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Interoperability: Grid Aspects

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1 Introduction

The SRNWP interoperability project aims at facilitating the exchange and cross-usage of model output data between the different European NWP consortia (Arpege/Aladin, COSMO, UKMO and Hirlam). The first stages of the project consisted of the exchange of documentation and test data.

This report gives a brief summary of the documentation regarding the horizontal and vertical coordinate systems used by the different consortia. Next, the problems of destaggering, horizontal interpolation and vertical interpolation are investigated.

2 A short description of the coordinate systems used by the different consortia

2.1 Horizontal coordinate systems

2.1.1 Arpege/Aladin

Arpege is a spectral global model. Its gridpoints ly on a stretched and tilted reduced Gauss grid [ARPEGE, 2008, sections II.3 and III.3].

Aladin is a spectral LAM model working on a rectangular area (bi-Fourier). In principle, it could act on any conformal geographic projection, but in practice, it is limited to the systems that can be described by the FA file format. These are polar stereographic, conical Lambert [Gril, 2004] and rotated Mercator [Bénard, 2004].

Arpege and Aladin work on an Arakawa A-grid, meaning that there is no horizontal staggering of wind components w.r.t. other variables.

2.1.2 COSMO

The global DWD model (actually not COSMO) uses an icosahedral grid system. This grid can be contained in a GRIB2 file [WMO-GRIB2, 2008]. Detailed information (e.g. on staggering) is not yet available.

[Personal note: although distributed as GRIB2 documentation, [WMO-GRIB2, 2008] does not give a technical description of how to specify the icosahedral grid in a GRIB2 file; it merely describes an icosahedron, gives some basic explanation on how to refine the mesh, and how the gridpoints can be numbered.]

The COSMO LAM model (COSMO-LM) uses rotated spherical (longitude/latitude) coordinates [COSMO, 2002, section 3.3]. The wind components are staggered according to an Arakawa C-grid [COSMO, 2002, section 4.1.1].

2.1.3 Hirlam

Hirlam is prepared for any general conformal horizontal coordinate system [HIRLAM, 2007]. Until now, rotated or unrotated spherical coordinates have been used.

The wind components are staggered according to an Arakawa C-grid.

2.1.4 UKMO-UM

The UKMO UM (Universal Model) can be used as a global model or as a LAM model. Both configurations use rotated spherical coordinates [UKMO, 2009a, section 4.1]. A particularity is that the grids need not to be uniform, i.e. variable grid spacing is allowed. This is done by explicitly specifying an array of grid cell spacings $\Delta\xi_i$, for $i = 1, \ldots, n$ with n the number of grid cells in the ξ-direction.

The wind components are staggered according to an Arakawa C-grid.

[Personal note: I wonder if the non-uniform grid spacing can be cast into a regular GRIB2 file.]

2.2 Vertical coordinate systems

2.2.1 Arpege/Aladin

Arpege and Aladin use a pressure-based terrain-following hybrid vertical coordinate [ARPEGE, 2008, section II.5]. The variables are vertically staggered on half-levels (index $\tilde{\ell}$) and full-levels (index ℓ). The (hydrostatic) pressure π is related to the hybrid coordinate η through

$$\pi_{\tilde{\ell}} = A(\eta_{\tilde{\ell}}) + B(\eta_{\tilde{\ell}})\pi_s \qquad \tilde{\ell} = 0, \ldots, L. \tag{1}$$

With π_s the surface pressure, and the functions A and B discretely specified for the different layers.

Temperature and humidity variables are defined on full levels. The geopotential is primarily defined on half-levels. To determine the pressure and the geopotential on the full levels, following identities can be discretized [ARPEGE, 2008, section II.5.2]:

$$\ln \pi = \frac{\partial}{\partial \pi}(\pi \ln \pi) - 1 \tag{2}$$

$$\Phi = \frac{\partial}{\partial \pi}(\pi \Phi) - \pi \frac{\partial \Phi}{\partial \pi} = \frac{\partial}{\partial \pi}(\pi \Phi) + RT. \tag{3}$$

2.2.2 COSMO

The global DWD model uses a pressure-based terrain-following hybrid vertical coordinate. Variables are staggered vertically [COSMO, 2009, section 1]. A minor point of difference with the Arpege/Aladin vertical coordinate is that the pressure at the full levels is determined as the arithmetic mean of the pressures at the half-levels above and below (instead of the more refined method in eq.(2)). The functions A and B are discretely specified and are stored in the grid description section of the GRIB file.

