Turbulence closure for stably stratified geophysical flows

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Geophysical turbulence and planetary boundary layers (PBLs)

Physics

Revised geophysical turbulence paradigm: waves, self-organisation

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Revised energetics, turbulence-closure and PBL theory

→

Geo-sciences

PBLs link atmosphere, hydrosphere, lithosphere and cryosphere within weather & climate systems

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Improved “linking algorithms” in weather & climate models

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Progress in understanding and modelling

weather & climate systems

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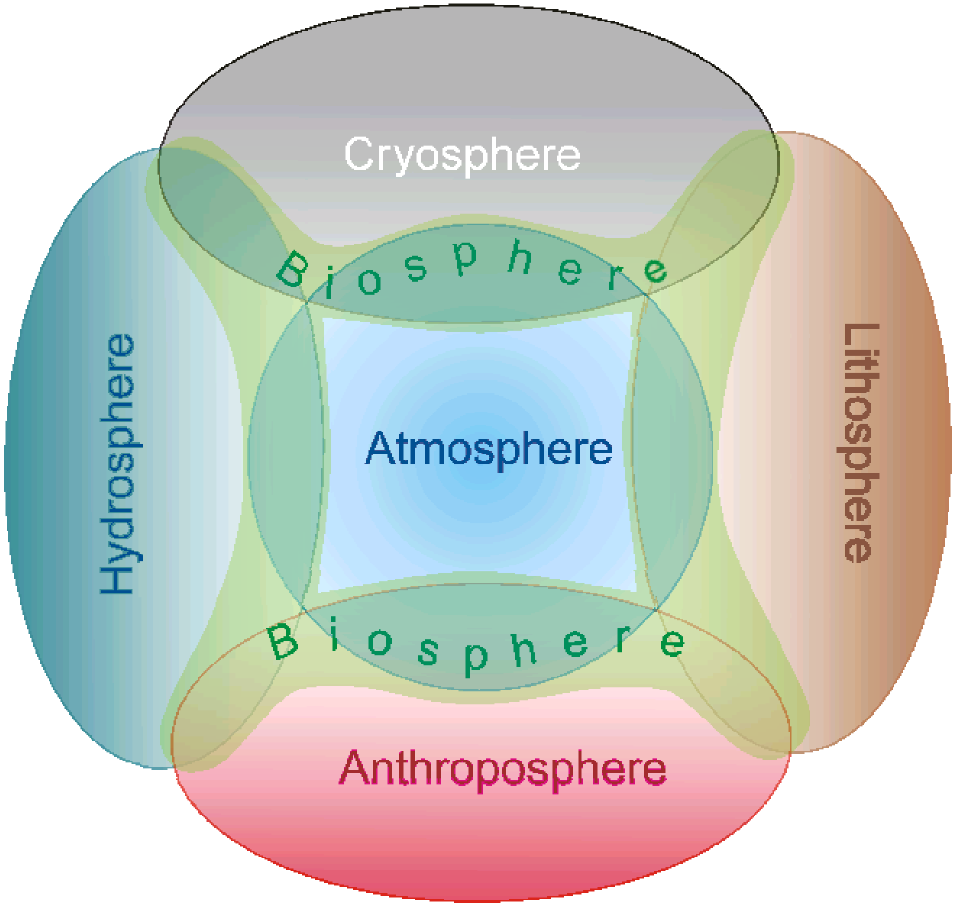
Geospheres in climate system

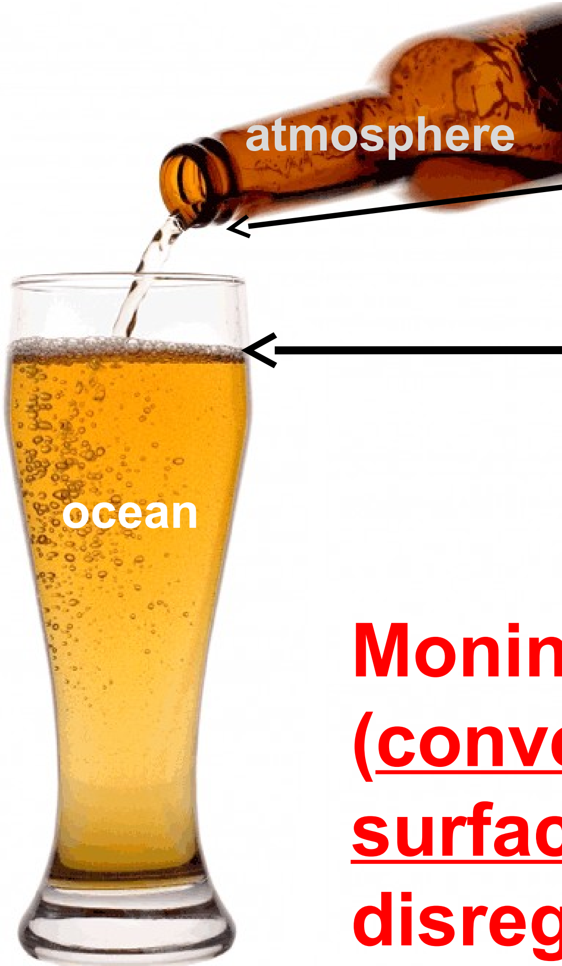
Turbulence performs vertical transports of energy, matter and momentum in fluid geospheres

Atmosphere, hydrosphere, lithosphere and **cryosphere** are coupled through turbulent **planetary boundary layers PBLs** (dark green lenses)

PBLs include 90% biosphere and entire anthroposphere



Role of planetary boundary layers (PBLs): TRADITIONAL VIEW



Surface fluxes at interface between
AIR
and
WATER (or **LAND**)
fully characterise interaction between
ATMOSPHERE and OCEAN/LAND

Monin-Obukhov similarity theory (1954) (<u>conventional framework for determining surface fluxes in operational models</u>) disregards non-local features of convective and long-lived stable PBLs

Role of PBLs: MODERN VIEW

Because of very stable stratification in atmosphere and ocean beyond PBLs and convective zones, density increments inherent at PBL outer boundaries prevent entities delivered by surface fluxes (or emissions) to efficiently penetrate from PBL into free atmosphere or deep ocean.

Hence PBL heights and fluxes due to entrainment at PBL outer boundaries essentially control extreme weather events
(e.g., heat waves associated with convection;
or strong stable stratification triggering air pollution).

This concept (equally relevant to hydrosphere) requires knowledge of PBL height/depth
and turbulent entrainment
in numerical weather prediction, air/water quality and climate modelling.

