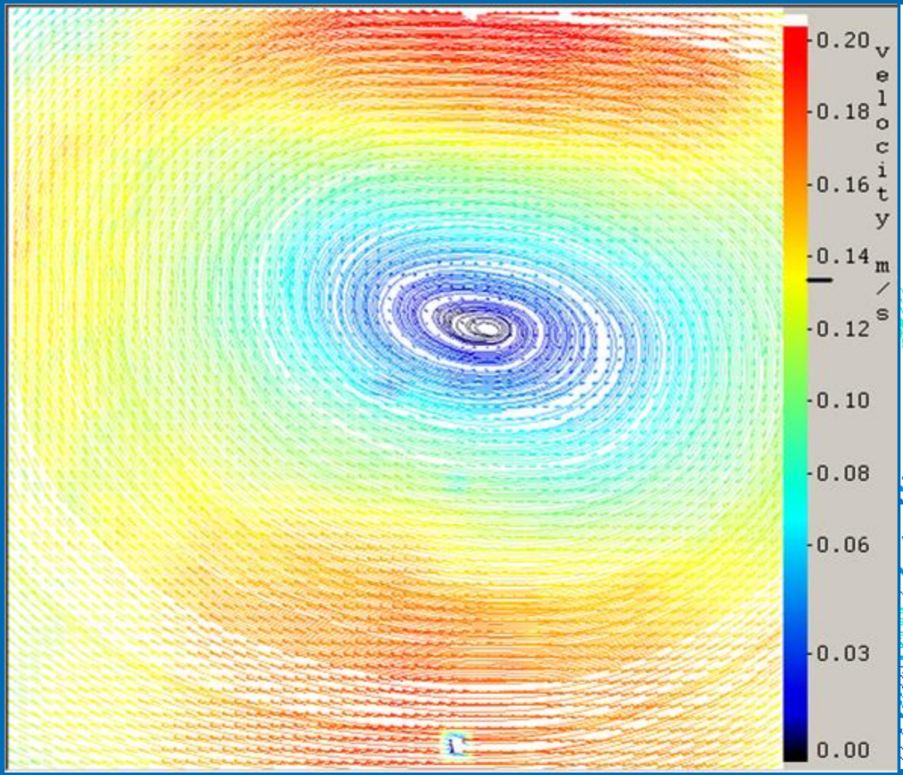
Laboratory Turbulent Convection

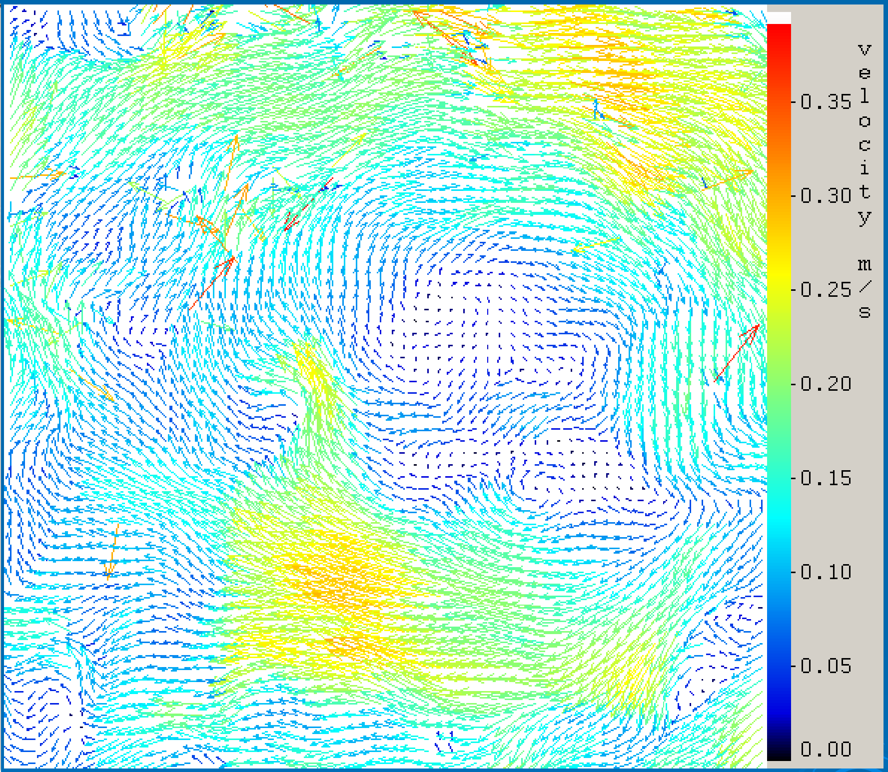
- In laboratory turbulent convection several organized features of motion, such as plumes, jets, and large-scale circulation patterns are observed.
- The large-scale circulation in a closed box with the heated bottom (in the Rayleigh-Benard apparatus) is often called the "mean wind".

There are several unsolved theoretical questions concerning these flows:

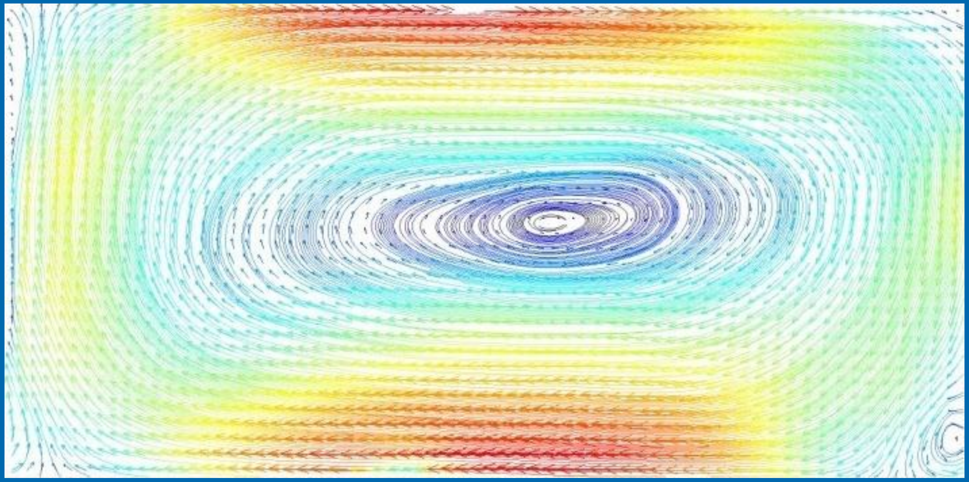
- How do they arise ?
- What are their characteristics and dynamics?

Coherent Structures (Mean Wind) in Laboratory Turbulent Convection

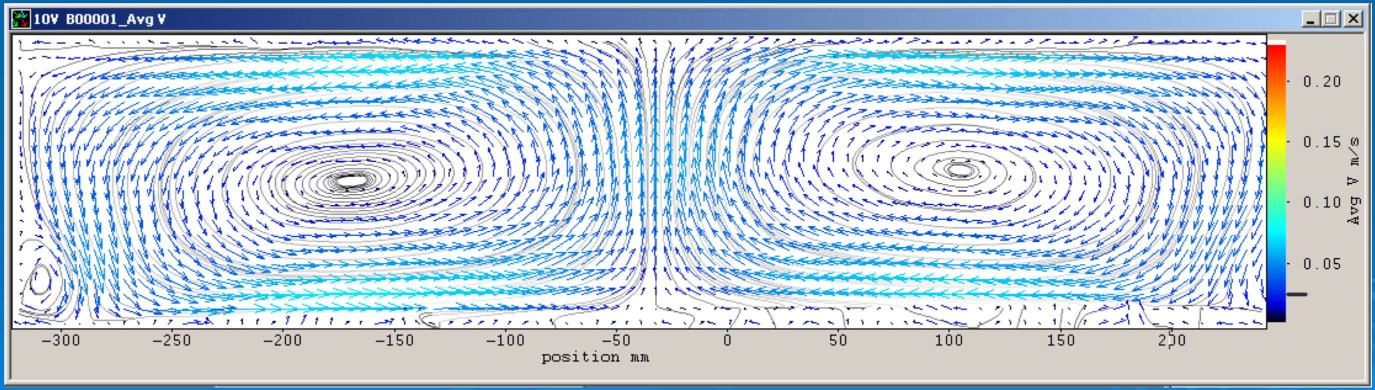




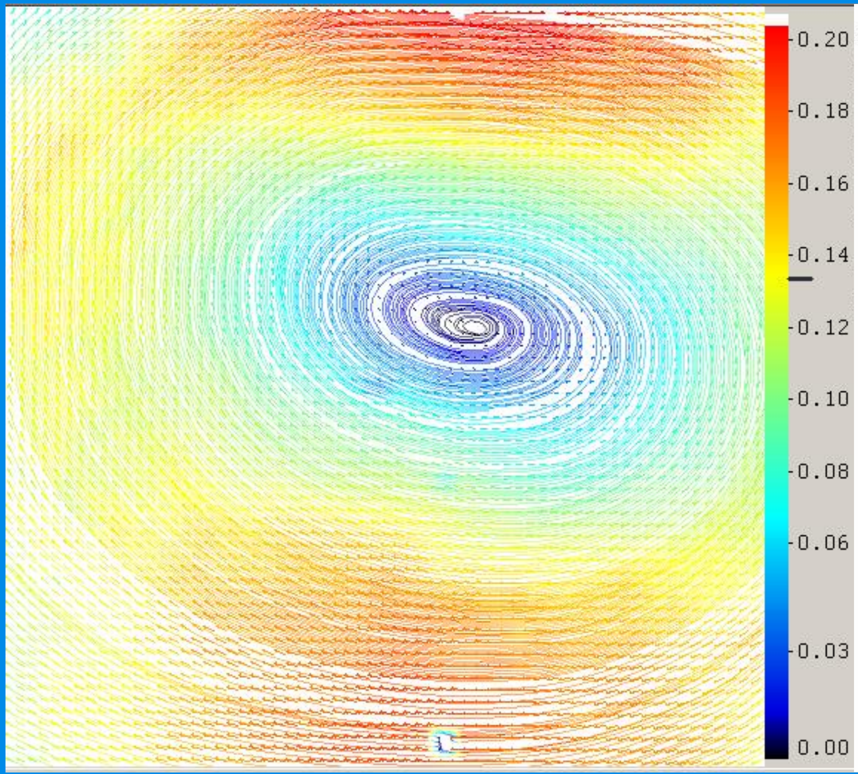
Preferential Coherent structures for A=2