The limited area COSMO-LM model is a nonhydrostatic model. Some model variables (T, p and ρ) are expressed as a base state value plus a deviation (see COSMO [2009, section 2.1] and COSMO [2002, section 3.4]). The atmosphere base state is defined as thermodynamically and hydrostatically balanced ($p_0 = \rho_0 R T_0$ and $\partial p_0/\partial z = -g\rho_0$), and is further characterized by a constant increase in temperature with the logarithm of pressure ($\partial T_0/\partial \ln p_0 = \beta$). Numeric values of the reference sea-level pressure p_{SL}, reference sea-level temperature T_{SL} and the coefficient β are:

$$p_{SL} = 1000\,\mathrm{hPa}, \qquad T_{SL} = 288.15\,\mathrm{K}, \qquad \beta = 42\,\mathrm{K}.$$

Three options exist for the vertical coordinate system: (i) base state pressure-based, (ii) height-based, and (iii) height-based with smoothing of small-scale orography (SLEVE). Common features of all systems are:

- terrain-following hybrid
- time-independent, nondeformable, fixed in physical space. This is different from the systems used by the other consortia.
- vertical grid-stretching (i.e. a non-uniform η-, μ- or μ_s-grid) is possible.

A more detailed description of the three options [COSMO, 2009, section 2.2]:

1. pressure-based η:
$$p_0(\eta) = A(\eta) + B(\eta)p_0^s. \tag{4}$$
Note that p_0 is the pressure of the base-state atmosphere, not the actual pressure. Another notable difference with the Arpege/Aladin system is that the functions $A(\eta)$ and $B(\eta)$ are not discretely specified, but are defined as prescribed piecewise linear functions.
2. height-based μ:
$$z(\mu) = a(\mu) + b(\mu)h, \tag{5}$$
with h the orography and $a(\mu)$ and $b(\mu)$ prescribed piecewise linear functions.
3. SLEVE coordinate μ_s:
$$z(\mu_s) = a(\mu_s) + b_1(\mu_s)h_1 + b_2(\mu_s)h_2, \tag{6}$$
with h_1 and h_2 the large-scale and small-scale components of the orography ($h_1 + h_2 = h$) and $a(\mu_s)$, $b_1(\mu_s)$ and $b_2(\mu_s)$ prescribed nonlinear functions of μ. The purpose of this system is to introduce a stronger decay for small-scale features than for large-scale features.

A list of parameters written to the GRIB2 grid description section is given in COSMO [2009], section 2.4.

2.2.3 Hirlam

Hirlam uses a general pressure-based terrain-following hybrid coordinate [HIRLAM, 2007], similar to the Arpege/Aladin coordinate. A minor difference is that the pressure values at the full levels are obtained as the arithmetic mean of the enclosing half-level pressures.

2.2.4 UKMO-UM

The Universal Model (UM) uses a height-based terrain-following hybrid coordinate η [UKMO, 2009b]. Different possibilities to relate height z to the coordinate η are considered:

- linear:
$$z = z^S + \eta(z^T - z^S), \tag{7}$$
with z^T the height of the atmosphere, and z^S the surface level.
- quadratic in the lower atmosphere and linear in the upper atmosphere to obtain a faster decay of orographic influence. This is quite similar to the COSMO-LM μ-variable (which is linear in both zones), only here an additional constraint of first-order continuity at the interface between the zones is imposed.
- further refinement by dividing the atmosphere into more than two parts and writing a quadratic or cubic spline-based relation between z and η for each part. This requires some special considerations on derivative continuity and mononoricity.

3 Destaggering

Destaggering means the conversion of an Arakawa C-grid, in which the wind velocities are staggered with respect to the other atmospheric variables, to an Arakawa A-grid, where all atmospheric variables (including wind components) are available on the same grid.

Generally speaking, destaggering can be treated as a (horizontal) interpolation problem. For instance, the Hirlam `gl` tool includes a destaggering procedure that performs a linear interpolation between the two surrounding grid points (see also section 4.1). In view of the particular characteristics of destaggering, however, this section investigates if more efficient/accurate methods can be envisaged.

3.1 Fourier transform

Aladin is a spectral model, meaning that many of its variables are described by a decomposition in harmonic functions. The Fourier transforms in the Arpege/Aladin model are defined as [`fft992.F`]

$$x_j = \sum_{k=0}^{n-1} (a_k + i b_k) \exp\left(\frac{2\pi i}{n} j\, k\right), \tag{8}$$

and

$$a_k = \frac{1}{n} \sum_{j=0}^{n-1} x_j \cos\left(\frac{2\pi}{n} j\, k\right), \qquad \text{for } k = 0, \ldots, n/2 \tag{9a}$$

$$b_k = -\frac{1}{n} \sum_{j=0}^{n-1} x_j \sin\left(\frac{2\pi}{n} j\, k\right), \qquad \text{for } k = 0, \ldots, n/2 \tag{9b}$$

where $a_{n-k} = a_k$ and $b_{n-k} = -b_k$.