Very shallow boundary layer separated form the free atmosphere by capping inversion

PBL height visualised by smoke blanket (Johan The Ghost, Wikipedia)

Capping inversion prevents PBL – free flow exchange

Main stream in turbulence-closure theory

Boussinesq (1877) **Turbulent transfer is basically similar to molecular transfer** but much more efficient → **down-gradient transport** → K-theory → eddy viscosity, conductivity, diffusivity

Richardson (1920, 1922) stratification (Ri), concept of forward energy cascade

Keller & Fridman (1924) a chain of budget equations for statistical moments
Problem: to express the higher-order moments through the lower-order moments

Prandtl (1930s) **mixing length** $l \sim z$, velocity scale $uT \sim ldU/dz$, viscosity $K \sim luT$

Kolmogorov (1941) quantified the cascade, closure as a problem of energetics:

- budget equation for **turbulent kinetic energy** (TKE)
- **TKE dissipation rate** expressed through the turbulent-dissipation **length scale**

$uT \sim (КЭТ)1/2, K \sim l\varepsilon uT$ **underlies further developments through 20th century**

Obukhov (1946) TKE-closure extended to stratified flows, **Obukhov length scale** L

Monin & Obukhov (1954) alternative → **similarity theory** for the surface layer z/L

Mellor & Yamada (1974) hierarchy of K-closures → **turbulence cut-off problem**

Turbulence cut-off problem

Buoyancy $b = (g/\rho 0)\rho$ (g – acceleration due to gravity, ρ –density)

Velocity shear $S = dU/dz$ (U – velocity, z – height)

$$Ri = \frac{db/dz}{(dU/dz)^2}$$

Richardson number characterises static stability:

the higher Ri (or z/L), the stronger suppression of turbulence

Key question **What happens with turbulence at large Ri?**

Traditional answer **Turbulence degenerates, and at Ri exceeding a critical value (Ricritical< 1) the flow inevitably becomes laminar (Richardson, 1920; Taylor, 1931; Prandtl, 1930,1942; Chandrasekhar, 1961;…)**

In fact **field, laboratory and numerical (LES, DNS) experiments show that GEOPHYSICAL (very high Re) turbulence is maintained up to $Ri \sim 102$**

Modellers were forced to VIOLENTLY preclude the turbulence cut-off

Milestones

Prandtl-1930's followed Boussinesq's idea of the down-gradient transfer (K-theory), determined $K \sim luT$, and expressed uT heuristically through the mixing length l

Kolmogorov-1942 (**<u>for neutrall stratication</u>**) followed Prandtl's concept of eddy viscosity $KM \sim luT$; determined uT $=$ (TKE)1/2 through TKE budget equation with dissipation $\varepsilon \sim$ (TKE)/tT $\sim$ (TKE)3/2/$l\varepsilon$; and assumed $l\varepsilon \sim l$ (grounded **<u>in neutral stratification</u>**)

Obukhov-1946 and then the entire turbulence community extended Kolmogorov's closure to **<u>stratified flows</u>** **keeping it untouched**, except for inclusion of the buoyancy term in the TKE equation. **<u>Its sole use has caused cutting off TKE in supercritical stable stratification</u>**

This approach, **missed** **<u>turbulent potential energy</u>** (TPE) and its interaction with TKE); **overlooked** inapplicability **<u>of Prandtl's relation</u>** $K \sim luT$ to the eddy conductivity KH; and **disregarded** **<u>principal deference between</u>** $l\varepsilon$ and l

For practical applications Mellor and Yamada (1974) developed **<u>corrections preventing unacceptable turbulence cut-off</u>** in "supercritical" static stability

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Energy- & flux-budget (EFB) closure (2007-12)

Budget equations for major statistical moments

Turbulent kinetic energy (TKE) EK

Turbulent potential energy (TPE) EP

Vertical flux of temperature $Fz = <\theta w>$ [or buoyancy $(g/T)Fz$]

Vertical flux of momentum $\tau iz = <uiw>$ $(i = 1,2)$

Relaxation equation for the dissipation time scale $tT = EK/\varepsilon K = l(EK)$-1/2

Accounting for TPE → vertical heat flux (that "killed" TKE in Kolmogorov type closures) **drops out from the equation for total turbulent energy (TTE = TKE + TPE)**

Heat-flux budget equation → imposes a limit on the vertical heat flux and assures **self-preservation of turbulence** → **no** Ri-**critical in the energetic sense**

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Disclosed two principally different regimes of stably stratified turbulence

"Strong turbulence" in boundary layer flows with $KM \sim KH$ at $Ri < Ric$ **"Weak turbulence" in the free atmosphere** with $PrT = KM/KH \sim 4\,Ri$ at $Ri >> Ric$

MOS theory disregards weak turbulence at $z/L >> 1$ **and yields artefact** Ric **PBL height = the boundary between *strong*- and *weak-turbulence* regimes**

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Turbulent potential energy – analogy to Lorenz (1955) available potential energy

Buoyancy fluctuation proportional to displacement of fluid particle

$$b' = \frac{g}{\rho_0}\rho' = \frac{g}{\rho_0}\frac{\partial\langle\rho\rangle}{\partial z}z' = N^2 z'$$

Potential energy (per unit mass) proportional to **squared** temperature

$$(E_P)' = \frac{1}{z'}\int_z^{z+z'} b' z\, dz$$

$$= \frac{1}{2}\frac{(b')^2}{N^2} = \frac{1}{2}\left(\frac{\beta}{N}\right)^2(\theta')^2 = \left(\frac{\beta}{N}\right)^2(E_\theta)'$$

Turbulent energy budgets

Kinetic energy

$$E_K = \frac{1}{2}\langle u_i u_i \rangle$$

$$\varepsilon_K = \frac{E_K}{t_T}$$

$$\frac{DE_K}{Dt} + \frac{\partial \Phi_K}{\partial z} = \tau S + \beta F_z - \varepsilon_K$$

Potential energy

$$E_P = \frac{1}{2}\left(\frac{\beta}{N}\right)^2 \langle \theta^2 \rangle$$

$$\varepsilon_P = \frac{E_P}{C_P t_T}$$

$$\frac{DE_P}{Dt} + \frac{\partial \Phi_P}{\partial z} = -\beta F_z - \varepsilon_P$$

Total energy

$$E = E_K + E_P$$

$$\frac{DE}{Dt} + \frac{\partial (\Phi_K + \Phi_P)}{\partial z} = \tau S - (\varepsilon_K + \varepsilon_P)$$