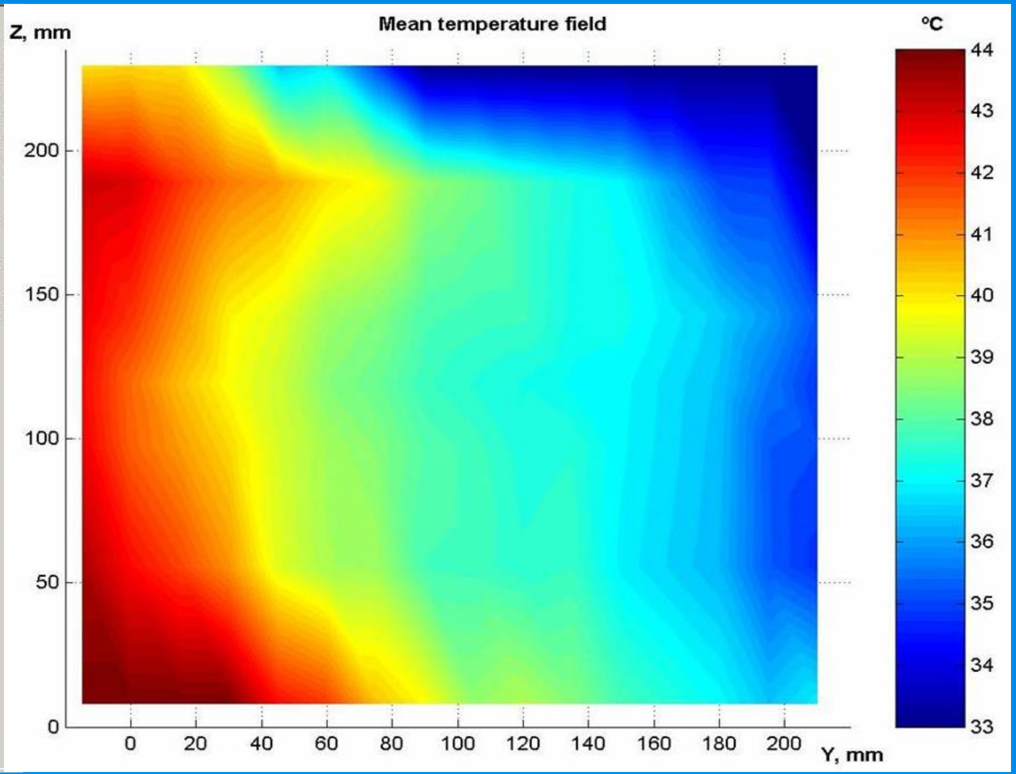


Preferential Coherent structures for A=4



Unforced Convection: A = 1

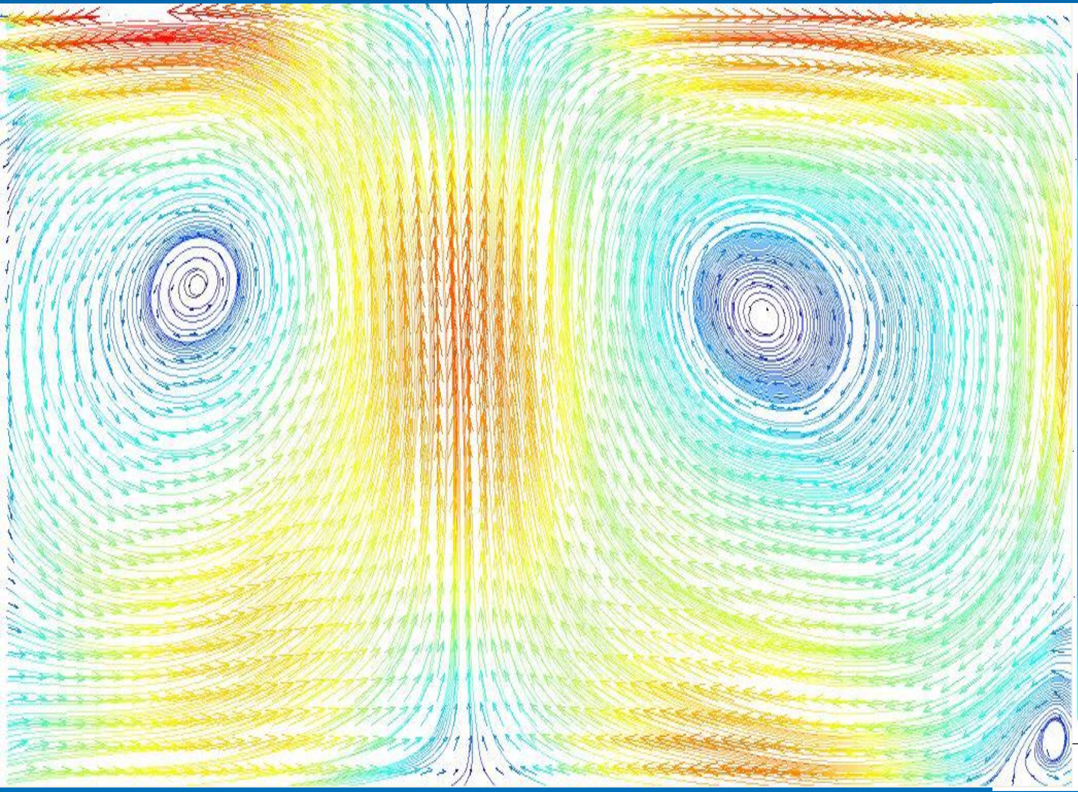


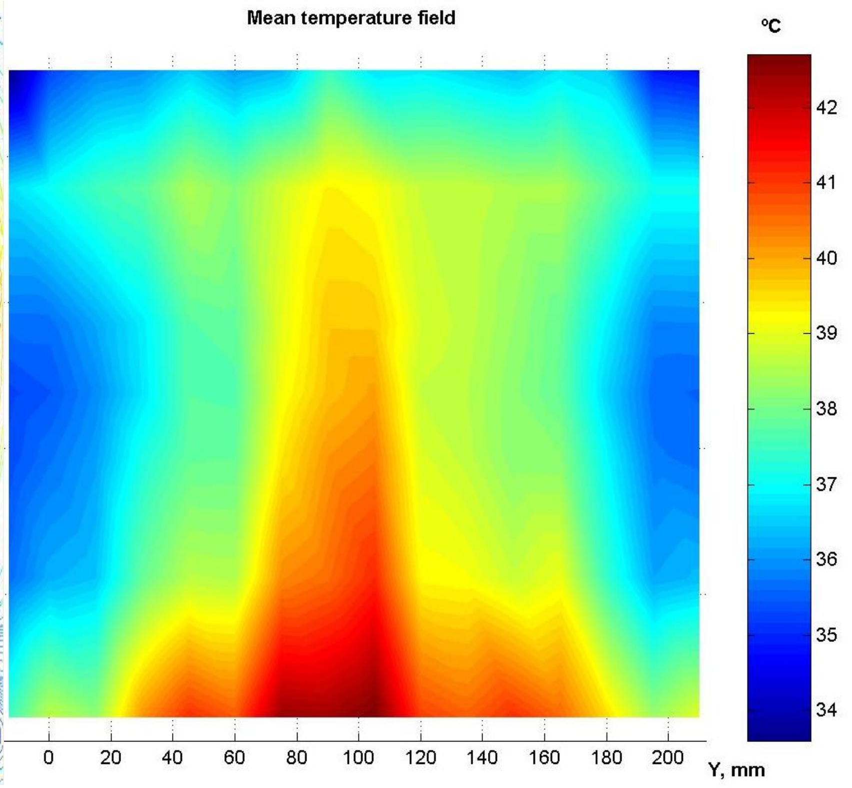


$\overline{U}(y,z)$

$\overline{T}(y,z)$

Unforced Convection: A=2





$\overline{U}(y,z)$

$\overline{T}(y,z)$

Problems

- The Rayleigh numbers based on the molecular transport coefficients are very large:

$$Ra = \frac{g\,\beta\,\Delta T\,L^3}{\nu\ \kappa} \approx 10^{11} \div 10^{13}$$

This corresponds to fully developed turbulent convection in atmospheric and laboratory flows.

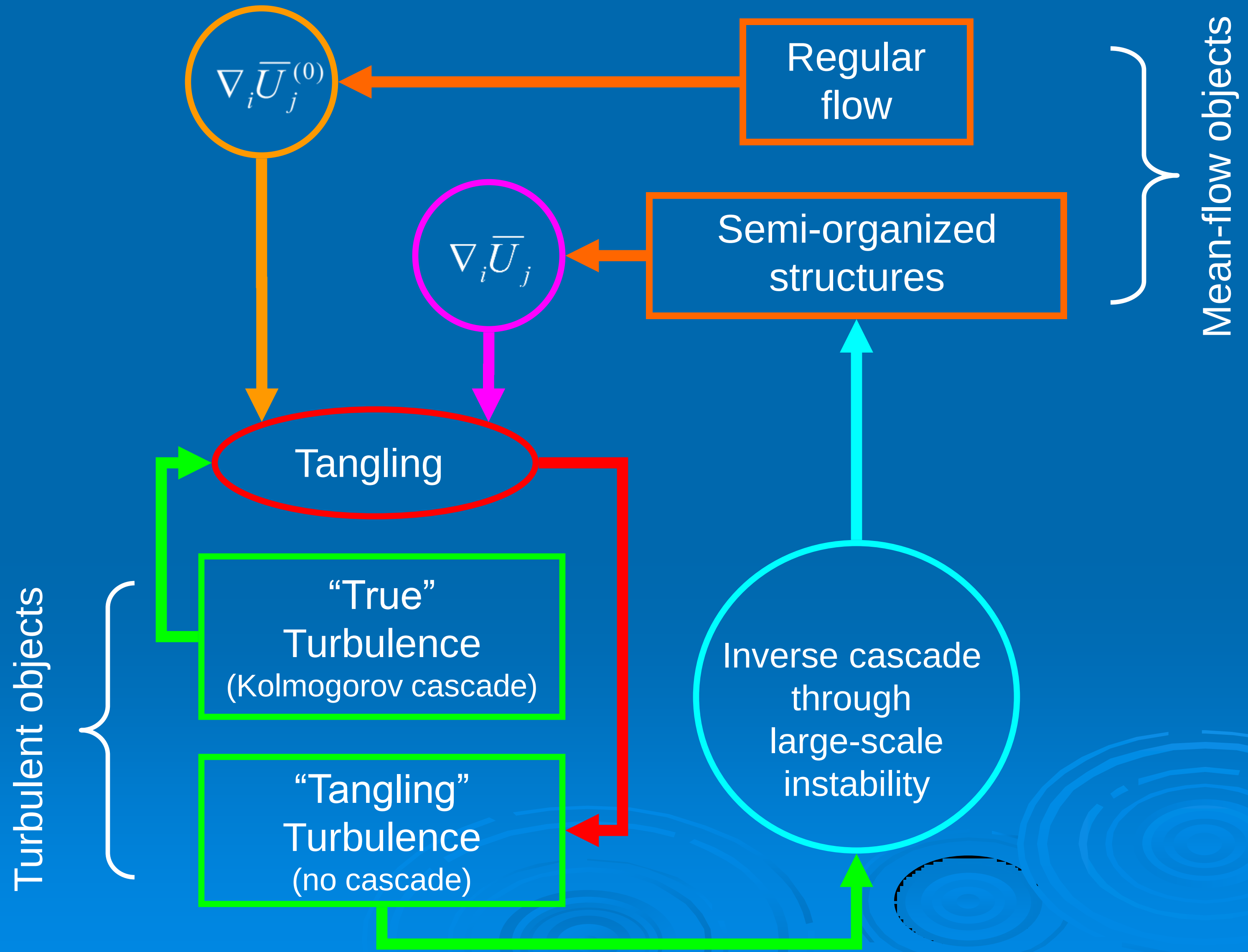
- The effective Rayleigh numbers based on the turbulent transport coefficients (the turbulent viscosity and turbulent diffusivity) are not high.

$$Ra^{eff} = \frac{g\,\beta\,\Delta T\,L^3}{\nu_T\ \kappa_T}$$

They are less than the critical Rayleigh numbers required for the excitation of large-scale convection.

Hence the emergence of large-scale convective flows (which are observed in the atmospheric and laboratory flows) seems puzzling.

Interaction between mean-flow and turbulent objects



Tangling turbulence in sheared mean flow

Sheared mean flow

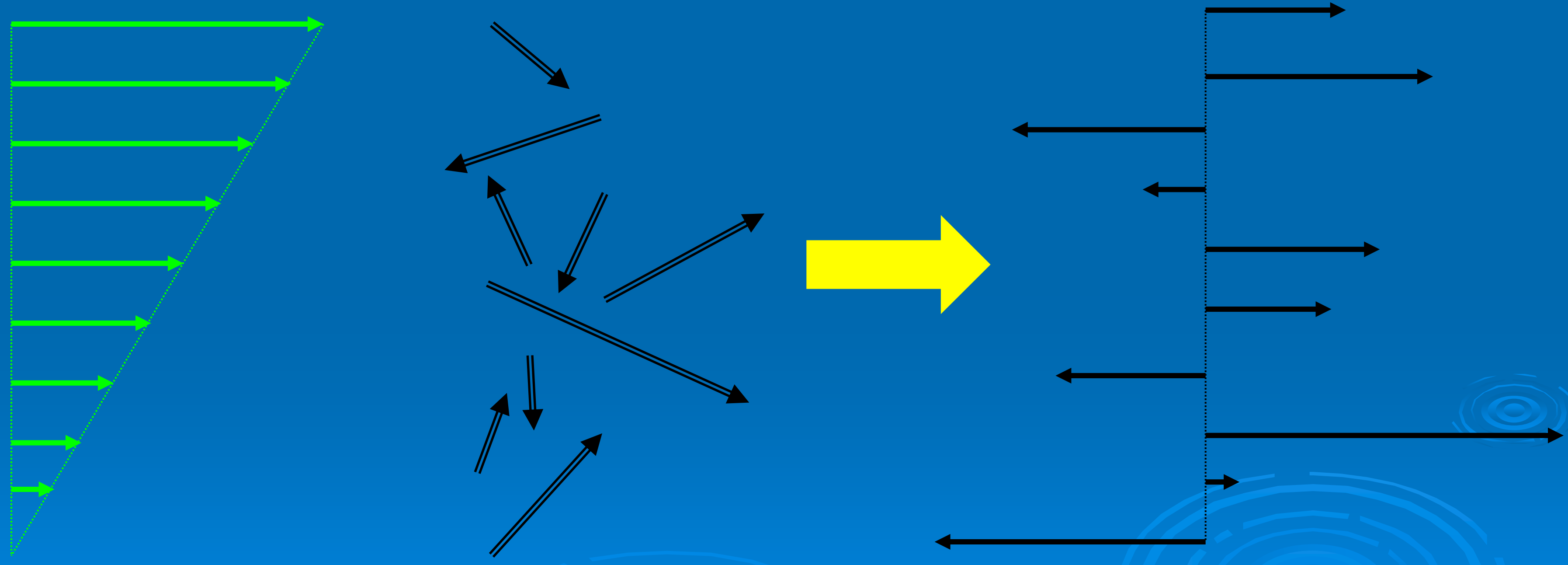
$$\nabla \overline{U}^{(0)} \neq 0$$

Kolmogorov turbulence

$$\vec{u}$$

"Tangling" turbulence

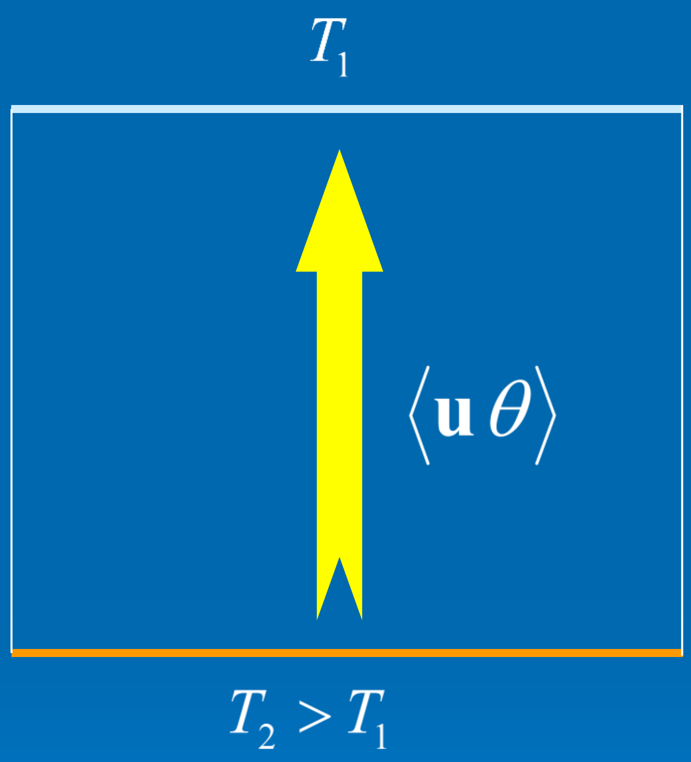
$$\delta u \propto (\vec{u}\cdot\vec{\nabla})\,\overline{U}^{(0)}$$



