The alternative variable d_4 for vertical divergence in ALADIN-NH model

Petra Smolíková
Czech Hydrometeorological Institute, e-mail:petra.smolikova@chmi.cz

1. Introduction

Motivation

It was shown that the choice of prognostic variables has an important impact on the stability and accuracy of the model. In addition to the standard prognostic variables, temperature T, horizontal divergence D and logarithm of surface pressure $\ln \pi_s$, two additional non-hydrostatic (NH) variables may be chosen in several different ways. There are three choices available in the current ALADIN-NH code for the variable representing pressure departure, $(p-\pi)/\pi^*$, $(p-\pi)/\pi$ and $q=\ln(p/\pi)$, with different effect on the stability of integration.

The last NH prognostic variable is based on vertical divergence. There were already six different variants (denoted d_0, d_1,... d_5) of this variable implemented and tested in ALADIN-NH. We mention only some of them. The remaining variants are somehow on the "half-way" between them. In 2001, the vertical divergence itself

$$d_3 = -g \cdot \frac{p}{mRT} \cdot \frac{\partial w}{\partial \eta} = \frac{\partial w}{\partial z} \tag{1}$$

has been implemented in the ALADIN-NH model and has been proven to ensure better stability properties than the previously used pseudo-vertical divergence

$$d_0 = -g \cdot \frac{\pi^*}{m^* R T^*} \cdot \frac{\partial w}{\partial \eta} \tag{2}$$

with * denoting values of the semi-implicit (SI) reference state (see [4]). Let us consider the atmosphere as dry. We know that in the presence of steep orography, the stability is strongly affected by the so-called X-term, the nonlinear part of the 3-dimensional divergence D_3, i.e.

$$X = \frac{p}{mRT} \cdot \nabla\Phi \cdot \frac{\partial \mathbf{V}}{\partial \eta} \tag{3}$$

$$D_3 = D + d_3 + X$$

This term was treated in an iterative process which results in a temporal average applied not only to the linear part of D_3, but to the whole D_3; see [2].

An idea has emerged (P. Bénard according to MC2 model) that X could be included into the variable used for vertical divergence representation. Then D_3 will be purely linear, and the corresponding instability could be cured in 3-time-level (3TL) scheme without a need for expensive iterations, while in 2-time-level (2TL) we cannot get rid of the iterative process and need to cure the instability by the so-called predictor/corrector scheme anyway; see [6]. Experiments in the 2d NH version of ALADIN were made which showed that the choice of

$$d_4 = d_3 + X \tag{4}$$

instead of d_3 may prevent the model from blowing up if no iterative procedure is used in 3TL scheme; see [3]. Hence the decision was made to implement d_4 into the (3d) code as a prognostic variable. It revealed that for the implementation of d_4, an auxiliary prognostic variable is needed. The reason for this is the following :

For the computation of the right hand side of the prognostic equation (6) for the variable d_4, the term ∇w is needed. To compute this term from (9) we need indispensably ∇d_3. And to compute ∇d_3 from the prognostic variables we have to calculate ∇X. But ∇X cannot be obtained in gridpoint space diagnostically from the other variables, since it needs second derivatives $\nabla^2 T$, $\nabla^2 p$, ... (through $\nabla^2\Phi$) and it would be too expensive. On the other hand, ∇X cannot be calculated in spectral space from the other variables, since (3) is nonlinear.

Thus the necessity of the introduction of an additional spectral variable is justified. We decided to define the auxiliary spectral variable as the previous d_3. Then ∇d_3 is obtained easily in spectral space. The auxiliary spectral field is biperiodicized in both horizontal directions by cubic splines (in x direction in gridpoint space before direct transformations when we have access to the whole row for a given latitude, and in y direction in Fourier space after transposition when we have access to all latitudes for a given zonal wavenumber).

In this short description of d_4 implementation, we first mention how the moisture is included, which prognostic equations are used and how they are time discretised. Then we present some results obtained in the 2d vertical-plane version of ALADIN and in 3 dimensions on PYREX case. We make some conclusions.

Inclusion of moisture

All "d"-variables are defined as "dry", while moisture is included in the term X through the gas constant R, i.e. the following formulas are used

$$d_3 = -g\cdot\frac{p}{mR_dT}\cdot\frac{\partial w}{\partial\eta} = \frac{\partial w}{\partial z}$$

$$X = \frac{p}{mRT}\cdot\nabla\Phi\cdot\frac{\partial\mathbf{V}}{\partial\eta}$$

$$d_4 = d_3 + X$$

Since the true 3d-divergence is defined as

$$D_3 = D + X - g\cdot\frac{p}{mRT}\cdot\frac{\partial w}{\partial\eta}$$

we get the following relationship

$$D_3 = D + \frac{R_d}{R}\cdot d_4 + \left(1 - \frac{R_d}{R}\right)\cdot X \qquad (5)$$

Thus if moisture is included there is still a small nonlinear part of D_3, equal to $(R_d/R - 1)\,d_3$. We expect that this small part may not cause any instability.