Buoyancy flux βFz drops out from the turbulent total energy budget

Budget equation for the vertical turbulent flux of potential temperature

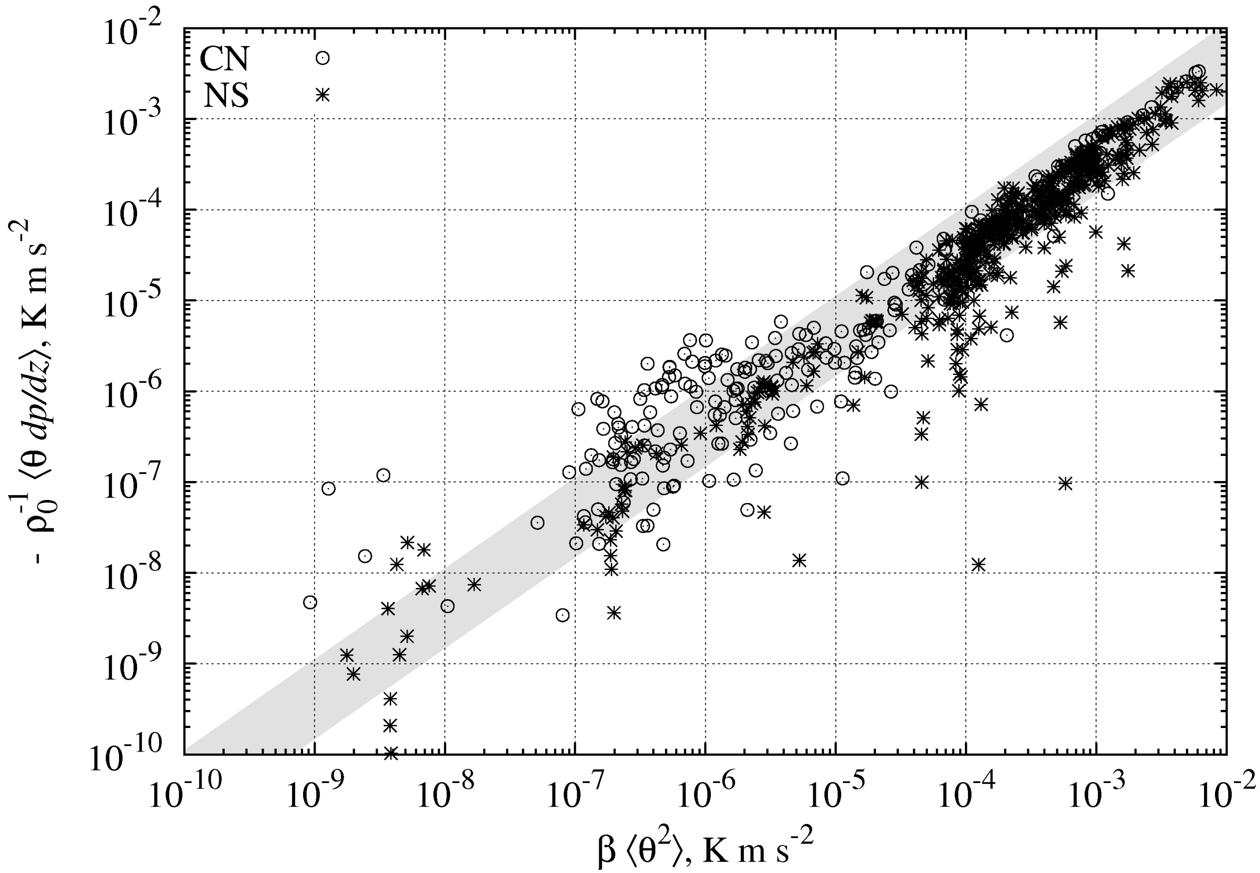
$F_z = \langle \theta w \rangle$

$$\frac{DF_z}{Dt} + \frac{\partial}{\partial z}\Phi_z^{(F)} = C_\theta \beta \langle \theta^2 \rangle - 2E_z \frac{\partial \Theta}{\partial z} - \frac{F_z}{C_F \, t_T}$$

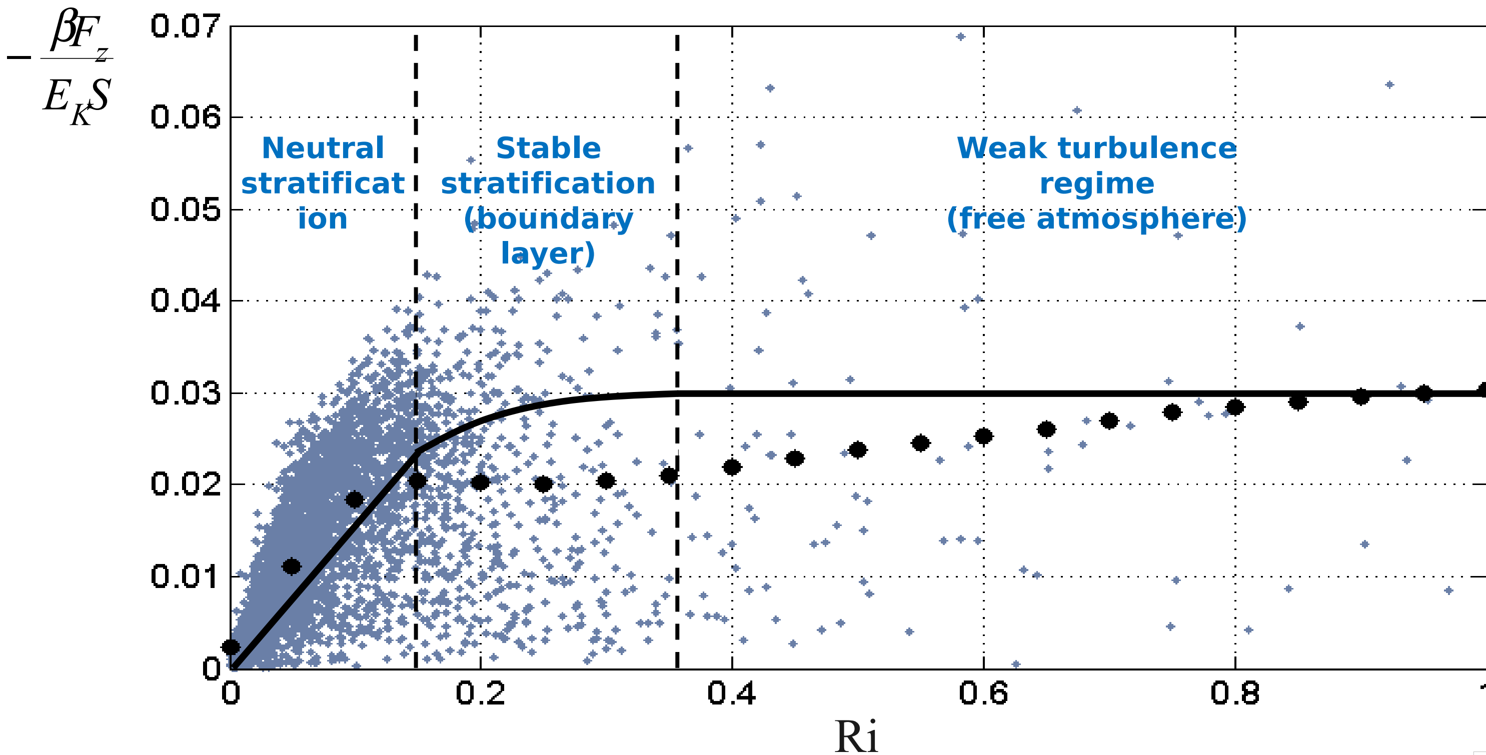
The “pressure term” is shown to be proportional to the mean squared fluctuation of potential temperature: $\frac{1}{\rho_0}\left\langle \theta \frac{\partial p}{\partial z} \right\rangle \sim \beta \langle \theta^2 \rangle$