$k^{-5/3}$

Lumley (1967) $k^{-7/3}$

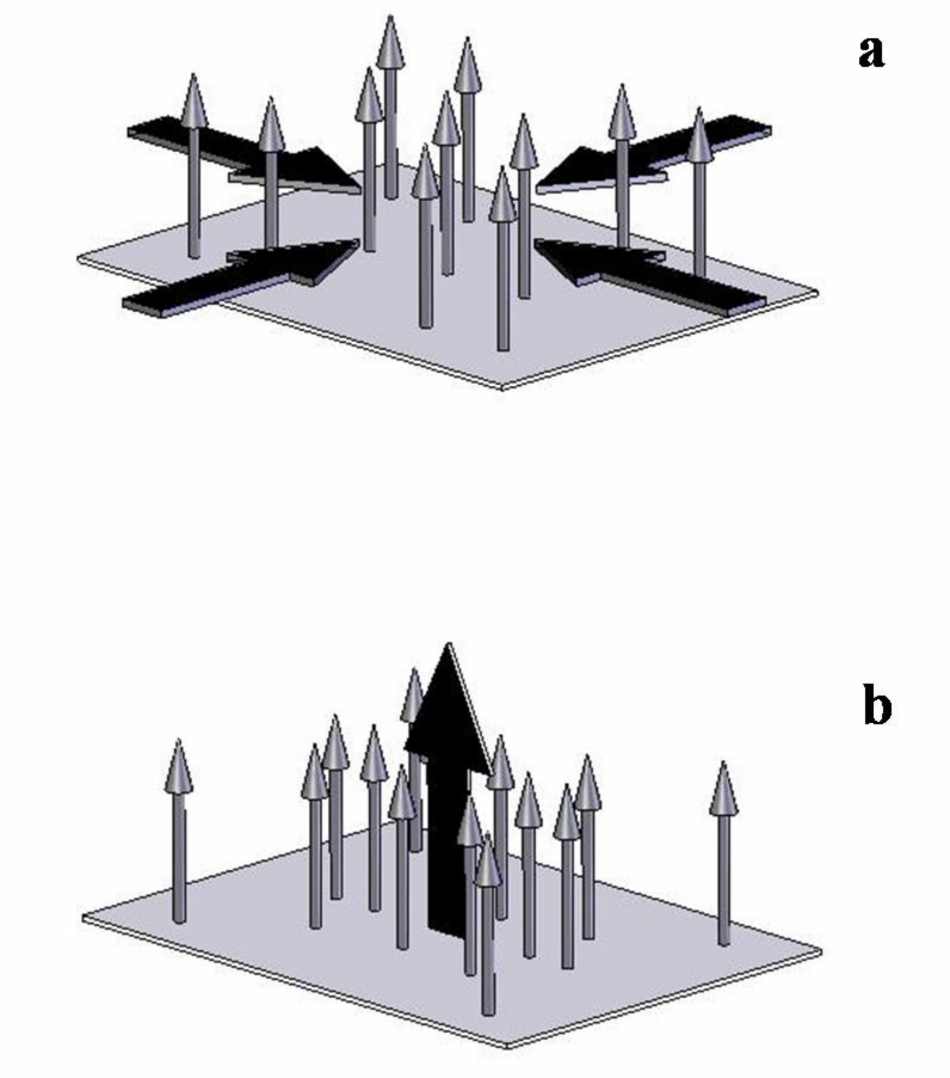
Heat flux



$$\langle \mathbf{u}\,\theta \rangle = -\kappa_T \vec{\nabla} \overline{\Theta}$$

$$\kappa_T \cong \frac{u_0\, l_0}{3}$$

Redistribution of a homogeneous vertical turbulent heat flux by a converging horizontal mean flow



$$\langle \theta \mathbf{u} \rangle = -\kappa_T \vec{\nabla} \overline{\Theta}$$

$$\vec{\nabla} \cdot \overline{\mathbf{U}}_\perp < 0$$

$$\langle \theta \mathbf{u} \rangle \approx -\kappa_T \vec{\nabla} \overline{\Theta} \left[1 - \tau_0 \left(\vec{\nabla} \cdot \overline{\mathbf{U}}_\perp \right) \right]$$

Heat flux

Traditional parameterization

$$\langle \theta\, \mathbf{u} \rangle = -\kappa_T\, \vec{\nabla}\, \overline{\Theta}$$

Modification of the heat flux by tangling turbulence

$$\langle \theta\, \mathbf{u} \rangle \approx -\kappa_T\, \vec{\nabla}\overline{\Theta} \left[1 - \tau_0 \left(\vec{\nabla} \cdot \overline{\mathbf{U}}_{\perp} \right) \right]$$

Mean field equations

$$\left(\frac{\partial}{\partial t}+\overline{\mathbf{U}}\cdot\vec{\nabla}\right)\overline{U}_i=-\nabla_i\left(\frac{\overline{P}}{\rho_0}\right)-\nabla_j\left\langle u_i u_j\right\rangle-g_i\overline{\Theta}+\nu\Delta\overline{\mathbf{U}},$$

$$\left(\frac{\partial}{\partial t}+\overline{\mathbf{U}}\cdot\vec{\nabla}\right)\overline{\Theta}=-\nabla_i\left\langle\theta\, u_i\right\rangle+\kappa\Delta\overline{\Theta}$$

$\boldsymbol{\Phi}\equiv\left\langle\theta\,\mathbf{u}\right\rangle$ is the heat flux

$\left\langle u_i u_j\right\rangle$ are the Reynolds stresses

Method of Derivation

Equations for the correlation functions for:

- **The velocity fluctuations** $(M_{ij}^{(II)})_u \equiv \langle u_i u_j \rangle$
- **The temperature fluctuations** $M_\theta^{(II)} \equiv \langle \theta\, \theta \rangle$
- **The heat flux** $(M_i^{(II)})_\Phi \equiv \langle \theta\, u_i \rangle$

The spectral τ-approximation (the third-order closure procedure)

$$\hat{D}M^{(III)}(\mathbf{k}) - \hat{D}M_K^{(III)}(\mathbf{k}) = -\frac{M^{(II)}(\mathbf{k}) - M_K^{(II)}(\mathbf{k})}{\tau_c(\mathbf{k})}$$

$$\left(\hat{D}M_{ij}^{(III)}\right)_u = -\langle u_j (\mathbf{u}\cdot\nabla) u_i \rangle - \langle u_i (\mathbf{u}\cdot\nabla) u_j \rangle$$

$$\left(\hat{D}M_{ij}^{(III)}\right)_\theta = -2\langle \theta (\mathbf{u}\cdot\nabla)\theta \rangle$$

$$\left(\hat{D}M_i^{(III)}\right)_\Phi = -\langle u_i (\mathbf{u}\cdot\nabla)\theta \rangle - \langle \theta (\mathbf{u}\cdot\nabla) u_i \rangle$$

Heat flux

Traditional parameterization

$$\langle \theta \, \mathbf{u} \rangle = -\kappa_T \, \vec{\nabla} \, \overline{\Theta}$$

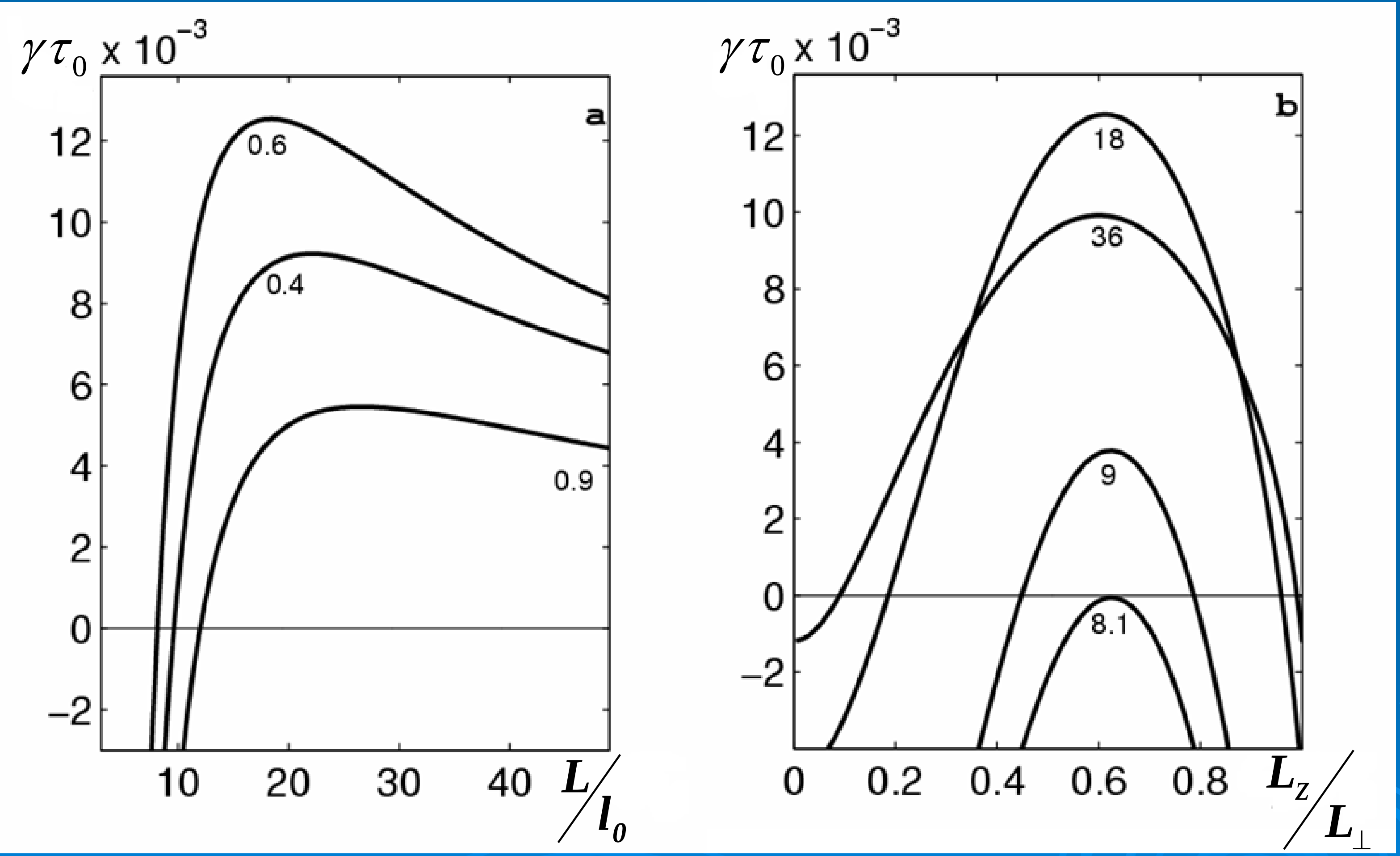
Modification of the heat flux by tangling turbulence

$$\langle \theta \, \mathbf{u} \rangle = \mathbf{\Phi}^* + \frac{\tau_0}{6}\left[-5\alpha\left(\vec{\nabla}\cdot\overline{\mathbf{U}}_\perp\right)\mathbf{\Phi}_z^* + \left(\alpha + \frac{3}{2}\right)\left(\overline{\mathbf{W}}\times\mathbf{\Phi}_z^*\right) + 3\left(\overline{\mathbf{W}}_z\times\mathbf{\Phi}^*\right)\right]$$