A domain shift over a distance Δj w.r.t. the normalized domain $j = 0, \ldots, n-1$ is obtained by modifying the coefficients a_k and b_k according to

$$a'_k = \cos\left(\frac{2\pi}{n} \Delta j\, k\right) a_k - \sin\left(\frac{2\pi}{n} \Delta j\, k\right) b_k \tag{10a}$$

$$b'_k = \sin\left(\frac{2\pi}{n} \Delta j\, k\right) a_k + \cos\left(\frac{2\pi}{n} \Delta j\, k\right) b_k \tag{10b}$$

for $k = 0, \ldots, n/2$, while maintaining the relations $a'_{n-k} = a'_k$ and $b'_{n-k} = -b'_k$.

This means that the destaggered wind field can be obtained without performing a direct interpolation by the following steps:

1. Calculate the Fourier transform of the staggered wind field.
2. Modify the spectral coefficients according to eq. (10) with $\Delta j = \frac{1}{2}$.
3. Perform an inverse Fourier transform.

Note that the first step is not necessary if the input model is a spectral model, and that the last step is not strictly necessary since Aladin ultimately needs the wind components in spectral representation.

3.2 Legendre transform

Arpege is a global spectral model, with some of its fields decomposed into global harmonics that consist of Fourier harmonics in the zonal direction and Legendre polynomials in the meridional direction.

Theoretically, it is possible to employ a similar destaggering strategy for the Legendre decomposition as for the Fourier decomposition, i.e. determine the spectral coefficients of the destaggered field by modifying the spectral coefficients of the staggered field.

However, such an approach seems difficult and not even very useful for the following reasons:

- Arpege is the only spectral global model in the I-SRNWP project: UKMO-UM and DWD-GME are gridpoint models.
- The latitude circles are not uniformly spaced, so one cannot fix $\Delta j = \frac{1}{2}$ as was the case for the Fourier transform.
- The modifications to the spectral coefficients are coupled. In the Fourier-transform, each pair (a'_k, b'_k) is fully characterized by (a_k, b_k) and Δj. For the Legendre transform, each modified coefficient would depend on all original coefficients.

Notwithstanding these difficulties, it may still be possible to determine the spectral decomposition of a staggered global field without performing explicit interpolations. This will be discussed in the next section.

4 Regridding

Regridding means the conversion of a field from one geographical projection to another projection. In gridpoint space, this becomes an interpolation problem. As will be shown in section 4.1, interpolation damps the high frequencies. Therefore, two alternative regridding methods will be considered in sections 4.2 and 4.3. These methods directly determine the spectral representation of the regridded field.

4.1 Regridding in gridpoint space

Most NWP consortia have developed designated interpolation software. For instance, the Hirlam `gl` tool includes nearest-neighbour and bilinear interpolations to transform between different projections. With the Arpege/Aladin FullPOS software, more advanced interpolation methods (e.g. accounting for climatology) can be used, but this requires that both the original and the target projection can be described by the FA file header (see section 6.1).

Although interpolation in gridpoint space is arguably the most easy and general method to regrid, one should be aware of the fact that it acts as a filter that damps the high frequency features. To illustrate this, let's consider the following destaggering problem. The harmonic function f is given on a uniform grid between 0 and 1:

$$f_j = \exp\left(i\,k\,x_j\right), \qquad x_j = j/N, \qquad j = 0, \ldots, N-1. \tag{11}$$

Applying linear interpolation to obtain the function values $f'_{j+\frac{1}{2}}$, one gets

$$f'_{j+\frac{1}{2}} = \frac{1}{2}\left(\exp\left(i\,k\,x_j\right) + \exp\left(i\,k\,x_{j+1}\right)\right) \tag{12}$$

The spectral filter $\mathcal{F}$ can be defined as the ratio of the interpolated function values to the exact function values $f_{j+\frac{1}{2}} = \exp\left(i\,k\,x_{j+\frac{1}{2}}\right)$:

$$\mathcal{F}^{linear} = \frac{f'_{j+\frac{1}{2}}}{f_{j+\frac{1}{2}}} = \frac{1}{2}\left(\exp\left(-i\,\frac{k}{2N}\right) + \exp\left(i\,\frac{k}{2N}\right)\right) = \cos\left(\frac{k}{2N}\right) \tag{13}$$

This shows that linear interpolation damps the high frequency features; the $2\Delta x$ wave ($k = \pi/\Delta x = N\pi$) is even completely removed.