On the r.h.s. of the equation, 1st term (generation of positive heat flux) counteracts to 2nd term (generation of negative heat flux) and yields **self-preservation of turbulence in very stable stratification**

LES verification of our parameterization of the pressure term $\rho_0^{-1}\langle \theta \partial p/\partial z \rangle \sim \beta\langle \theta^2 \rangle$



Ri-dependence of the buoyancy flux $B = \beta F z$



$Ri = (N/S)2$

Data (Sheba) and theory disprove <u>very concept of eddy-conductivity</u>

$$B = K_H N^2$$

Almost neutral stratification ($0 < z/L < 0.5$) **MOS OK**

$$B \sim E_K^{1/2} z N^2, \quad K_H \sim E_K^{1/2} z$$

z-less stratification” ($0.5 < z/L << 10$) **MOS OK**

$$B \sim E_K N, \quad K_H \sim E_K / N$$

Very stable stratification ($z/L >> 10$) **MOS fails**

$$B \sim E_K S, \quad K_H \sim E_K S / N^2$$

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Turbulent dissipation time and length scales

By definition, time scale $t_T \equiv E_K / \varepsilon_K$ and length scale $l \equiv E_K^{1/2} t_T$

The steady-state TKE budget

$$\tau S + \beta F_z \equiv \tau S (1 - Ri_f) = \varepsilon_K \equiv -\frac{E_K}{t_{TE}}$$

Flux Ri number $$Ri_f \equiv \frac{-\beta F_z}{\tau S} = \frac{\tau^{1/2}}{SL}$$

Obukhov length $$L = \frac{\tau^{3/2}}{-\beta F_z}$$

$$Ri_f \to R_\infty < 1$$

Shear: **neutral** $S = \frac{\tau^{1/2}}{kz}$, **extreme stable** (TKE) $S \to \frac{-\beta F_z}{R_\infty \tau} = \frac{\tau^{1/2}}{R_\infty L}$

Interpolation yields **empirical law valid in any stratification**

$$S = \frac{\tau^{1/2}}{kz}\left(1 + \frac{k}{R_\infty}\frac{z}{L}\right)$$

$$k / R_\infty = 1.6$$

Combining this law with the TKE equation yields

$$t_{TE} = \frac{kz}{E_K^{1/2} + C_\Omega \Omega z}\left(\frac{E_K}{\tau}\right)^{3/2} \frac{1 - Ri_f / R_\infty}{1 - Ri_f}$$

where kz plays the role of a “**master length scale**”

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Relaxation equation for dissipation time scale

Evolution of tT is controlled by

tendency towards equilibrium $t_T \to t_{TE}$

counteracted by **distortion** due to non-stationary processes and heterogeneity causing mean-flow and turbulent transports.

This **counteraction** is described by **RELAXATION EQUATION**

$$\frac{Dt_T}{Dt} - \frac{\partial}{\partial z} K_T \frac{\partial t_T}{\partial z} = -C_R \left(\frac{t_T}{t_{TE}} - 1 \right)$$

$C_R \sim 1$ relaxation constant (differs for increasing/decreasing regimes)

KT is the vertical turbulent exchange coefficient (~ to eddy viscosity)

EFB closure and M-O similarity theory (MOST)

New physics behind known relation

$$S = \frac{dU}{dz} = \frac{\tau^{1/2}}{kz}\left(1 + \frac{k}{R_\infty}\frac{z}{L}\right)$$

Combined with the flux Richardson number

$$Ri_f = \frac{-\beta F_z}{\tau S}$$

Yields **CONVERTOR** between Rif and z/L

$$Ri_f = \frac{kz/L}{1 + kR_\infty^{-1} z/L}$$

EFB theory yields **CONVERTOR** between Rif and Ri:

$$Ri_f = \frac{Ri}{Pr_T(Ri)}$$

where $Pr_T \approx Pr_0 = 0.8$ at $Ri << 1$, $Pr_T \approx Ri/R_\infty \approx 4Ri$ at $Ri >> 1$

See below empirical Ri-dependence of the turbulent Prandtl number Pr_T

Major results

- The concept of ***turbulent potential energy*** (Z et al., 2007) analogous to Lorenz's ***available potential energy*** (both ~ **squared** density)

- New vision and relaxation equation for **dissipation time scale**

- Disproving **erroneous conclusion** that at high-Re the flow becomes laminar at Ri exceeding critical $Ric \sim 0.25\text{-}1$. In fact, it demarcates: - reknown **Strong turbulence** with $KM \sim KH$ at $Ri < Ric$ typical of PBLs - new **weak turbulence** with $PrT = KM / KH \sim 4Ri$ at $Ri >> Ric$ in free flow

- **Hierarchy of closure models** of different complexity – for use in research and operational modelling

- Revision of Monin-Obukhov similarity theory

- Field, lab and numerical (LES, DNS) experiments confirm EFB theory for conditions typical of free atmosphere and deep ocean up to $Ri \sim 103$