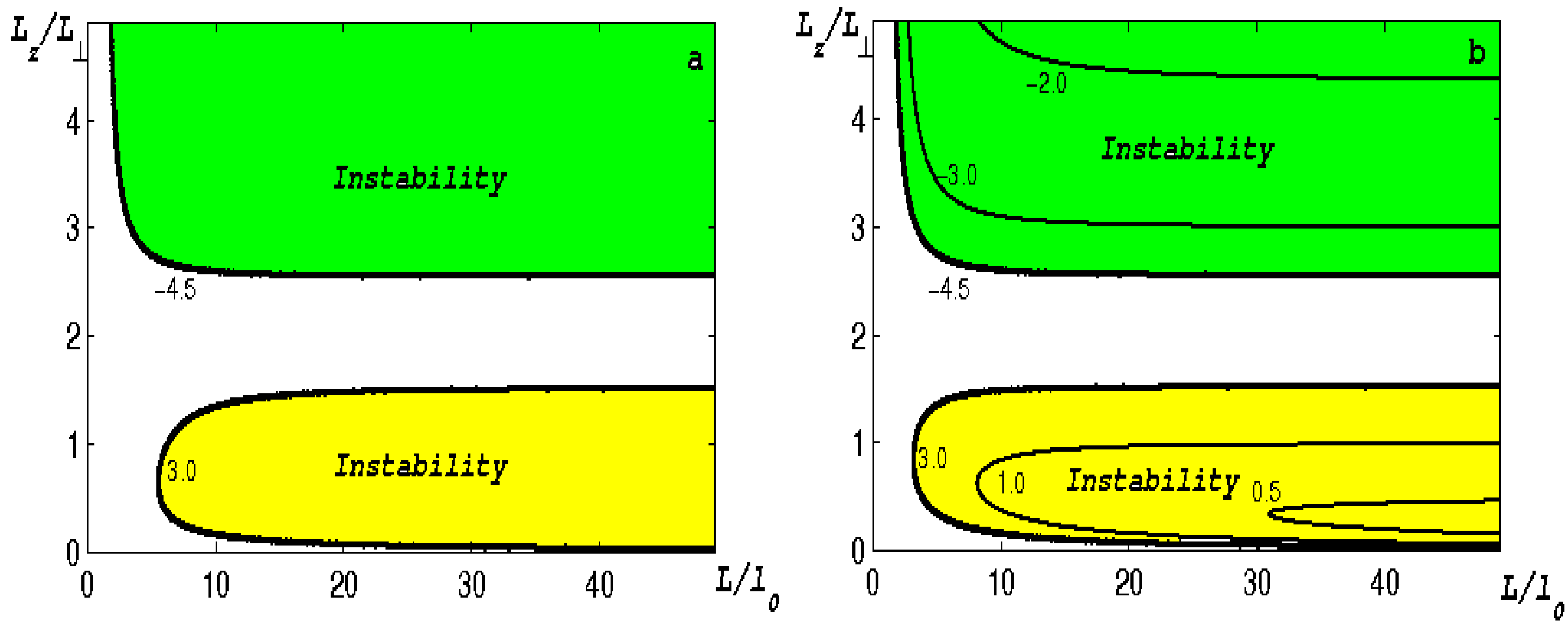
$$\mathbf{\Phi}^* = -\kappa_T \vec{\nabla}\overline{\Theta} - \tau_0\left(\mathbf{\Phi}_z^*\cdot\vec{\nabla}\right)\overline{\mathbf{U}}^{(0)}(z)$$

$$\overline{\mathbf{W}} = \vec{\nabla}\times\overline{\mathbf{U}}$$

The growth rate of convective wind instability

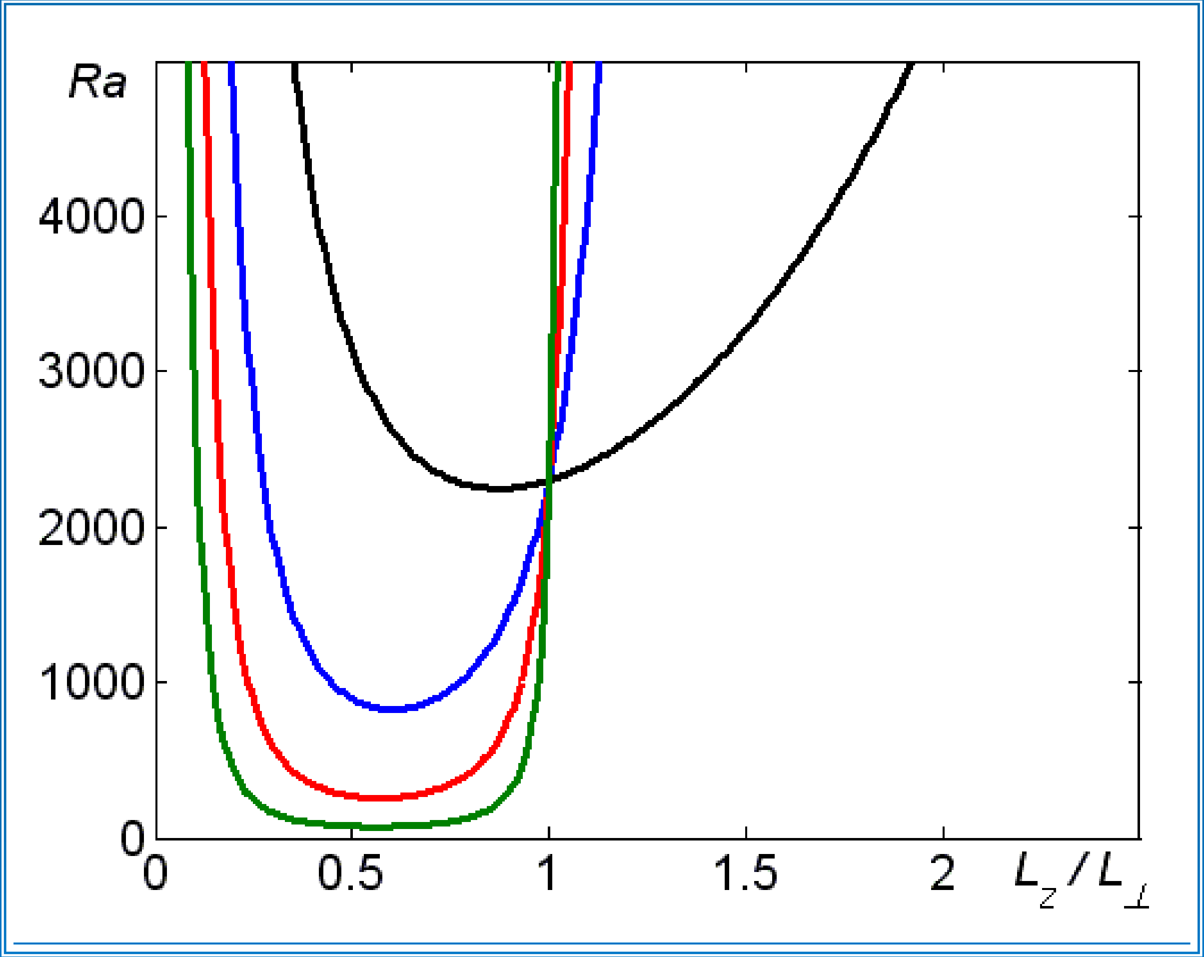


Convective-wind instability



The range of parameters for which the convective-wind instability occurs for different anisotropy of turbulence.

Critical Rayleigh Number



$Ra^{cr} = 2247$

$\mu = 0.7$ $Ra^{cr} = 826$

$\mu = 2$ $Ra^{cr} = 256$

$\mu = 5$ $Ra^{cr} = 72$

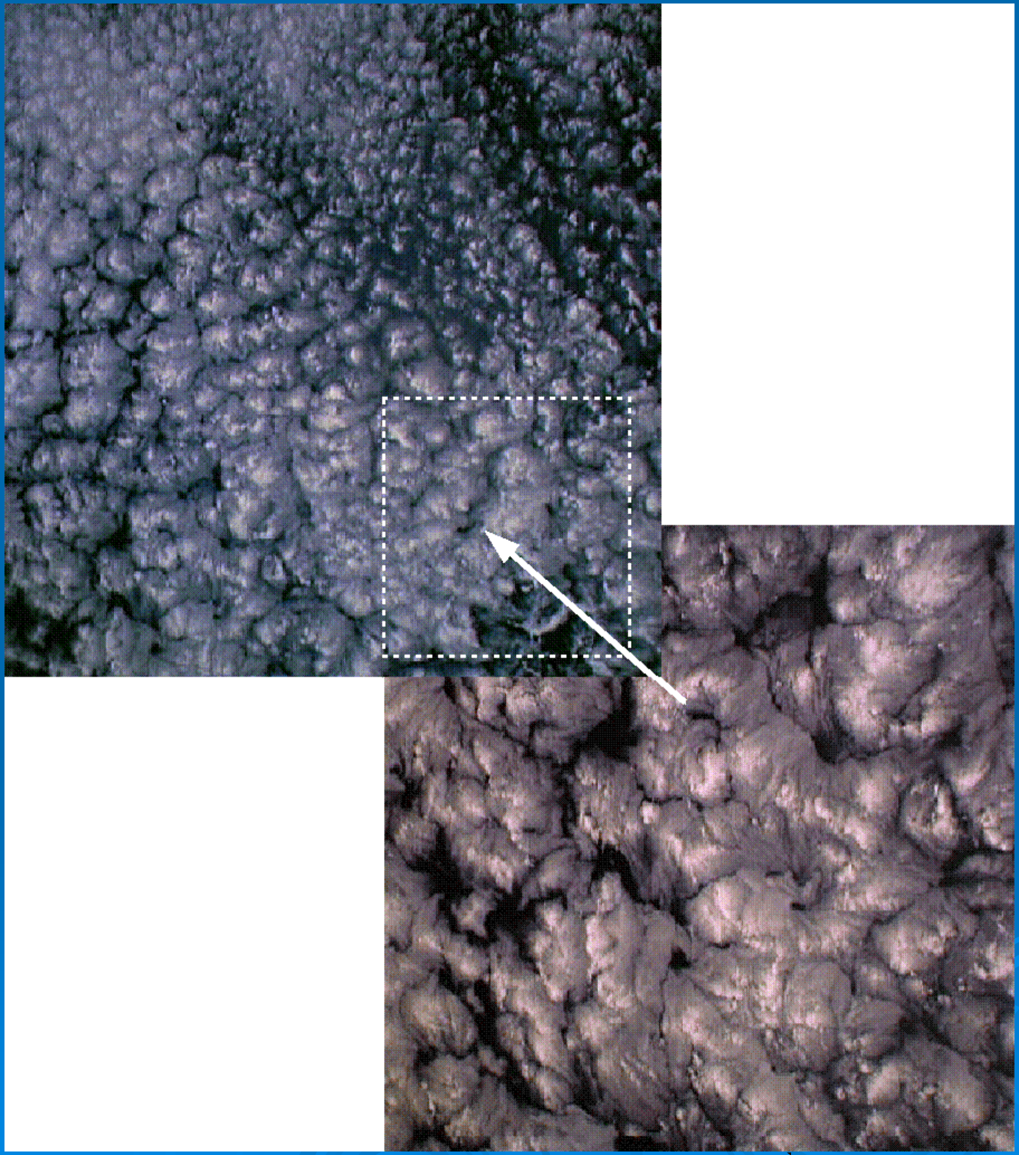
$$\mu = \frac{4 g \tau \langle u_z \theta \rangle}{L_z^2 |N^2|} \left(\frac{\mathrm{Ra}}{\mathrm{Pr_T}} \right)^{1/3}$$

$$N^2 = -\mathbf{g} \cdot \vec{\nabla} \Theta$$

In laminar convection:

$$Ra^{cr} = 657.5$$

Closed Cloud Cells over the Atlantic Ocean



Cloud cells

	Observations	Theory
$L_z / L_\perp$	$0.05 \div 1$	$0 \div 1$
L / l_0	$5 \div 20$	$5 \div 15$
$T_{lifetime}$	Several hours	$\gamma^{-1} = (25 \div 100)\tau_0$ $= 1 \div 3\,h$

Budget Equations

$$\frac{DE_K}{Dt} + \nabla \cdot \mathbf{\Phi}_K = -\tau_{ij}\frac{\partial U_i}{\partial x_j} + \beta F_z - \varepsilon_K ,$$

$$\frac{DE_\theta}{Dt} + \nabla \cdot \mathbf{\Phi}_\theta = -(\mathbf{F}\cdot\nabla)\Theta - F_z\frac{\partial \overline{\Theta}}{\partial z} - \varepsilon_\theta ,$$

$$\frac{DF_i}{Dt} + \frac{\partial}{\partial x_j}\Phi_{ij}^{(F)} = \beta\delta_{i3}\langle\theta^2\rangle - \frac{1}{\rho_0}\langle\theta\nabla_i p\rangle - \tau_{ij}\frac{\partial(\Theta+\overline{\Theta})}{\partial x_j} - F_j\frac{\partial U_i}{\partial x_j} - \varepsilon_i^{(F)} ,$$

$$\tau_{ij} \equiv \langle u_i u_j\rangle = \delta_{ij}\frac{\langle \mathbf{u}^2\rangle}{3} - K_M\left(\nabla_i U_j + \nabla_j U_i\right),$$

$$\varepsilon_K = \frac{E_K}{t_T}, \quad \varepsilon_i^{(F)} = \frac{F_i}{C_F t_T}, \quad \varepsilon_\theta = \frac{E_\theta}{C_P t_T},$$

Cloud Cells

$$\frac{D\boldsymbol{\omega}}{Dt} = K_M \Delta \boldsymbol{\omega} - \beta\left(\mathbf{e}\times\nabla\right)\Theta + (\boldsymbol{\omega}\cdot\nabla)\mathbf{U},$$

$$\frac{D\Theta}{Dt} = -\nabla\cdot\mathbf{F} \ ,$$

$$\boldsymbol{\omega} = \nabla \times \mathbf{U}$$

$$D/Dt = \partial/\partial t + U_k \partial / \partial x_k \ , \quad \beta = g / T_0$$

$$\nabla\cdot\mathbf{F} = -t_T \sigma F_z^* (\mu\Delta_h - \Delta_z) U_z - K_H \Delta\Theta,$$

Solution for Cloud Cells

$$U_r = -A_* U_{z0} J_1\left(\lambda \frac{r}{R}\right)\cos\left(\frac{\pi z}{L_z}\right),$$

$$U_z = U_{z0} J_0\left(\lambda \frac{r}{R}\right)\sin\left(\frac{\pi z}{L_z}\right),$$

$$\Theta = \Theta_0 J_0\left(\lambda \frac{r}{R}\right)\sin\left(\frac{\pi z}{L_z}\right),$$

$$\boldsymbol{\omega} = \mathbf{e}_\varphi \lambda \frac{U_{z0}}{R}\left(1 + A_*^2\right) J_1\left(\lambda \frac{r}{R}\right)\sin\left(\frac{\pi z}{L_z}\right).$$

$$A_* = \pi R / \lambda L_z$$

$$\frac{U_{z0}}{\Theta_0} = \frac{\beta L_z^2}{\pi^2 K_M} \frac{A_*^2}{\left(1 + A_*^2\right)^2},$$

$$\frac{K_M^2}{\beta F_z t_T R^2 \Pr_T} = \frac{\sigma}{\lambda^2} \frac{A_*^2 - \mu}{\left(1 + A_*^2\right)^3}.$$