A similar calculation shows that destaggering with cubic interpolation is equivalent to applying the following filter:

$$\mathcal{F}^{cubic} = \frac{1}{4}\left(5\cos\left(\frac{k}{2N}\right) - \cos\left(\frac{3k}{2N}\right)\right) \tag{14}$$

As visible in figure 1, cubic interpolation amplifies the midrange frequency content, and damps the high frequency features. Although the damping is less pronounced than for the linear interpolation, the $2\Delta x$ wave is still completely filtered out.

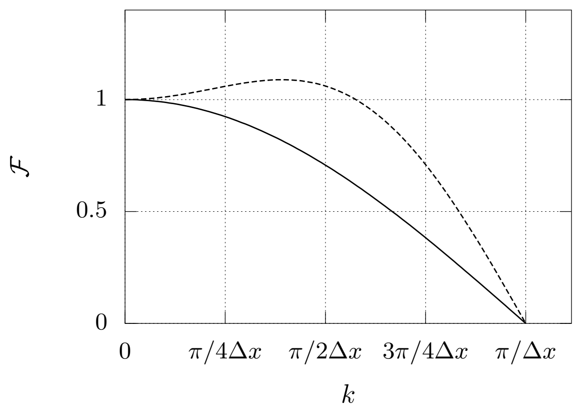


Figure 1: Influence of linear (solid line) and cubic (dashed line) interpolation on the frequency spectrum.

4.2 Regridding in spectral space by direct decomposition

When taking some distance from the problem and considering the core of spectral decomposition (see appendix A), it becomes apparent that it is not strictly required that the gridpoint data be available on a regular grid. In fact, two approaches to determine the spectral coefficients of a field from data on a non-regular grid, can be distinguished: direct decomposition (discussed in this section) or direct integration (discussed in the next section).

The direct decomposition method simply fills in the available information in eq. (24). Suppose data f_j are given at arbitrarily positioned points x_j , for $j = 1, \ldots, N$, then we can write

$$\begin{bmatrix} \Phi_1(x_1) & \ldots & \Phi_n(x_1) \\ \vdots & \ddots & \vdots \\ \Phi_1(x_N) & \ldots & \Phi_n(x_N) \end{bmatrix} \begin{Bmatrix} \varphi_1 \\ \vdots \\ \varphi_n \end{Bmatrix} = \begin{Bmatrix} f_1 \\ \vdots \\ f_N \end{Bmatrix} \tag{15}$$

or

$$\boldsymbol{\Phi}\boldsymbol{\varphi} = \mathbf{f} \tag{16}$$

Solving this linear system yields the spectral coefficients φ_k.

Some remarks on this method:

- If the number of available points N is equal to the number of spectral coefficients n, then the interpolating function goes through the data points. If $N > n$ then the system becomes overdetermined and the interpolating function is not guaranteed to cross the data points. If $N < n$ then the system is underdetermined and the solution is not unique. This situation leads to arbitrary results and should certainly be avoided. This is illustrated in figure 2, where $N = 32$ and $n = 64$. Although the interpolating function (solid line) goes through the data points (markers), it does not approximate the original function (dashed line) very well.

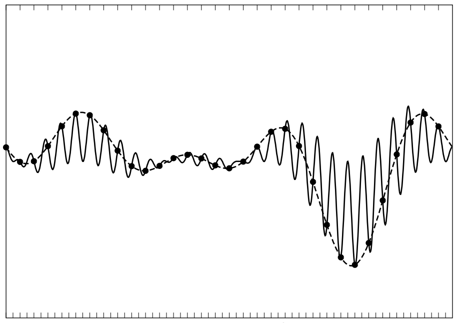


Figure 2: Underdetermined system for direct decomposition method: actual function (dashed line), which is given on the source grid (markers); interpolating function (solid line).

- Even when $N \leq n$, the direct decomposition method may yield in poor results when the source grid is very irregular. This is illustrated in figure 3, where $n = 64$ and $N = 60$. In the region where many data points are available, the interpolating function follows the original function closely, but in regions where data points are further apart, all correspondence is lost.