Prognostic equations

There are several possible algorithmic approaches to the problem; see document [1] of Pierre Bénard for details. The analysis shows that some of them could result in a more stable solution than the others. Here only the variant denoted as "variant 2" is described for 3TL Eulerian scheme; see [5] for the specificity of other variants and temporal schemes. We restrict ourselves to purely adiabatic case. We start from the time-continuous system of prognostic equations derived in terms of d_4 and $q = \ln(p/\pi)$:

$$\frac{d(d_4)}{dt} = -g^2 \cdot \frac{p}{mRT} \cdot \frac{\partial}{\partial\eta}\left(\frac{1}{m} \cdot \frac{\partial p}{\partial\eta}\right) + g \cdot \frac{p}{mRT} \cdot \frac{\partial \mathbf{V}}{\partial\eta} \cdot \nabla w$$

$$-d_4 D_3 + d_4 D + X D_3 - X D + \frac{dX}{dt} \quad (6)$$

$$\frac{dq}{dt} = -\frac{C_p}{C_v} \cdot D_3 - \frac{1}{\pi}\frac{d\pi}{dt} \quad (7)$$

$$\frac{dT}{dt} = -\frac{RT}{C_v} \cdot D_3 \quad (8)$$

where $g\nabla w$ is given by

$$g\nabla w = g\nabla w_s + \int_\eta^1 \nabla\left(\frac{mRT}{p} \cdot d_3\right) d\eta \quad (9)$$

The remaining equations (for D and $\ln \pi_s$) are not touched by our considerations.

Time discretisation

Let $\mathcal{M}$ denote the matrix form of the r.h.s. of the fully compressible Euler equation system, and $\mathcal{L}^*$ its linearization around a dry, resting, horizontally homogenous isothermal reference stay ψ^* in hydrostatic equilibrium that is fully defined by the values of reference temperature T^* and reference surface pressure π_s^*. The linear model $\mathcal{L}^*$ may be written in the following form

$$\frac{\partial d_4}{\partial t} = -\frac{g^2}{R_d T^*} \cdot \mathbf{L}^* q$$

$$\frac{\partial q}{\partial t} = -\frac{C_{pd}}{C_{vd}} \cdot (D + d_4)$$

$$\frac{\partial T}{\partial t} = -\frac{R_d T^*}{C_{vd}} \cdot (D + d_4)$$

plus two equations (for D and $\ln \pi_s$) not touched by our problem; see [2] for the definition of the vertical operator $\mathbf{L}^*$. Let ψ^-, ψ^0, ψ^+ denote the state vector at time t-Δt, t and t+Δt. If the 3TL SI time scheme is employed, we have

$$\frac{\psi^+ - \psi^-}{2\Delta t} = \mathcal{L}^*\left(\frac{\psi^+ + \psi^-}{2}\right) + (\mathcal{M} - \mathcal{L}^*)(\psi^0)$$

We use a first order approximation to dX/dt and get the following discretised prognostic equation for d_4

$$\frac{d_4^+ - d_4^-}{2\Delta t} = -\frac{g^2}{R_d T^*}\cdot \mathbf{L}^*\left(\frac{q^+ + q^-}{2} - q^0\right) - g^2\left(\frac{p}{mRT}\right)^0 \frac{\partial}{\partial\eta}\left(\frac{1}{m^0}\cdot\frac{\partial p^0}{\partial\eta}\right)$$

$$+\left(\frac{p}{mRT}\right)^0 \left(\frac{\partial \mathbf{V}}{\partial\eta}\right)^0 \cdot g\nabla w^0 - d_4^0 D_3^0 + d_4^0 D^0 + X^0 D_3^0 - X^0 D^0 + \frac{X^0 - X^-}{\Delta t}$$

The term X is calculated by (3) and D_3 by (5), both from prognostic variables at corresponding time level. Moreover, $g\nabla w^0$ is calculated as

$$g\nabla w^0 = g\nabla w_s^0 + \int_\eta^1 \left(\frac{mRT}{p}\right)^0 \cdot \nabla d_3^0 \cdot d\eta - \int_\eta^1 d_4^0 \cdot \nabla\left(\frac{\partial\phi}{\partial\eta}\right)^0 \cdot d\eta + \int_\eta^1 X^0 \cdot \nabla\left(\frac{\partial\phi}{\partial\eta}\right)^0 \cdot d\eta$$

Hence, d_3 appears only in derivatives ∇d_3^0 in prognostic equation for vertical divergence (d_4) and nowhere else.

We derive a "pseudo-prognostic" equation for d_3 from

$$\frac{d_3^+ + d_3^-}{2} = \frac{d_4^+ + d_4^-}{2} - X^0$$

as

$$d_3^+ = d_4^+ + [d_4^- - d_3^- - 2\,X^0]$$

The part in brackets is calculated in gridpoint space and then the final addition is made in spectral space after d_4^+ is obtained.

Variable d_4 has been introduced in cycle 25T2, and it will be enabled to use it with 3TL predictor-corrector (PC) scheme in the next cycle (26).