Budget Equations

$$\frac{DE_K}{Dt} + \nabla \cdot \mathbf{\Phi}_K = -\tau_{ij} \frac{\partial U_i}{\partial x_j} + \beta F_z - \varepsilon_K ,$$

$$\frac{DE_\theta}{Dt} + \nabla \cdot \mathbf{\Phi}_\theta = -(\mathbf{F} \cdot \nabla)\Theta - F_z \frac{\partial \overline{\Theta}}{\partial z} - \varepsilon_\theta ,$$

$$\frac{DF_i}{Dt} + \frac{\partial}{\partial x_j} \Phi_{ij}^{(F)} = \beta \delta_{i3} \langle \theta^2 \rangle - \frac{1}{\rho_0} \langle \theta \nabla_i p \rangle - \tau_{ij} \frac{\partial (\Theta + \overline{\Theta})}{\partial x_j} - F_j \frac{\partial U_i}{\partial x_j} - \varepsilon_i^{(F)} ,$$

$$\tau_{ij} \equiv \langle u_i u_j \rangle = \delta_{ij} \frac{\langle \mathbf{u}^2 \rangle}{3} - K_M (\nabla_i U_j + \nabla_j U_i),$$

$$\varepsilon_K = \frac{E_K}{t_T}, \quad \varepsilon_i^{(F)} = \frac{F_i}{C_F t_T}, \quad \varepsilon_\theta = \frac{E_\theta}{C_P t_T},$$

Dimensionless Ratios for Cloud Cells

$$\frac{E_K}{E_U} = 3C_\tau A_z \left(\frac{l}{L_z}\right)^2 \frac{\Phi_1(A_*)}{1-\hat{F}},$$

$$\hat{F} \equiv \frac{\beta F_z t_T}{E_K} = \frac{(2\pi C_\tau A_z)^2}{\sigma \mathrm{Pr}_T} \frac{l^2}{L_z^2} \Phi_2(A_*),$$

$$\frac{E_\theta}{E_\Theta} = \frac{C_P C_F A_r}{7A_*^2}\left(\frac{l}{L_z}\right)^2,$$

$\langle \dots \rangle_V$ **implies the averaging over the volume of the semi-organized structure.**

$$\frac{F_z}{\langle \Theta U_z \rangle_V} = \frac{1}{40}\left(\frac{l}{L_z}\right)^2.$$

$$\Phi_1(A_*) = \left(4A_*^2 + 4A_*^{-2} - 1\right)J_2^2(\lambda) + J_3^2(\lambda) - \frac{4}{\lambda^2}\left(1 - J_0^2(\lambda)\right) \approx A_*^2 + A_*^{-2} - \frac{1}{3},$$

$$\Phi_2(A_*) = \frac{\left(1+A_*^2\right)^3}{A_*^2\left(A_*^2-\mu\right)^3}. \quad \Phi_8(A_*) = \frac{(1+A_*^2)^4}{A_*^6}\frac{\Phi_1(A_*)}{\Phi_4^{4/3}(A_*)}.$$

$$\Phi_4(A_*) = \sqrt{\frac{5}{2}}\pi^2 J_2^2(\lambda)\frac{1}{A_*^2}\Phi_1^{1/2}(A_*)\left(1+A_*^2\right)^2.$$

$$\frac{E_\Theta}{\Theta_D^2} = \frac{6C_P C_F C_\tau A_r A_z}{\left(1-\hat{F}\right)^{1/3}}\left(\frac{l}{L_z}\right)^{10/3}\left(1-\frac{F_z}{F_{\mathrm{tot}}}\right)^{4/3}\Phi_8(A_*),$$

$$\Theta_D = \left(F_z + \langle \Theta U_z \rangle_V\right)/U_D$$

$$U_D = \left[\left(F_z + \langle \Theta U_z \rangle_V\right)\beta L_z\right]^{1/3}$$

Energies of Coherent Structures

The kinetic energy of the semi-organized structures (cloud cells):

$$E_U \equiv \frac{1}{2} U_{z0}^2 = \frac{1}{3 C_\tau A_z} \left(\frac{L_z}{l} \right)^{4/3} \left(\langle \Theta U_z \rangle_V \beta L_z \right)^{2/3} \frac{\Phi_9(A_*)}{\Phi_1(A_*)} \left(1 - \hat{F} \right)^{1/3},$$

The thermal energy of the semi-organized structures (cloud cells):

$$E_\Theta \equiv \frac{1}{2} \Theta_0^2 = \frac{83}{2} \left(\frac{l}{L_z} \right)^{4/3} \left(\frac{\langle \Theta U_z \rangle_V^2}{\beta L_z} \right)^{2/3} C_\tau A_z \left(1 - \hat{F} \right)^{-1/3} A_*^2 \Phi_8(A_*).$$

The mean vertical temperature gradient:

$$A_* = \pi R / \lambda L_z$$

$$\Phi_7(A_*) = \frac{A_*^2}{\Phi_5(A_*)}.$$

$$\frac{\partial \overline{\Theta}}{\partial z} = 12.7 \frac{\left(\langle \Theta U_z \rangle_V l \right)^{2/3}}{\beta^{1/3} L_z^2} \frac{C_F A_r}{A_z} \left(1 - \hat{F} \right)^{1/3} \Phi_6(A_*) \left[1 - \frac{A_z^2 \Pr_T \Phi_7(A_*)}{3 A_r \sigma (1 - \hat{F})} \right],$$

$$\Phi_5(A_*) = \frac{A_*^2 (A_*^2 - \mu)}{(1 + A_*^2)(3 A_*^4 - A_*^2 + 1)}.$$

$$\Phi_6(A_*) = \frac{1}{A_*^2} \Phi_1^{1/2}(A_*) \Phi_4^{1/3}(A_*),$$

$$\Phi_8(A_*) = \frac{(1 + A_*^2)^4}{A_*^6} \frac{\Phi_1(A_*)}{\Phi_4^{4/3}(A_*)}.$$

$$\Phi_9(A_*) = \frac{\Phi_1(A_*)}{\Phi_4^{2/3}(A_*)}.$$

The vertical flux of entropy transported by the cloud cells

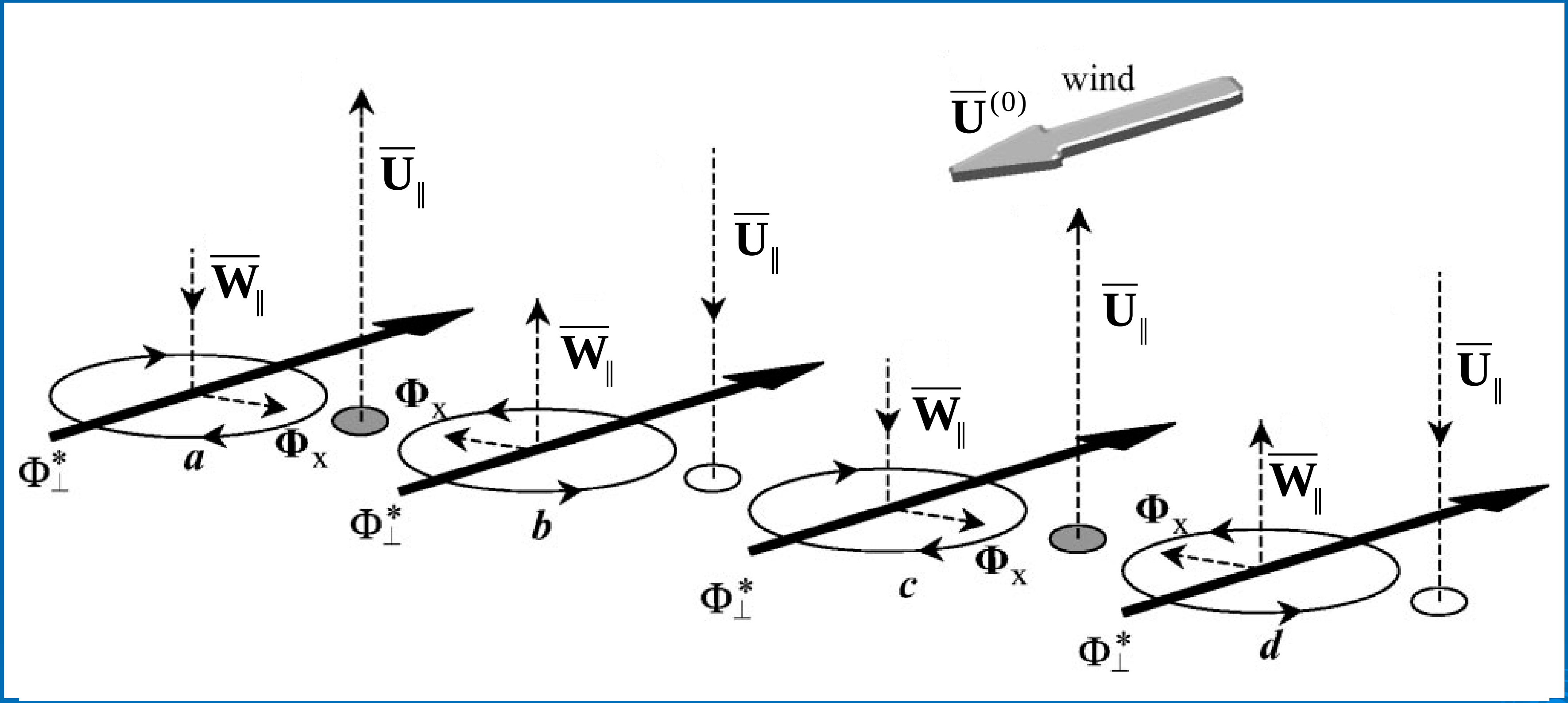
The vertical flux of entropy transported by the semi-organized structures:

$$\left\langle \Theta U_z \right\rangle_V = \frac{1}{2}\Theta_0 U_{z0} J_2^2(\lambda) = \left(C_\tau A_z\right)^{3/2} \frac{l^2 U_{z0}^3}{\beta L_z^3} \frac{\Phi_4\left(A_*\right)}{\left(1-\hat{F}\right)^{1/2}},$$

The ratio of fluxes of entropy :

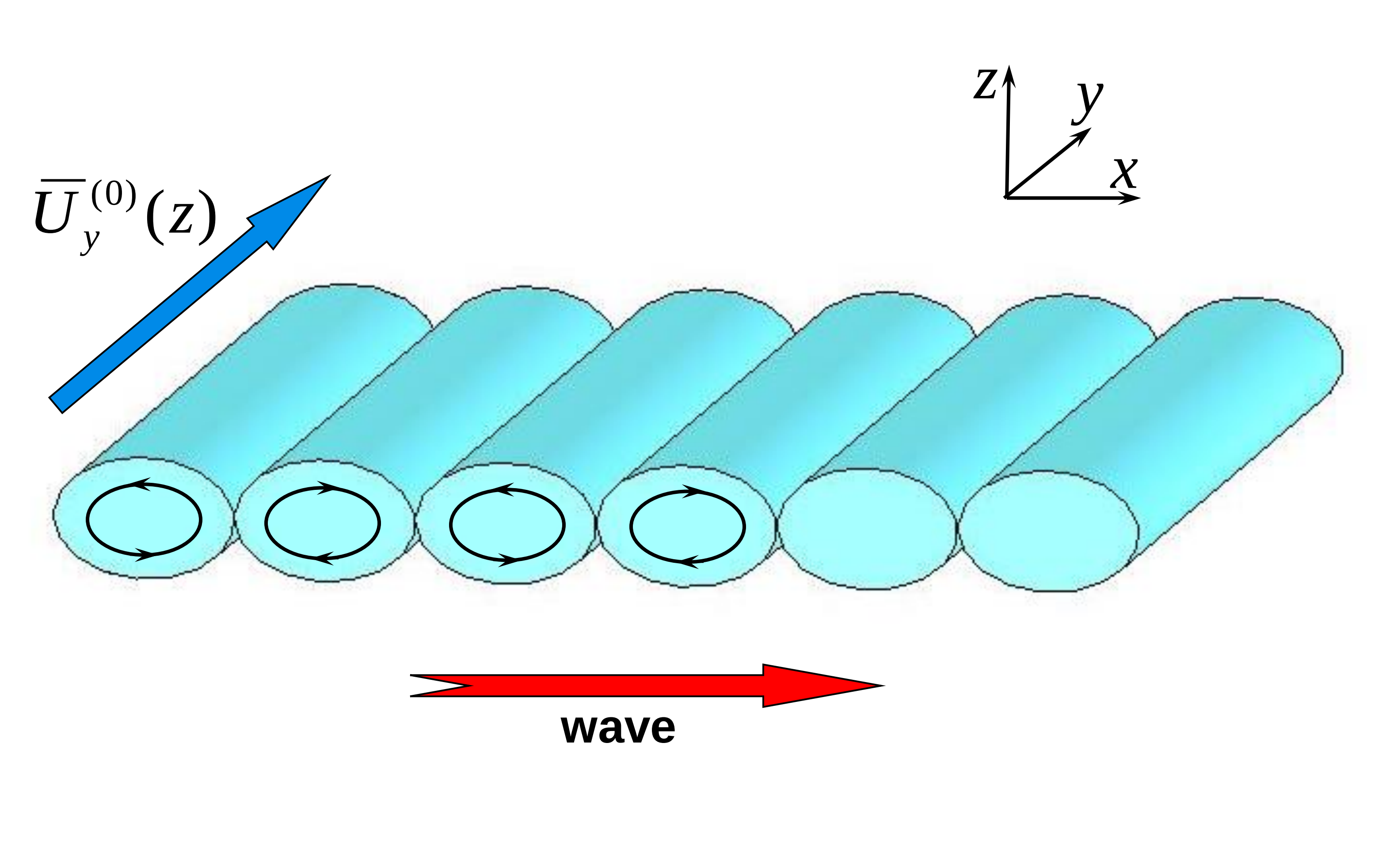
$$\frac{\left\langle \Theta U_z \right\rangle_V}{F_z} = \frac{\sigma \,\mathrm{Pr}_T}{\left(C_\tau A_z\right)^2} \frac{L_z^2}{l^2} \left(1-\hat{F}\right) \Phi_5\left(A_*\right),$$