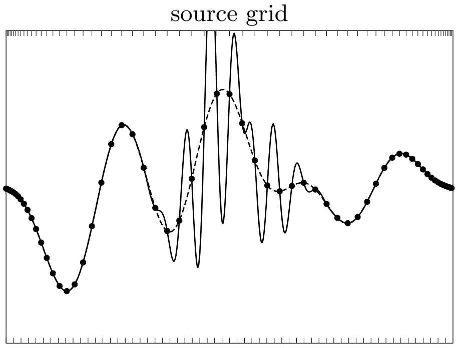


Figure 3: Direct decomposition method on a very irregular source grid: actual function (dashed line), which is given on the source grid (markers); interpolating function (solid line).

- The computational cost is probably too heavy if this method is applied on an irregular 2D grid ($N \sim 10^5$). However, many regridding problems can be decoupled into a zonal and a meridional regridding problem. This is the case for conversions between rotated spherical coordinates, rotated Mercator projection, and a Gauss grid (if the poles are aligned). For conversions to and from a stereographic polar projection, conical Lambert projection or the icosahedral grid, it does not seem to be possible to decouple both directions.

  If the problem can be decoupled, then the matrix operations and dimensions are:

  1. meridional decomposition:

  $$\underset{(n_x \times N_y)}{\boldsymbol{\psi}} \quad \underset{(N_y \times n_y)}{\boldsymbol{\Phi}_y^T} = \underset{(n_x \times n_y)}{\mathbf{f}} \tag{17}$$

  2. zonal decomposition:

  $$\underset{(n_x \times N_x^{(j)})}{\boldsymbol{\Phi}_x^{(j)}} \quad \underset{(N_x^{(j)} \times 1)}{\boldsymbol{\varphi}^{(j)}} = \underset{(n_x \times 1)}{\boldsymbol{\psi}^{(j)}}, \qquad j = 1, \ldots, N_y \tag{18}$$

where $n_x \times n_y$ is the dimension of the source grid, and $N_x^{(j)} \times N_y$ are the number of spectral coefficients on the target grid. $\boldsymbol{\psi}^{(j)}$ is the j^{th} column of $\boldsymbol{\psi}$. Note that the number of zonal wave components is not necessarily constant since its truncation may depend on the meridional wavenumber (e.g. triangular global truncation or elliptic LAM truncation).

- The matrix (or matrices for a problem that can be decoupled) that needs to be inverted is independent of the field values, so it's inverse (or it's LU decomposition) only has to be computed once.

4.3 Direct integration

This method calculates the spectral coefficients with eq. (25), but evaluates the integral directly. Suppose data f_j are given at arbitrarily positioned points x_j, for $j = 1, \ldots, N$. Further suppose that for each point x_j a weighting coefficient w_j is available (this could be the area represented by the point). Then the spectral coefficients can be determined as

$$\varphi_k = \int_{\mathcal{D}} f(x)\Phi_k(x)dx \approx \sum_{j=1}^{N} w_j f_j \Phi_k(x_j). \tag{19}$$

Some remarks:

- This method is generally applicable, even to convert to and from a Lambert projection or an icosahedral grid.
- The interpolating function obtained with this method does not necessarily go through the data points.
- When the number of data points N is lower than the number n of spectral coefficients that one tries to determine, aliasing will occur. This is illustrated in figure 4, where $N = 32$ and $n = 64$. Although this phenomenon is similar to the one encountered with the direct decomposition method (figure 2), a crucial difference is that the lower frequency coefficients are estimated well with the direct integration method, whereas they are 'contaminated' in the direct decomposition method. Simply applying a lowpass filter afterwards would therefore solve the problem.

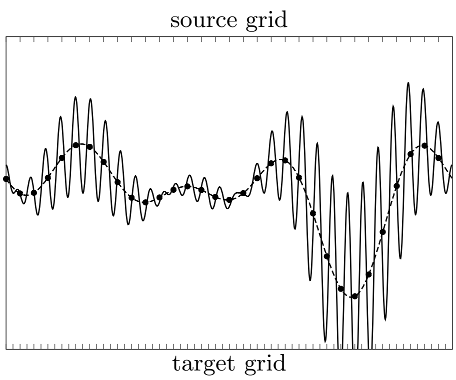


Figure 4: Aliasing with direct integration method when number of spectral coefficients is larger than number of data points: actual function (dashed line), which is given on the source grid (markers); interpolating function (solid line).

- As can be expected, the range of frequencies for which the coefficients are estimated well also depends on the degree of irregularity of the source grid. This is illustrated in figure 5.