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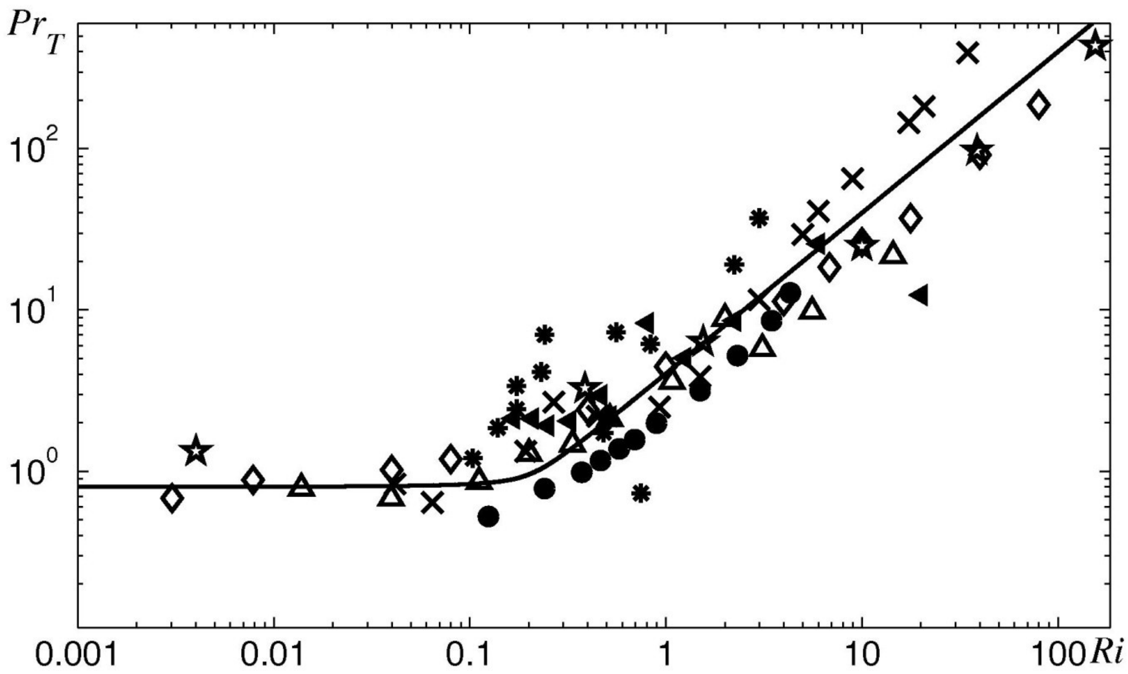
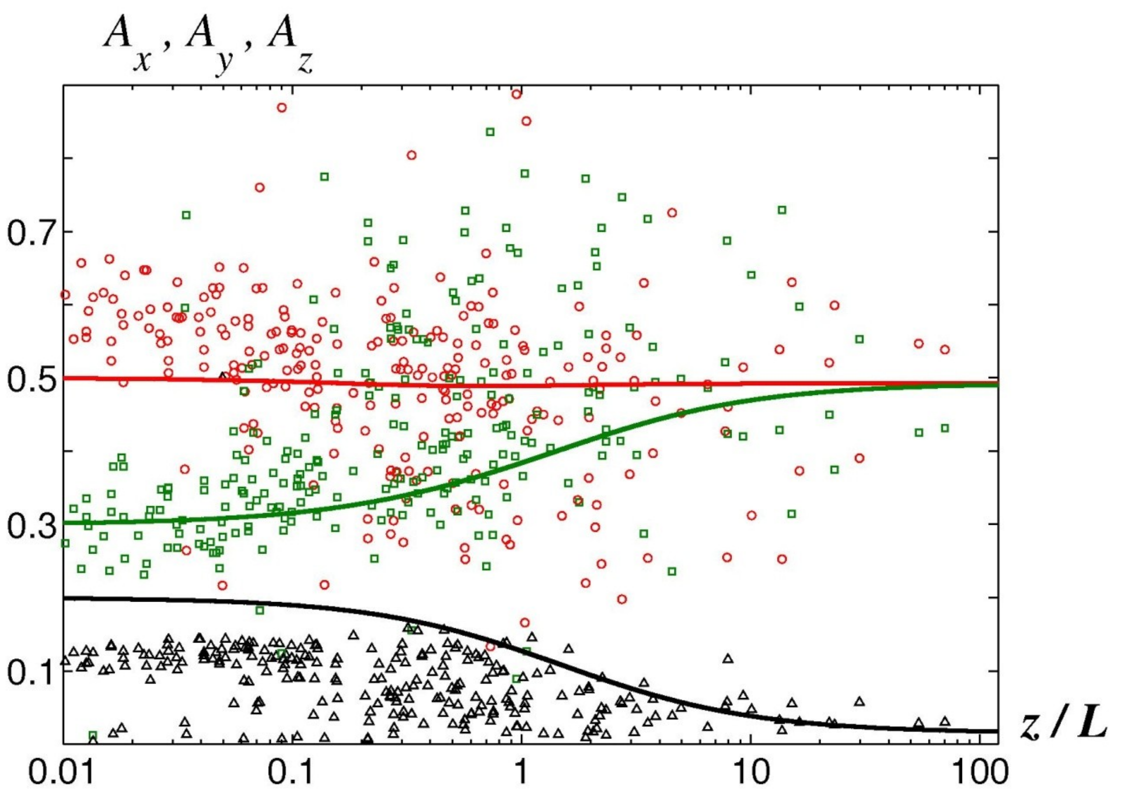
Turbulent Prandtl number $PrT = KM/KH$ versus Ri



Atmospheric data: ◀ (Kondo et al., 1978), ✱ (Bertin et al., 1997); laboratory experiments: ✕(Rehmann & Koseff, 2004), ◇(Ohya, 2001), ●(Strang & Fernando, 2001); DNS: ☆(Stretch et al., 2001); and LES: △(Esau, 2009). The curve sows our EFB theory. The "strong" turbulence ($PrT \approx 0.8$) and the "weak" turbulence ($PrT \sim 4\,Ri$) match at $Ri \sim 0.25$.

MOST assumes $PrT = constant$

Longitudinal *Ax*, transverse *Ay* & vertical *Az* TKE shares vs. *z/L*



Experimental data from Kalmykian expedition 2007 of the Institute of Atmospheric Physics (Moscow). Theoretical curves are plotted after the EFB theory. The **traditional "return-to-isotropy" model overlook the stability dependence of *Ay*** clearly seen in the Figure. The strongest stability, $z/L = 100$, corresponds to $Ri = 8$.