Mechanism of convective-shear instability



$$\mathbf{\Phi} \propto \tau_0 \left(\overline{\mathbf{W}}_z \times \mathbf{\Phi}^* \right)$$

Convective-shear waves

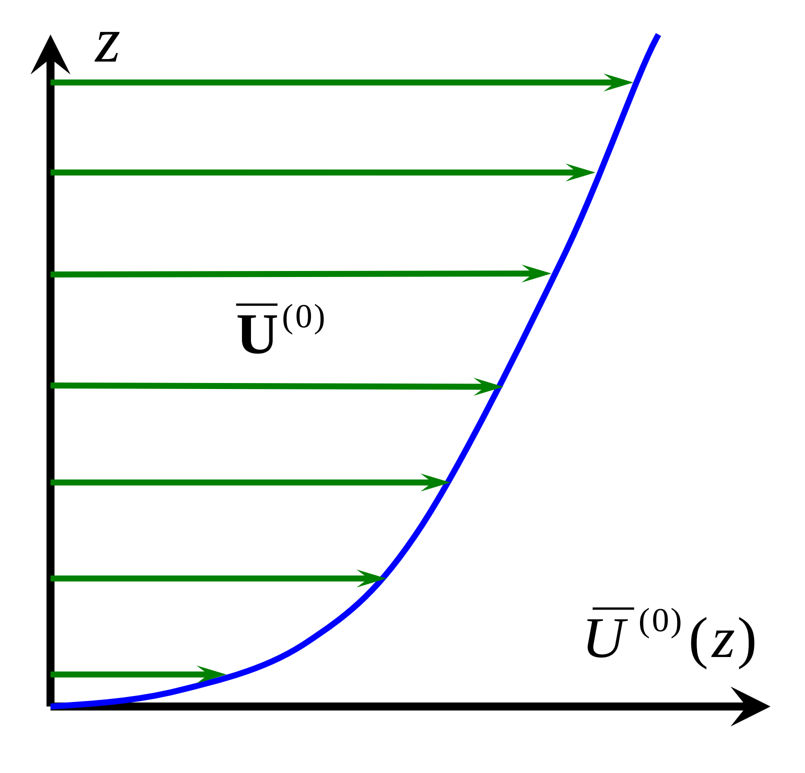


$$\overline{W}_z \propto \cos(\omega t - K x)$$

$$\overline{U}_z \propto \cos\left(\omega t - K x - \frac{\pi}{6}\right) \qquad \overline{\Theta} \propto \cos\left(\omega t - K x + \frac{\pi}{6}\right)$$

Counter wind flux

$$\mathbf{\Phi}^* = -\kappa_T \vec{\nabla}\overline{\Theta} - \tau_0 \left(\mathbf{\Phi}_z^* \cdot \vec{\nabla} \right) \overline{\mathbf{U}}^{(0)}(z)$$



$$\frac{\partial \mathbf{u}}{\partial t} \propto -\left(\mathbf{u} \cdot \vec{\nabla} \right) \overline{\mathbf{U}}^{(0)} + \ldots$$

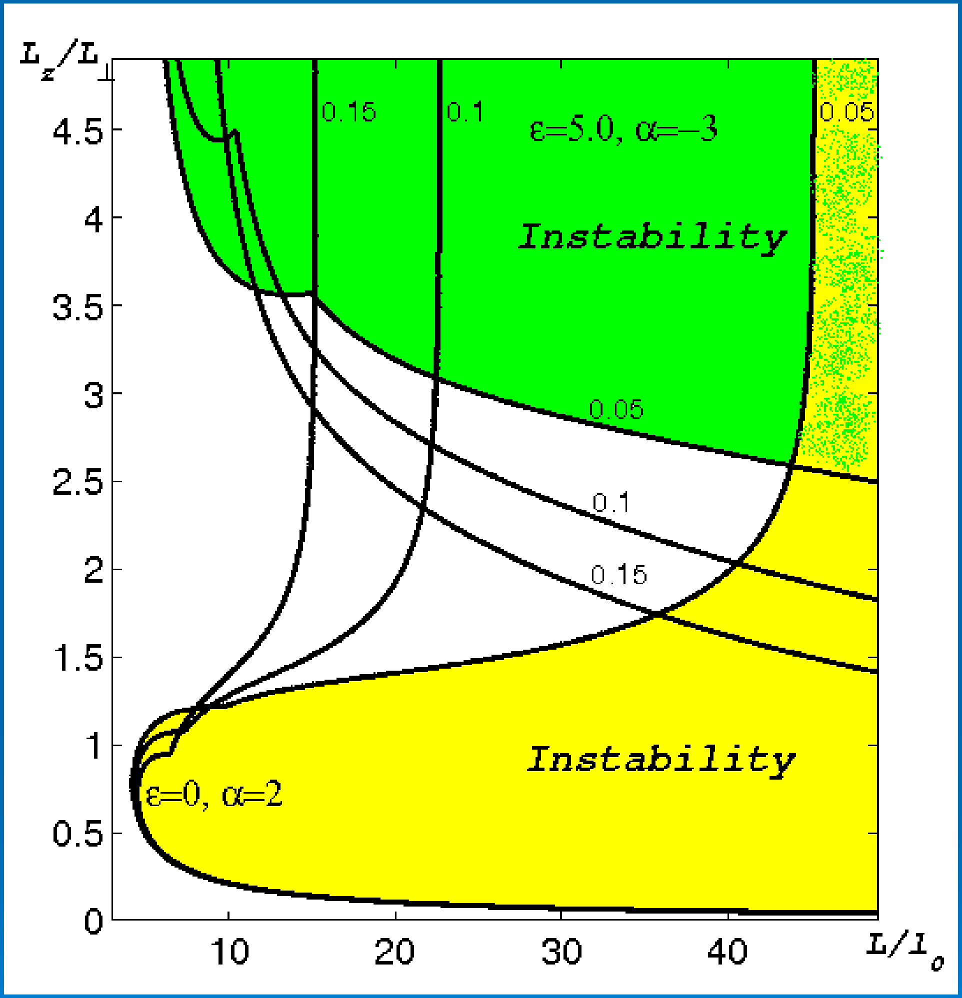
Tangling fluctuations

$$\delta \mathbf{u} \propto -\tau_0 \left(\mathbf{u} \cdot \vec{\nabla} \right) \overline{\mathbf{U}}^{(0)}$$

$$\langle \theta \, \delta \mathbf{u} \rangle \propto -\tau_0 \left(\mathbf{\Phi}_z^* \cdot \nabla \right) \overline{\mathbf{U}}^{(0)}(z)$$

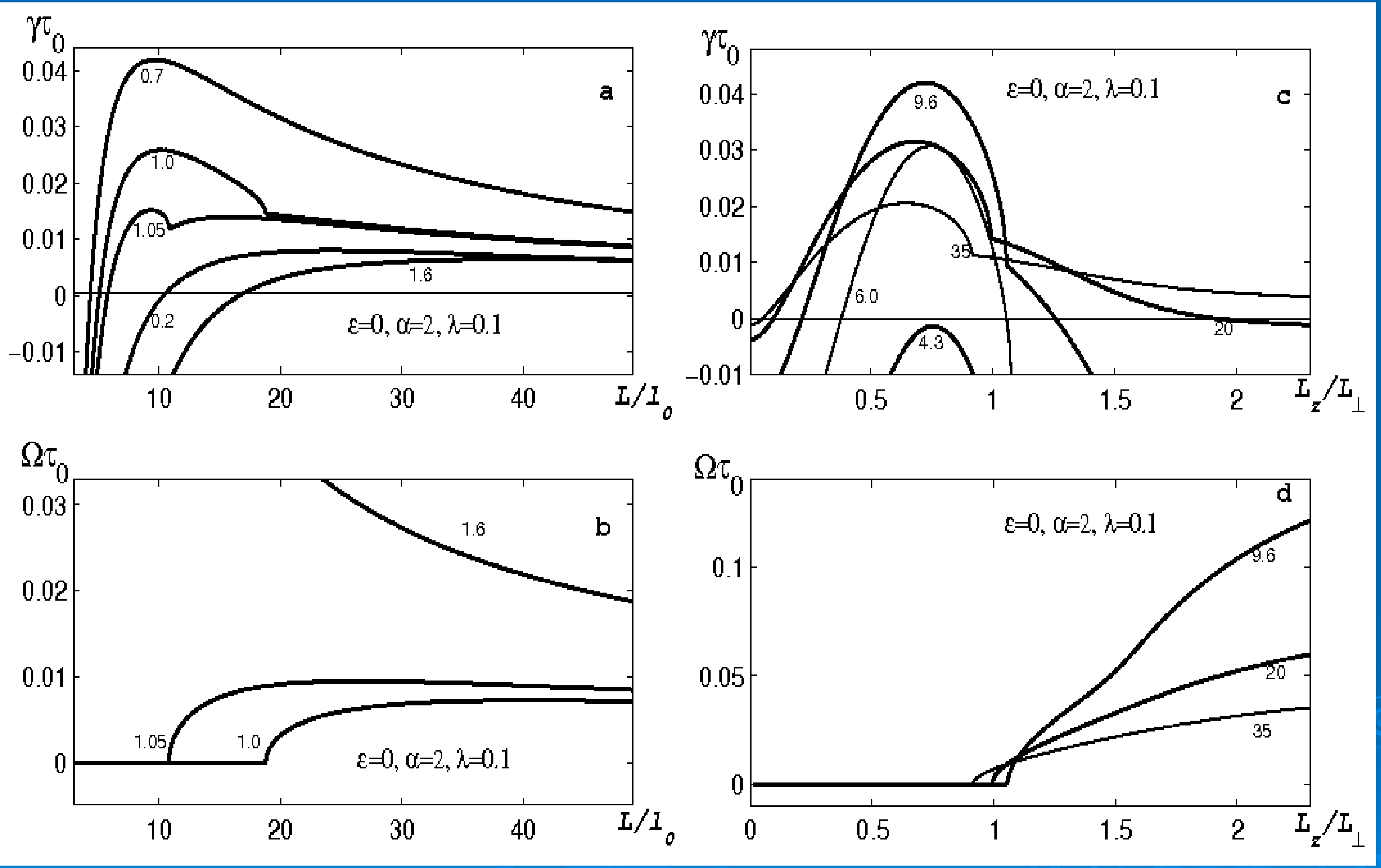
$$\mathbf{\Phi}_z^* = \langle \theta \, \mathbf{u}_z \rangle$$

Convective-shear instability



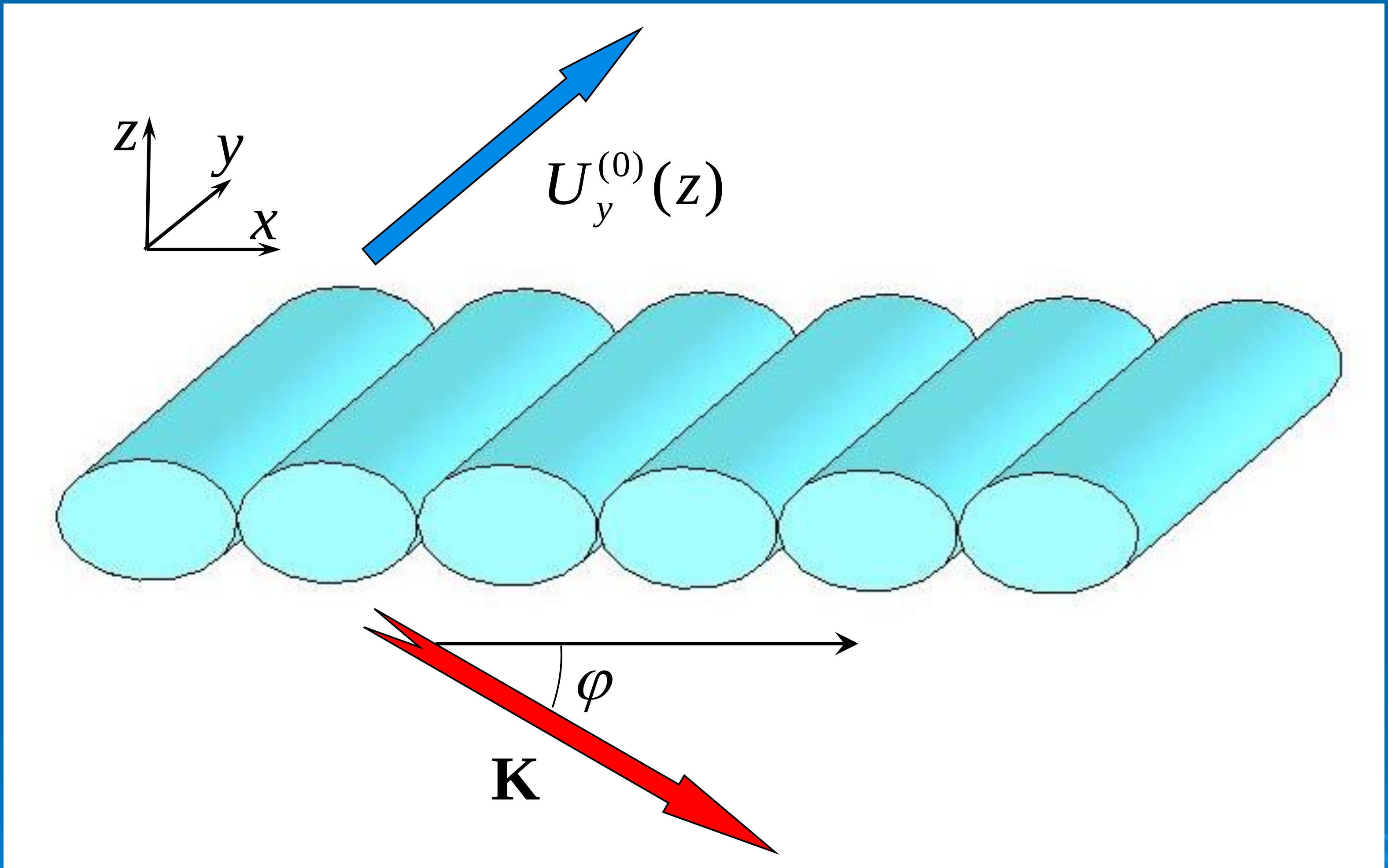
The range of parameters for which the convective-shear instability occurs for different values of shear and anisotropy.