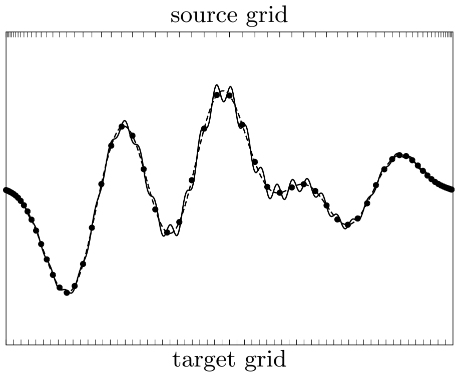


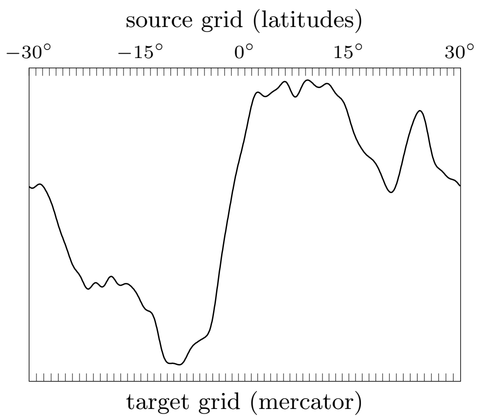
Figure 5: Direct integration method on a very irregular source grid: actual function (dashed line), which is given on the source grid (markers); interpolating function (solid line).

4.4 Example: regridding spherical coordinates to Mercator projection

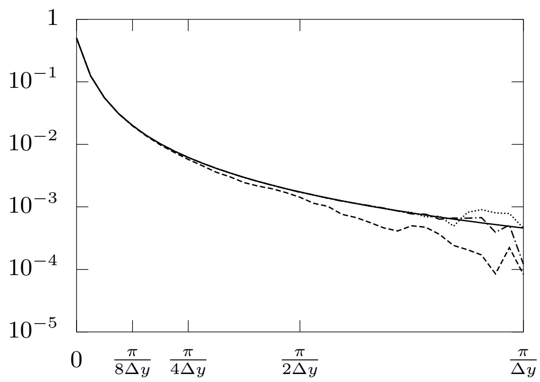
This regridding problem can be decoupled into a zonal regridding and a meridional regridding. In this example, we focus on the meridional regridding problem since it is nonlinear: the meridional coordinate y in a Mercator projection is related to the latitude through

$$y = a \ln \left[\tan \left(\frac{\theta}{2} + \frac{\pi}{4} \right) \right]. \tag{20}$$

Let's consider the situation where the data are available on 64 evenly distributed latitudes between $-30\,^\circ$ and $30\,^\circ$. Due to the nonlinear transformation in eq. (20), this uniform θ-grid will transform into a non-uniform y-grid. Figure 6(a) shows the function that is given on the irregular y-grid, and that we will try to regrid to the regular y-grid with different methods. The spectrum of this function is shown along with the spectra of the interpolating functions in figure 6(b). Apparently, all three methods (linear interpolation, direct decomposition and direct integration) yield very accurate results in the lower frequency range. The higher frequency content is conserved very well by the spectral interpolation methods, while the gridpoint interpolation seems to damp the higher frequencies. This damping behavior of gridpoint interpolation was expected (see figure 1).



(a) Gridpoint space



(b) Spectral space

Figure 6: Meridional regridding from spherical coordinates to mercator coordinates. (a) Gridpoint-space representation of function to be regridded; (b) Spectrum of this function (solid line), spectrum from linear interpolation (dashed line), spectrum from direct decomposition (dash-dotted line), and spectrum from direct integration (dotted line).

Finally, it should be noted that this example is not very challenging in the sense that the transformation between θ and y is nearly linear in the considered region between -30° and 30°. However, an extent of more than 60° does not seem to be common for LAM models.

4.5 Example: regridding a hexagonal grid to a rectangular grid

In this section, we consider the problem of regridding a hexagonal grid to a rectangular grid. A hexagonal grid may originate from the global DWD model. The considered grid, shown in figure 7 is obtained by calculating the voronoi cells for a quasi-uniform set of points, such that the resulting cells are nearly - *but not exactly* - regular hexagons. The gridpoints are chosen to be the centers of mass of these cells.

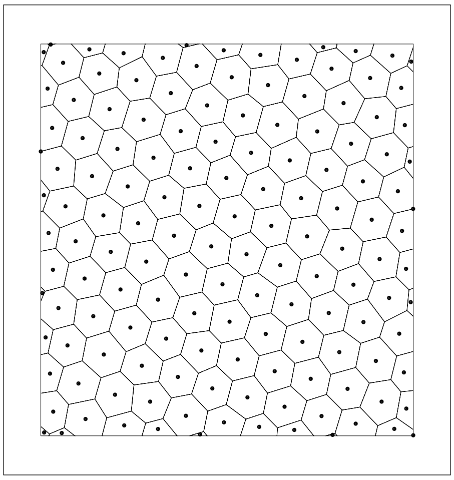


Figure 7: Quasi-regular hexagonal grid.