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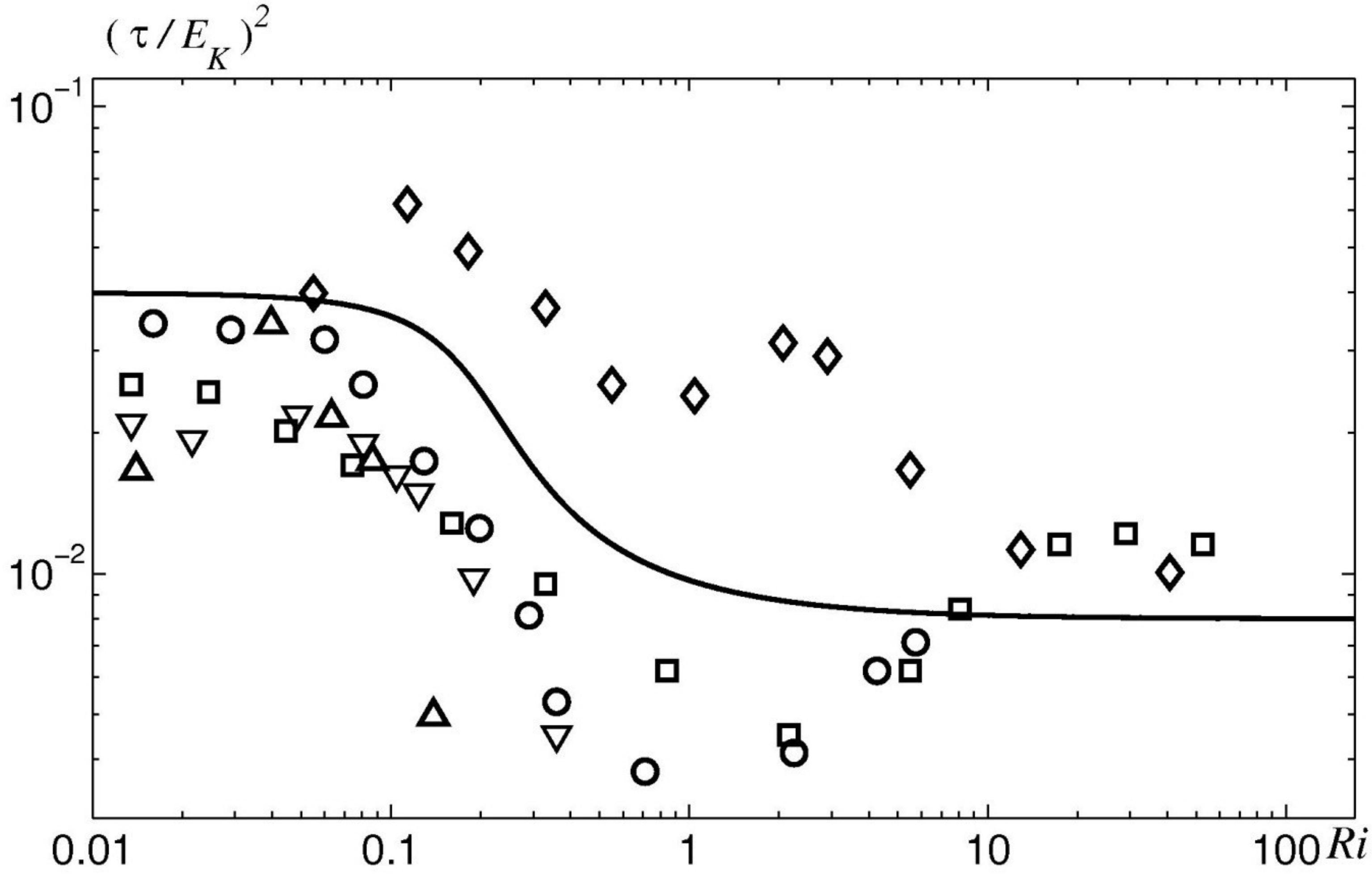
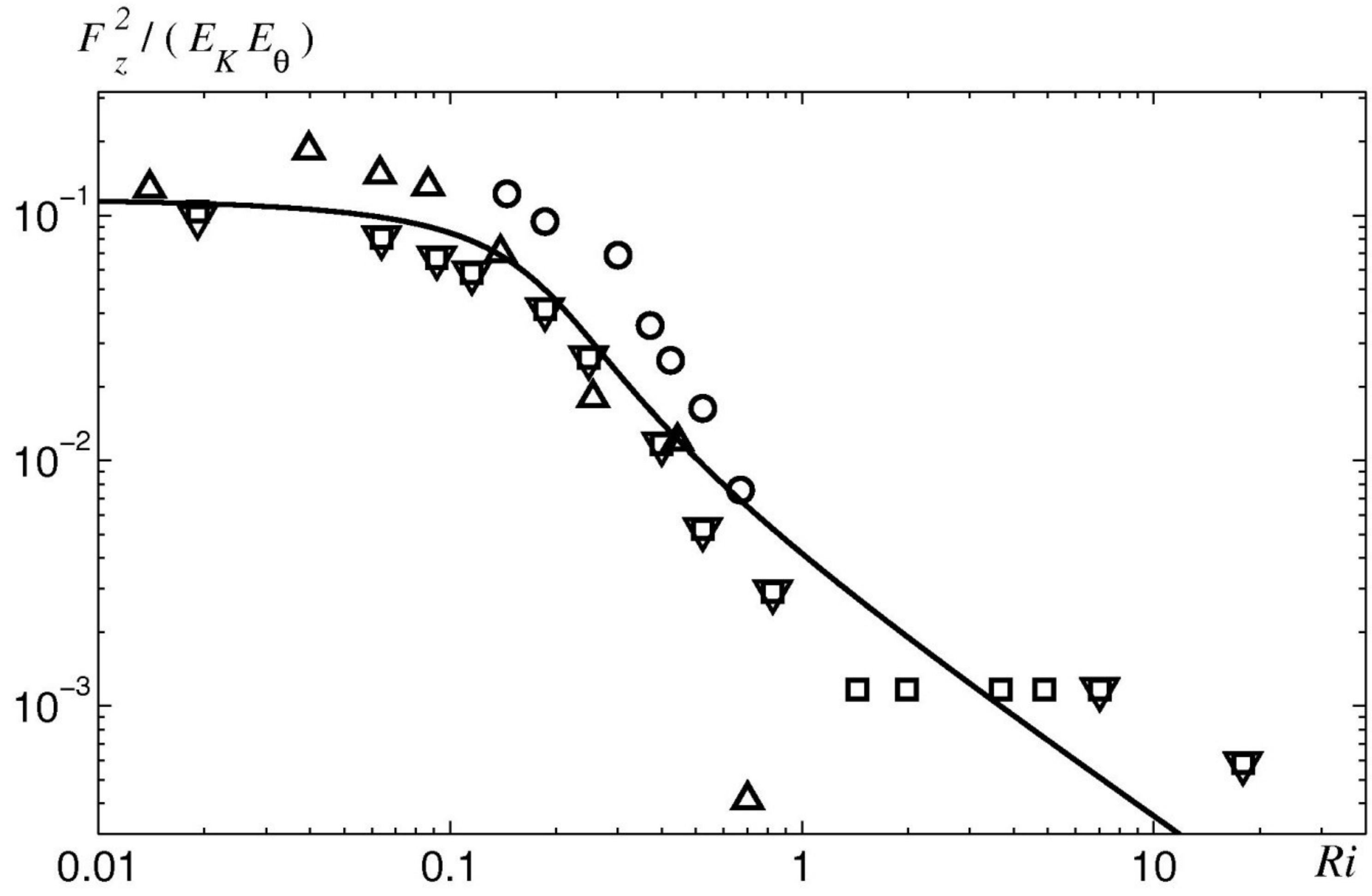
Dimensionless vertical flux of momentum: two plateaus corresponding to the *strong* and *weak* turbulence regime