Convective-shear instability

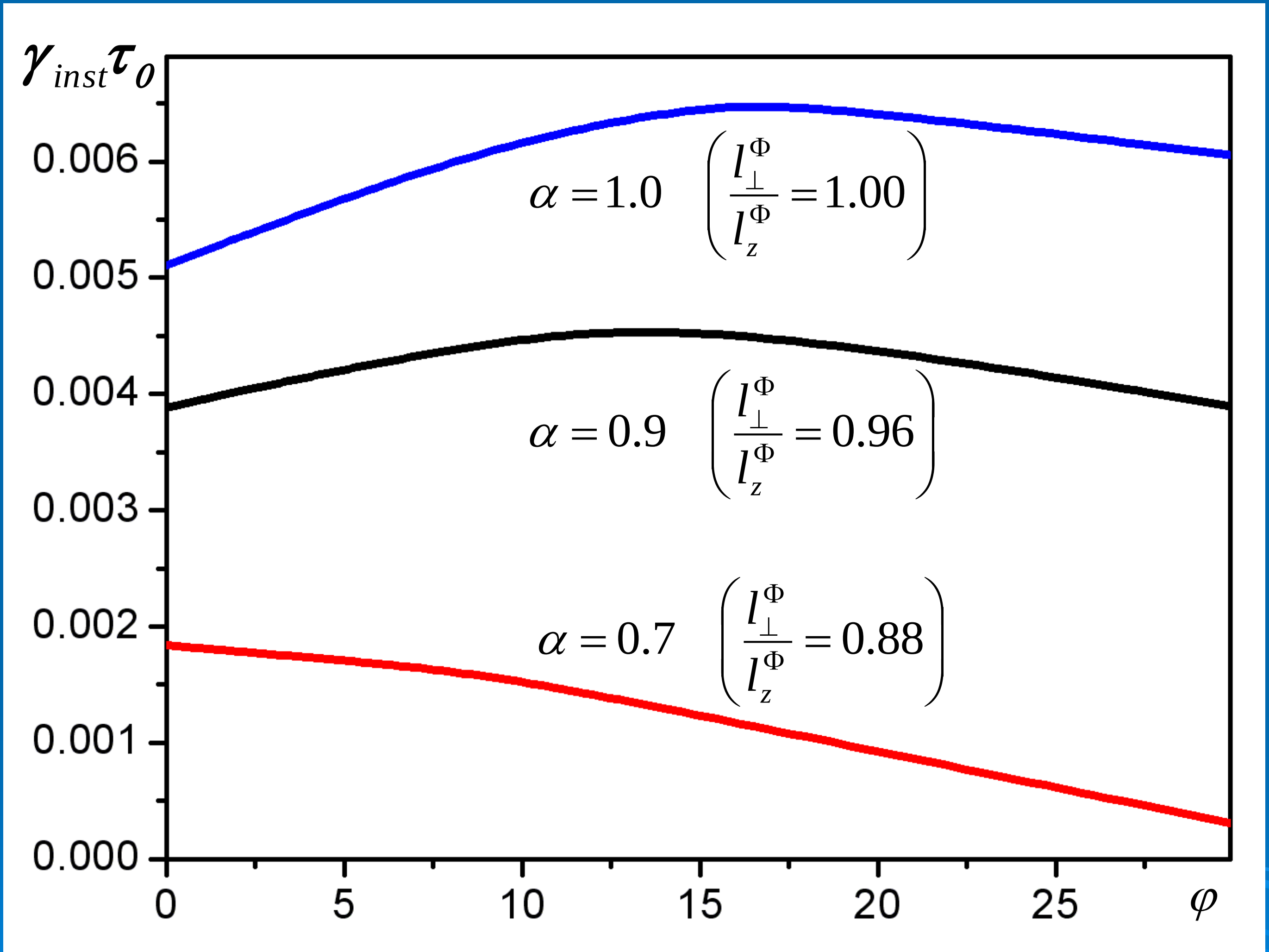


The growth rate of the convective-shear instability and frequencies of the generated convective-shear waves.

Convective-shear instability

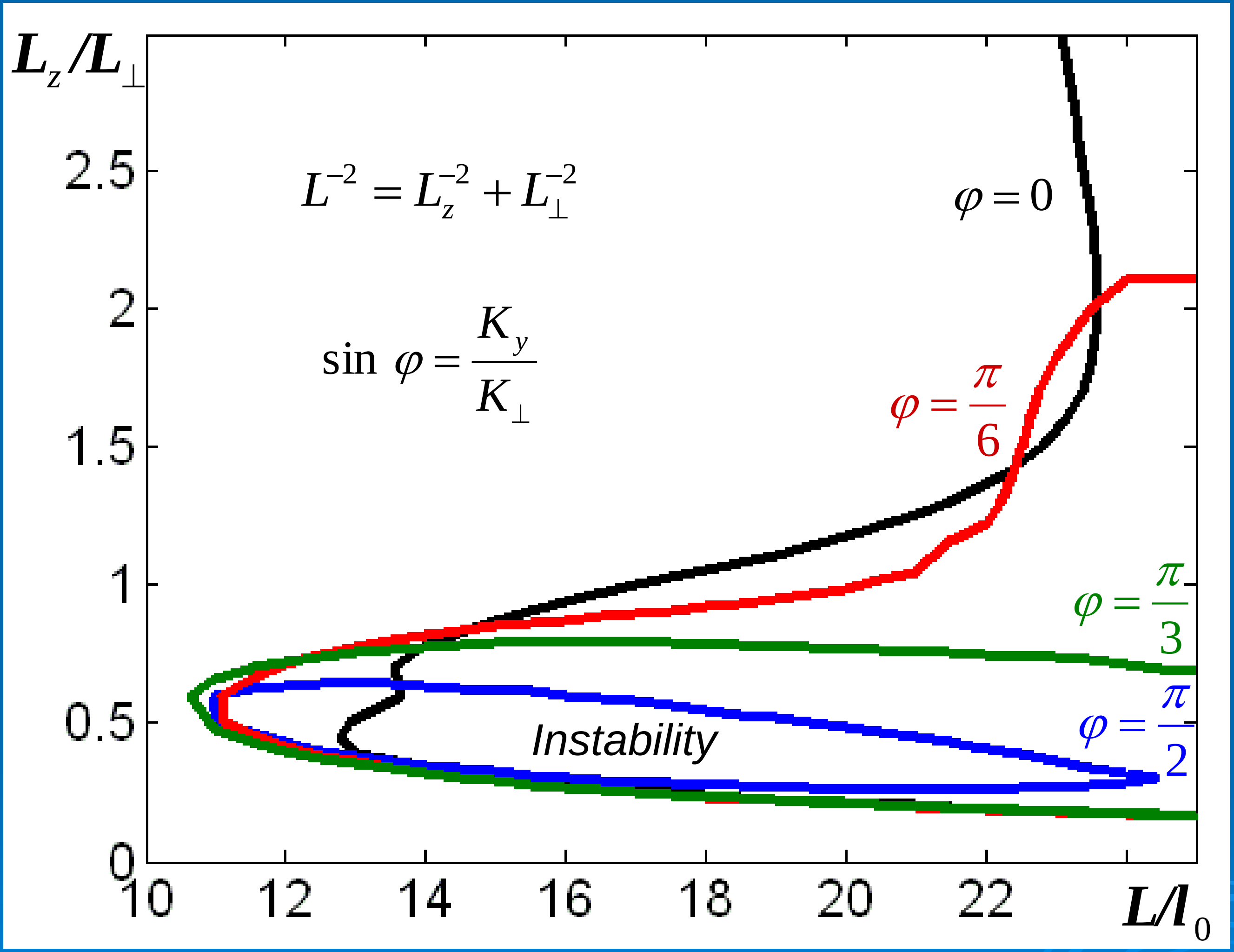


Maximum growth rate



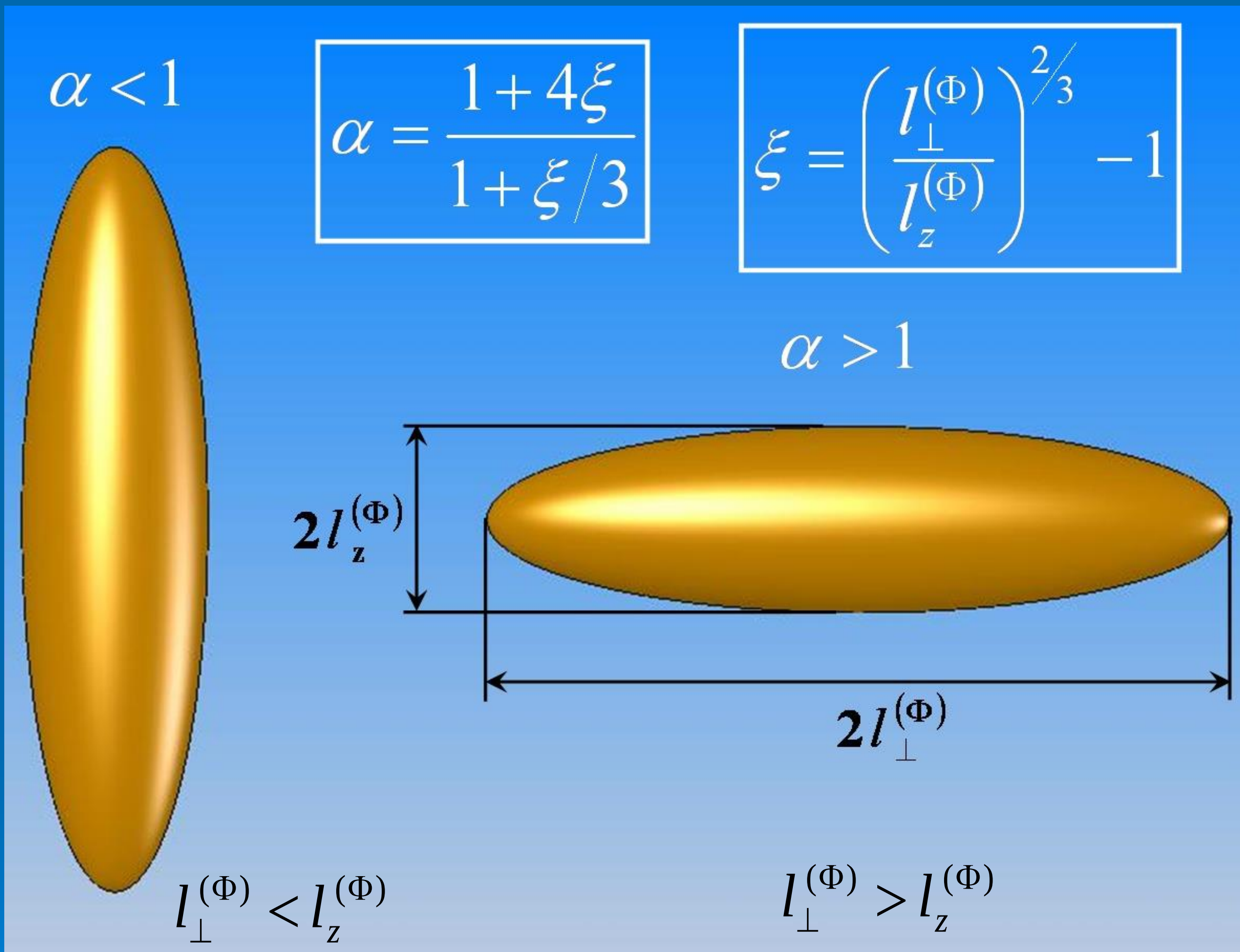
The growth rate of the convective shear instability
for different thermal anisotropy

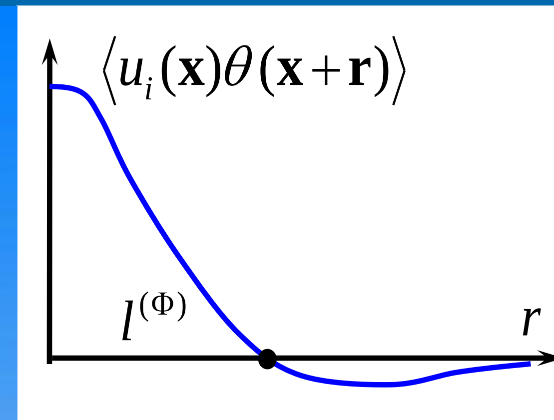
Conditions for the instability



The range of parameters $L_z / L_\perp$ and L / l_0 for which the convective shear instability occurs