On this grid, the function shown in figure 8(a) is sampled. As shown in figure 8(e), this function is characterized by a constant spectrum up to wavenumbers $k_x^2 + k_y^2 \leq 6^2$. Three regridding methods are considered: linear interpolation, direct decomposition and direct integration. The resulting regridded fields are shown in figures 8(b)–(d), while the spectra are shown in figures 8(f)–(h).

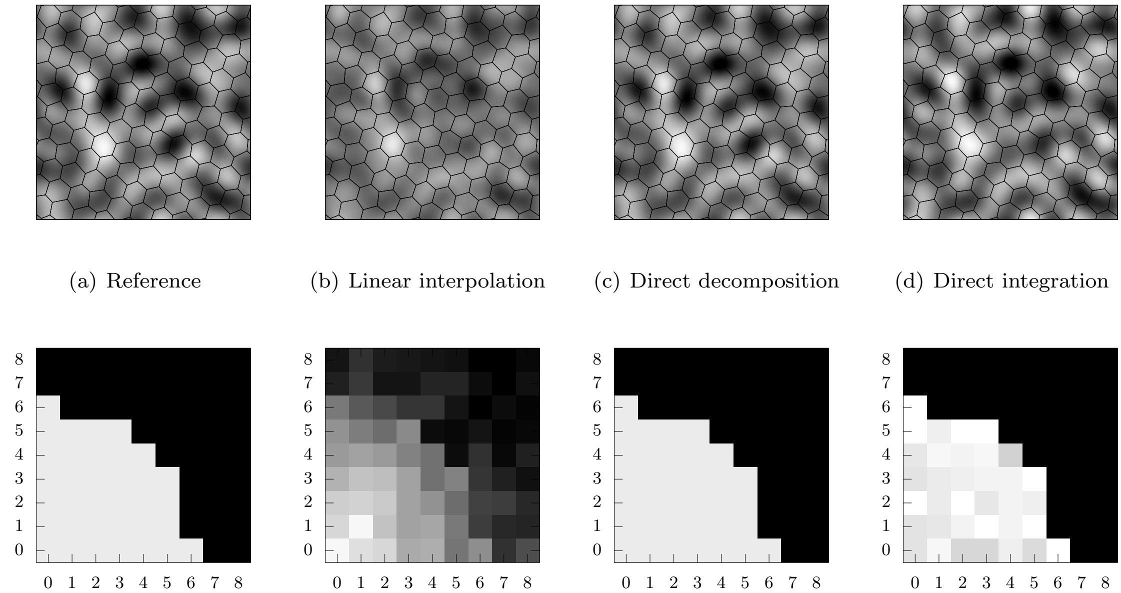
Similar to the previous example, the linear interpolation method clearly damps the higher frequencies. The direct decomposition method yields the exact result, but requires the inversion of a large matrix (which would be infeasible for a real-scaled problem). For the direct integration method, the cell areas were used as the weighting coefficients in eq. (19). This method yields a quite accurate result at a relatively low computational cost.

4.6 Preliminary conclusions

Linear or cubic interpolation in gridpoint space is quite attractive from a computational point of view: it is quite efficient, and can be generally applied. However, one should keep in mind that the higher frequency content of a signal is damped by gridpoint interpolation.

Spectral interpolation methods do not suffer from this drawback, but are computationally more expensive. If the problem can be decoupled into a zonal and a meridional regridding problem, the gain in accuracy may justify the additional cost. A caveat is that one should not try to estimate spectral components that simply are not available in the original sampled data: for instance in case of a strongly nonuniform grid, the maximum frequency should be chosen well below $N/2$.

Of the two spectral interpolation methods, direct decomposition is probably the more accurate one in most cases, but also the more expensive. The direct integration method has a broader application domain due to its higher efficiency.



(a) Reference (b) Linear interpolation (c) Direct decomposition (d) Direct integration

(e) Reference (f) Linear interpolation (g) Direct decomposition (h) Direct integration

Figure 8: Results of regridding from a hexagonal grid to a rectangular grid: (a)–(d): gridpoint space; (e)–(h): spectral space.