MOST assumes $\tau/EK = constant$



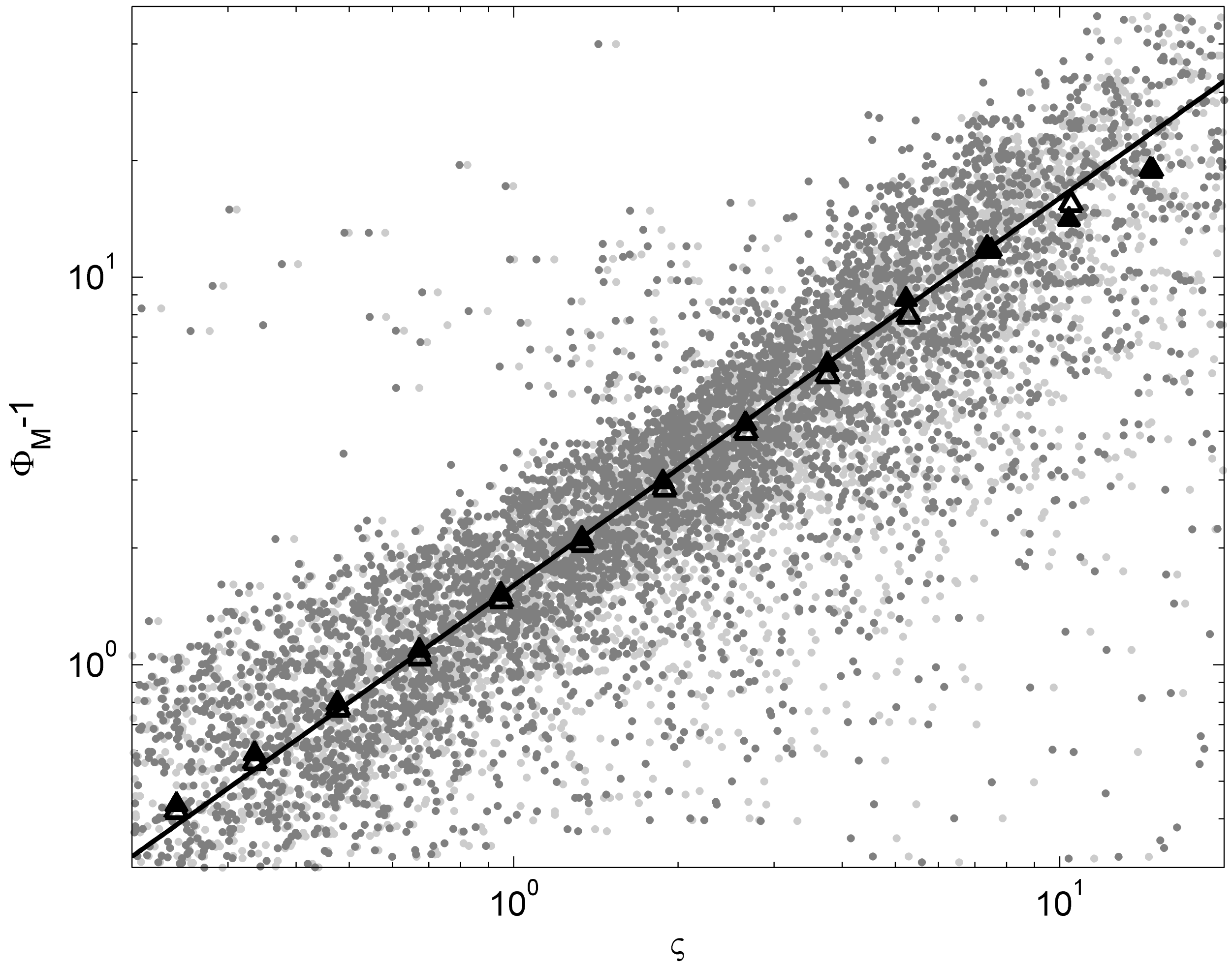
Dimensionless heat flux: practically constant in *strong* turbulence and sharply decreases in *weak* turbulence

MOST assumes *Fz*/(*EK E*θ)1/2 = *constant*



Dimensionless velocity gradient $\Phi_M = \left(kz / u_* \right) / \left(\partial U / \partial z \right)$ versus $\zeta = z/L$ after LES (dots) and the EFB model (curve)