Thermal anisotropy

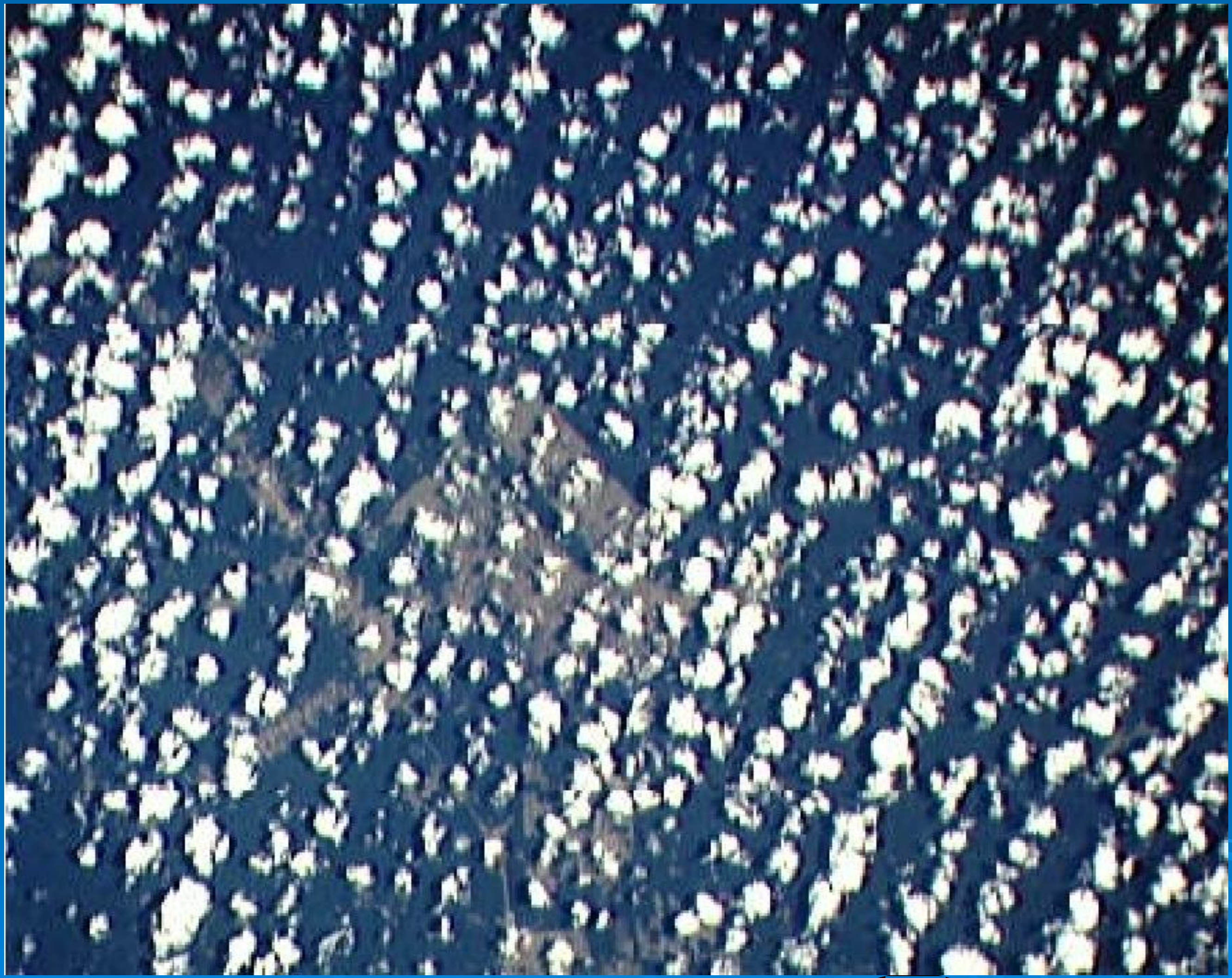




"Column"-like thermal structure

"Pancake"-like thermal structure

Cloud “streets” over the Amazon River



Cloud “streets”

	Observations	Theory
$L_z / L_\perp$	$0.14 \div 1$	$0 \div 1$
L / l_0	$10 \div 100$	$10 \div 100$
$T_{lifetime}$	$1 \div 72\ h$	$\gamma^{-1} = (25 \div 100)\tau_0$ $= 1 \div 3\,h$

Budget Equations

$$\frac{DE_K}{Dt} + \nabla \cdot \mathbf{\Phi}_K = -\tau_{ij}\frac{\partial U_i}{\partial x_j} + \beta F_z - \varepsilon_K ,$$

$$\frac{DE_\theta}{Dt} + \nabla \cdot \mathbf{\Phi}_\theta = -(\mathbf{F} \cdot \nabla)\Theta - F_z \frac{\partial \overline{\Theta}}{\partial z} - \varepsilon_\theta ,$$

$$\frac{DF_i}{Dt} + \frac{\partial}{\partial x_j}\Phi_{ij}^{(F)} = \beta \delta_{i3}\langle \theta^2 \rangle - \frac{1}{\rho_0}\langle \theta \nabla_i p \rangle - \tau_{ij}\frac{\partial(\Theta + \overline{\Theta})}{\partial x_j} - F_j \frac{\partial U_i}{\partial x_j} - \varepsilon_i^{(F)} ,$$

$$\tau_{ij} \equiv \langle u_i u_j \rangle = \delta_{ij}\frac{\langle \mathbf{u}^2 \rangle}{3} - K_M (\nabla_i U_j + \nabla_j U_i),$$

$$\varepsilon_K = \frac{E_K}{t_T}, \quad \varepsilon_i^{(F)} = \frac{F_i}{C_F t_T}, \quad \varepsilon_\theta = \frac{E_\theta}{C_P t_T},$$

Cloud Streets. Sheared Convection

$$\frac{D\boldsymbol{\omega}}{Dt} = K_M \Delta \boldsymbol{\omega} - \beta(\mathbf{e}\times\nabla)\Theta + (\boldsymbol{\omega}\cdot\nabla)\mathbf{U} + (\boldsymbol{\omega}^{(s)}\cdot\nabla)\mathbf{U},$$

$$\frac{D\Theta}{Dt} = -\nabla\cdot\mathbf{F} \;,$$

$$\nabla\cdot\mathbf{F} = -t_T\left(\sigma F_z^*(\mu\Delta_h - \Delta_z)U_z + \frac{8}{15}\mathbf{F}_x^*\cdot(\mathbf{e}\times\nabla)\omega_z\right) - K_H\,\Delta\Theta,$$

The shear velocity:

$$\mathbf{U}^{(s)} = (Sz,0,0)$$

$$\boldsymbol{\omega}^{(s)} = (0,S,0)$$

$$A_* = L_y / L_z$$

The solution of linearized equations:

$$U_x = -U_{x0}\cos\left(\frac{\pi y}{L_y}\right)\sin\left(\frac{\pi z}{L_z}\right),$$

$$U_y = -A_* U_{z0}\sin\left(\frac{\pi y}{L_y}\right)\cos\left(\frac{\pi z}{L_z}\right),$$

$$U_z = U_{z0}\cos\left(\frac{\pi y}{L_y}\right)\sin\left(\frac{\pi z}{L_z}\right),$$

$$\Theta = \Theta_0\cos\left(\frac{\pi y}{L_y}\right)\sin\left(\frac{\pi z}{L_z}\right),$$

$$\frac{U_{x0}}{U_{z0}} = \frac{L_y^2 S}{\pi^2 K_M}\left(1+A_*^2\right)^{-1},$$

$$\frac{\Theta_0}{U_{z0}} = \frac{L_y^2 S^2}{\pi^2 \beta K_M}\left(1+A_*^2\right)^{-1}\left[1+\frac{\pi^4 K_M^2}{L_y^4 S^2}\left(1+A_*^2\right)^2\right].$$

Vorticily of Cloud Streets

$$\omega_x = -\frac{\pi U_{z0}}{L_y}\left(1 + A_*^2\right)\sin\left(\frac{\pi y}{L_y}\right)\sin\left(\frac{\pi z}{L_z}\right),$$

$$\omega_y = -\frac{\pi U_{x0}}{L_z}\cos\left(\frac{\pi y}{L_y}\right)\cos\left(\frac{\pi z}{L_z}\right),$$

$$\omega_z = -\frac{\pi U_{x0}}{L_y}\sin\left(\frac{\pi y}{L_y}\right)\sin\left(\frac{\pi z}{L_z}\right).$$

A large-scale helicity produced by the semi-organized structures:

$$\chi_U \equiv \mathbf{U}\cdot\boldsymbol{\omega} = \frac{\pi U_{x0} U_{z0}}{2L_y} A_*^2 \sin\left(\frac{2\pi y}{L_y}\right) = \frac{\pi U_{z0}^2 A_*^2}{2C_* \, l \, \hat{E}_K^{1/2}(1 + A_*^2)} .$$

Budget Equations

$$\frac{DE_K}{Dt} + \nabla \cdot \mathbf{\Phi}_K = -\tau_{ij}\frac{\partial U_i}{\partial x_j} + \beta F_z - \varepsilon_K ,$$

$$\frac{DE_\theta}{Dt} + \nabla \cdot \mathbf{\Phi}_\theta = -(\mathbf{F}\cdot\nabla)\Theta - F_z \frac{\partial \overline{\Theta}}{\partial z} - \varepsilon_\theta ,$$

$$\frac{DF_i}{Dt} + \frac{\partial}{\partial x_j}\Phi_{ij}^{(F)} = \beta\delta_{i3}\langle\theta^2\rangle - \frac{1}{\rho_0}\langle\theta\nabla_i p\rangle - \tau_{ij}\frac{\partial(\Theta+\overline{\Theta})}{\partial x_j} - F_j\frac{\partial U_i}{\partial x_j} - \varepsilon_i^{(F)} ,$$

$$\tau_{ij} \equiv \langle u_i u_j\rangle = \delta_{ij}\frac{\langle \mathbf{u}^2\rangle}{3} - K_M\left(\nabla_i U_j + \nabla_j U_i\right),$$

$$\varepsilon_K = \frac{E_K}{t_T}, \quad \varepsilon_i^{(F)} = \frac{F_i}{C_F t_T}, \quad \varepsilon_\theta = \frac{E_\theta}{C_P t_T},$$

Production in Sheared Convection

The production of turbulence is caused by three sources:

a) the shear of the semi-organized structures: $\Pi^{(cs)} = -\left\langle \tau_{ij}\,\partial U_i / \partial x_j \right\rangle_V = K_M \left\langle S_{ij}\, S_{ji} \right\rangle_V$

b) the background wind shear: $\Pi^{(s)} = K_M\, S^2$

c) the buoyancy:

$$\hat{E}_K \left(\frac{l}{L_y} \right)^2 >> 1 .$$

$$\hat{E}_K = E_K / L_y^2\, S^2 ,$$

$$\frac{E_K}{E_U} = \frac{C_*}{4(1-\hat{F})} \left(\frac{l}{L_y} \right)^2 \left(\left(1 + A_*^2\right)^2 - 3\, A_*^2 \right),$$

$$C_* = 2\pi^2 C_\tau A_z ,$$

$$\frac{E_\theta}{E_\Theta} = \pi^2 C_P C_F\, A_z \left(\frac{l}{L_z} \right)^2 .$$

$$\frac{\left\langle \Theta U_z \right\rangle_V}{F_z} = \frac{\pi^2 \sigma \Pr_T}{4 C_*} \left(\frac{U_{z0}}{L_y\, S} \right)^2 \left(\frac{A_*^2 - \mu}{1 + A_*^2} \right),$$

Energies of Coherent Structures

The kinetic energy of the semi-organized structures (cloud streets):

$$\hat{F} = \frac{C_*^2}{\pi^2 \sigma \, \mathrm{Pr}_T \, \hat{E}_K} \left(\frac{l}{L_y} \right)^2 \frac{\left(1 + A_*^2\right)^2}{(A_*^2 - \mu)},$$

$$E_U \equiv \frac{1}{2} U_{z0}^2 = \frac{2^{1/3}}{C_*} \left(\frac{L_y}{l} \right)^{4/3} U_D^2 \left(1 - \hat{F}\right)^{1/3} \Phi_{10}(A_*),$$

The thermal energy of the semi-organized structures (cloud streets):

$$E_\Theta \equiv \frac{1}{2} \Theta_0^2 = C_* \left(\frac{l}{2L_y} \right)^{4/3} \left(\frac{U_D^2}{\beta L_y} \right)^2 \left(1 - \hat{F}\right)^{-1/3} \Phi_{12}(A_*),$$

The Deardorff velocity scale:

$$U_D = \left(\langle \Theta U_z \rangle_V \beta L_z \right)^{1/3}$$

$$\Phi_{10}(A_*) = \frac{A_*^{2/3}}{\left(\left(1 + A_*^2\right)^2 - 3 A_*^2 \right)^{1/3} \left(1 + A_*^2\right)^{4/3}}.$$

$$\Phi_{11}(A_*) = \left(\left(1 + A_*^2\right)^2 - 3 A_*^2 \right)^{1/3} \left(1 + A_*^2\right)^{7/3} A_*^{1/3} \left(A_*^2 - \mu\right)^{-1}.$$

The vertical turbulent flux of entropy:

$$F_z = \frac{2^{2/3} C_*^2}{\pi^2 \sigma \, \mathrm{Pr}_T} \left(\frac{l}{L_y} \right)^{4/3} \left(\frac{U_D L_y S^2}{\beta} \right) \left(1 - \hat{F}\right)^{-1/3} \Phi_{11}(A_*),$$

$$\Phi_{12}(A_*) = \left(\frac{\left(1 + A_*^2\right)^2 - 3 A_*^2}{\left(1 + A_*^2\right)^2} \right)^{1/3} A_*^{4/3}.$$

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- New, in preparation, …

THE END