It should be remarked that efficiency, generality and ease of use are probably more important features of an interpolation method than accuracy. Therefore, it seems unlikely that the presented spectral interpolation methods will find their way to operational NWP models.

5 Vertical interpolation

In view of the jungle of vertical coordinate systems that exist, it seems infeasible (and not very useful anyway) to integrate all systems into the FA file format.

Vertical interpolation in Arpege/Aladin is done linearly with pressure. In order to use the existing routines to vertically interpolate data delivered by other consortia, the pressure level of the fields thus needs to be calculated.

The **Hirlam model** and the **DWD global model** use pressure-based hybrid coordinates, so the pressure in a point is readily computable from the surface pressure and the η coordinate.

The **UKMO model** uses a height-based hybrid coordinate. The pressure variable in the UKMO model is the Exner function

$$\Pi = c_p \left(\frac{p}{p_s} \right)^{R/c_p}. \tag{21}$$

To determine pressure in a certain point, the surface pressure p_s and the distribution of the moist variables (which influence c_p and R) are required.

The pressure variable in the **COSMO LAM** model is the deviation p' from the pressure in the base state atmosphere. For the pressure-based hybrid coordinate η, the total pressure in a certain point is obtained as

$$p = p_0(\eta) + p', \tag{22}$$

with $p_0(\eta)$ defined in eq. (4). For the height-based coordinates μ and μ_s, the total pressure in a certain point is given by

$$p = p_0(z(\mu)) + p', \tag{23}$$

with $z(\mu)$ given in eqs. (5) and (6), and $p_0(z)$ prescribed by the definition of the base state of the atmosphere [COSMO, 2009, section 2.1]

6 Organization and coding

6.1 Configuration 901

The purpose of this configuration is to convert a GRIB2 file to an FA file. Therefore, following tasks should be carried out in configuration 901:

- read fields and grid description from GRIB2
- perform vertical interpolation to Arpege/Aladin vertical coordinate, using vertical interpolation methods of FullPOS.
- perform (spectral) destaggering of wind components
- perform spectral regridding (?)

To represent rotated spherical coordinates in the FA file format, some modifications are needed. It appears that spherical coordinates are already available (`ERPK` < 0) in the FA file format [Gril, 2006], and that some header parameters are not used in this configuration (`ELON0` $= 0$ and `ELAT0` $= 0$). The extension to rotated spherical coordinates by using these parameters to describe the position of the south pole seems to be straightforward.

6.2 Configuration 927

Horizontal interpolations will be carried out in this configuration. This will be necessary for the destaggering and regridding problems that cannot be treated in spectral space.

The interpolation methods will need to be adapted to enable the use of rotated spherical coordinates.

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A Basics of spectral decomposition

The core idea of spectral methods is to decompose a field into a set of orthogonal functions on a domain $\mathcal{D}$. For a Fourier transform, these are the harmonic functions e^{ikx}, for a Legendre transform, these are the (associated) Legendre polynomials $P_n^m(x)$. [1] Let's denote, in general, the decomposition of a field $f(x)$ as

$$f(x) = \sum_{k=1}^{n} \varphi_k \Phi_k(x) \tag{24}$$

For the sake of simplicity (but without loss of generality), let's assume that the function $\Phi_k(x)$ are scaled such that they are orthonormal:

$$\int_{\mathcal{D}} \Phi_j(x)\Phi_k(x)dx = \delta_{jk} = \begin{cases} 1 & \text{if } j = k \\ 0 & \text{if } j \neq k \end{cases} \tag{25}$$

In order to determine the spectral coefficients φ_k, the orthogonality of the function Φ_k is used:

$$\int_{\mathcal{D}} f(x)\Phi_k(x)dx = \sum_j \varphi_j \int_{\mathcal{D}} \Phi_j(x)\Phi_k(x)dx = \sum_j \varphi_j \delta_{jk} = \varphi_k. \tag{26}$$

In practice, the integral $\int_{\mathcal{D}} f(x)\Phi_k(x)dx$ is calculated with the FFT algorithm for a Fourier decomposition, or by Gauss-Legendre quadrature for a Legendre decomposition. Although very efficient, these algorithms impose constraints on the gridpoint layout: the FFT algorithm supposes that the grid points are uniformy distributed, and Gauss-Legendre quadrature integration requires that the gridpoints coincide with the roots of a Legendre polynomial (hence the non-uniform spacing of latitude circles in Arpege).

[1] Strictly speaking, the functions e^{ikx} are not orthogonal on the domain $\mathcal{D} = [0, 2\pi]$, but $\cos(kx)$ and $\sin(kx)$ are.