MOST OK



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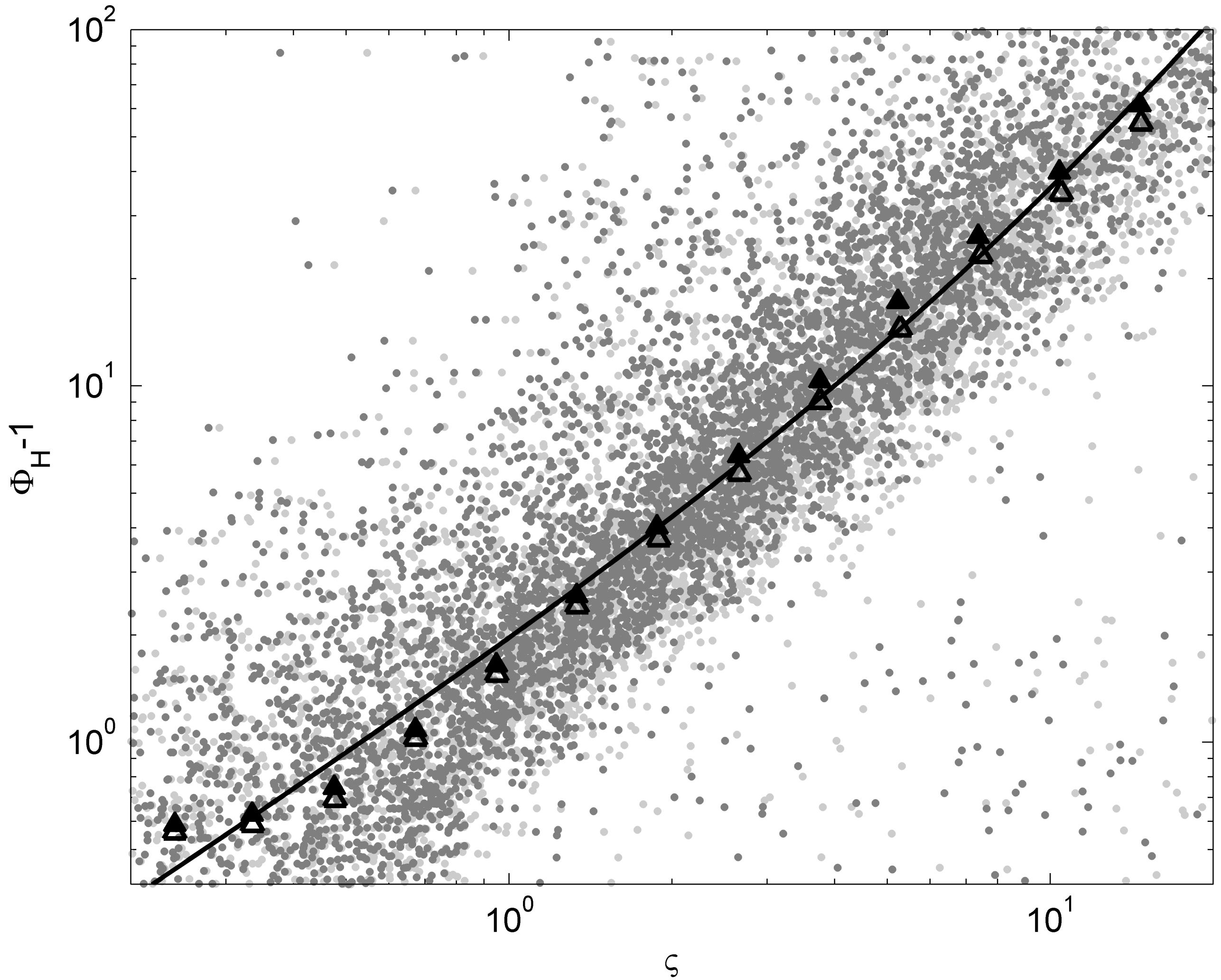
Dimensionless temperature gradient versus $\zeta = z/L$ after LES (dots) and the EFB model (curve)

$$\Phi_H = (-k_T z \tau^{1/2} / F_z)(\partial\Theta / \partial z)$$

MOST fails



General closure model: energy & flux equations

Kinetic energy

$$\frac{DE_K}{Dt} - \frac{\partial}{\partial z} K_E \frac{\partial E_K}{\partial z} = -\tau_{i3} \frac{\partial U_i}{\partial z} + \beta F_z - \frac{E_K}{t_T}$$

Potential energy

$$\frac{DE_P}{Dt} - \frac{\partial}{\partial z} K_E \frac{\partial E_P}{\partial z} = -\beta F_z - \frac{E_P}{C_P t_T}$$

Momentum flux

$$\frac{D\tau_{i3}}{Dt} - \frac{\partial}{\partial z} K_{FM} \frac{\partial \tau_{i3}}{\partial z} = -2E_z \frac{\partial U_i}{\partial z} - \frac{\tau_{i3}}{C_\tau t_T}$$

Θ-flux

$$\frac{DF_z}{Dt} - \frac{\partial}{\partial z} K_{FH} \frac{\partial F_z}{\partial z} = -2(E_z - C_\theta E_P) \frac{\partial \Theta}{\partial z} - \frac{F_z}{C_F t_T}$$

Turbulent exchange coefficients for energies and fluxes are taken proportional to the eddy viscosity

$$K_E / C_E = K_{FM} / C_{FM} = K_{FH} / C_{FH} = K_T / C_T = E_z t_T$$

General closure model: Vertical TKE

To characterise stability we use, instead of Rif, the energy-ratio $\Pi = E_P / E_K$ {in the steady-state $\Pi = C_P Rif /(1 - Rif)$ } and employ our steady-state solution to express E_z / E_K and E_K / τ as universal functions of Π determined from our prognostic equations

Dissipation time scale

Similarly, we express the equilibrium time scale t_{TE} through Π

$$t_{TE} = \frac{kz}{E_K^{1/2} + C_\Omega \Omega z} \cdot \left(\frac{E_K}{\tau} \right)^{3/2} \left(1 - \frac{\Pi}{\Pi_\infty} \right)$$

and determine t_T after our relaxation equation

$$\frac{Dt_T}{Dt} - \frac{\partial}{\partial z} K_T \frac{\partial t_T}{\partial z} = -C_R \left(\frac{t_T}{t_{TE}} - 1 \right)$$

Optimal closure model

For operational modelling we recommend as optimal the model based on 3 prognostic equations for: - the two turbulent energies EK and EP - and the dissipation time scale tT - in combination with diagnostic eddy viscosity & eddy conductivity

Advantages of the EFB closures:

- consistent energetics with no Ri-critical
- advanced concept of the turbulent dissipation time scale
- "energy stratification parameter" preventing artificial extremes
- essential anisotropy of turbulence
- generally non-gradient and non-local turbulent transports

Conclusions

- **EFB turbulence closure → new vision and modelling of geophysical stably stratified turbulence**

- **No Ric in the energetic sense: experimental data confirm this conclusion up to $Ri \sim 103$**

- **Instead: $Ri \sim 0.2\text{-}0.3$ (hydrodynamic instability limit) separates regimes of "strong" and "weak" turbulence → <u>the boundary between PBL and free atmsophere</u> → another view at the PBL height**

- **MOS is applicable to the "strong" turbulence regime typical of boundary layer flows but inapplicable to "weak turbulence" typical of free atmosphere / ocean**