2. 2d vertical plane experiments

Experiments settings

We are using the experimental setting that was prepared and used by Jan Masek for the evaluation of the newly proposed NH prognostic variables d_4 and d_5 ("easy" implementation in 2d vertical plane version of the cycle CY24T1/AL15). It will enable us to compare our results with previously obtained ones. Let us briefly summarize, see report [3] for details.

- initial state : temperature profile with Brunt-Väisälä frequency $N = 0.01\ s^{-1}$ up to tropopause (sea level temperature 293 K), isothermal above (133 K); constant wind profile $V = 10\ m.s^{-1}$; sea level pressure 1013.25 hPa
- orography : bell-shaped mountain with height h and half-width a depending on the regime
- geometry : 128 horizontal points (114 in C, 14 in I, 0 in E zone), quadratic grid (NSMAX=42), 100 regular z-levels, 30 or 60 levels above tropopause (depending on the regime), total vertical extent ≈30 km
- integration settings : T^*=220 K, π_s^*=900 hPa, no decentering, no horizontal diffusion, semi-Lagrangian (SL) advection treatment

■ regime dependent settings :

regime	Δx [m]	a [km]	h [m]	hill [gp]	C_{LH}	`REPONBT` [m]	`REPONTP` [m]	`REPONTAU` [s]	timestep [s] 3TL	timestep [s] 2TL
LH	2000	10	100	64	0.1	21 000	29 500	2 000	50.	100.
QLQH	1000	5	200	64	0.2	21 000	29 500	1 000	25.	50.
NLNH 1	400	2	500	32	0.5	12 000	29 500	400	10.	20.
NLNH 2	200	1	1000	32	1.0	12 000	29 500	200	5.	10.

LH : linear hydrostatic, QLQH : quasi-linear quasi-hydrostatic, NLNH : nonlinear non-hydrostatic

Table 1 : 2d experiments setting

We evaluate the dimensionless flow dependent parameter C_{LH} which measures the linearity and hydrostatism at the same time, $C_{LH} = N\,h/V = V\,/\,(N\,a)$. The growth of this parameter from 0 to 1 represents the move from a linear hydrostatic regime to a nonlinear non-hydrostatic one.

We consider an experiment as stable if 1000 timesteps are completed without respect to the length of timesteps. We adopt the convention that the numbers of completed timesteps of experiments with acceptable results are written in tables with bold fonts.

For 2TL scheme, we used PC scheme with one iteration (`LPC_FULL=TRUE`, `NSITER=1`), either extrapolating (`LPC_NESC=FALSE`), or not (`LPC_NESC=TRUE`). Without any iteration, the model is unstable even in LH regime independently from the choice of prognostic variable for vertical divergence (with our settings, only 10-13 completed timesteps).

Results

number of completed timesteps										
		3TL , SI		2TL , PC		2TL , PC , NE				
"d"								+`XIDT`=0.1		
var.	C_{LH}}	σ	η	σ	η	σ	η	σ	η
d_3	0.1 (LH)	**1000***	**1000***	**1000**	**1000**	**1000**	**1000**	—	—
	0.2 (QLQH)	**1000***	**1000***	**1000**	**1000**	**1000**	**1000**	—	—
	0.5 (NLNH1)	**1000***	**1000***	81	108	**1000**	**1000**	—	—
	1.0 (NLNH2)	44*	55*	14	14	20	21	53	72
d_4	0.1 (LH)	**1000**	**1000**	**1000**	**1000**	**1000**	**1000**	—	—
	0.2 (QLQH)	**1000**	**1000**	**1000**	**1000**	**1000**	**1000**	—	—
	0.5 (NLNH1)	**1000**	**1000**	**1000**	**1000**	**1000**	**1000**	—	—
	1.0 (NLNH2)	43	1000	104	109	1000	1000	1000	1000

* experiments done by Jan Masek, see [3]

Table 2 : 2d experiments

There is a clear indication of enhanced stability of 2TL PC scheme with d_4 in comparison to d_3 in NLNH regimes. In the most severe regime (NLNH2), for any chosen type of 2TL PC scheme and for 3TL SI scheme the d_4 implementation is more stable than d_3. Neither non-extrapolating 2TL PC scheme (previously in NLNH1 more stable than the full PC), nor 2TL PC with pseudo second-order decentering (ξ = 0.1) is able to stabilize the integration with d_3. For d_4 we got a stable integration with 2TL PC NE and 2TL PC NE with decentering (ξ = 0.1), but the accuracy of obtained solutions is touched.

3. 3d experiments - PYREX case

Experiments settings

For 3d experiments, we have chosen the real case named PYREX from 16.4.2002, 12 UTC, with domain placed over Pyrénées mountains and the following settings :

- geometry : 277×181 horizontal points (269×173 in C, 8 in each direction in I, 0 in E zone), quadratic grid (`NMSMAX`=92, `NSMAX`=60), Δx =2635 m, 41 vertical levels with hybrid η-coordinate
- integration settings : T*=350 K for 3TL and T*=300 K for 2TL schemes, π_s*= 1013.25 hPa, horizontal diffusion applied, SL advection
- timestep : Δt =15 s for 3TL scheme, Δt =30 s for 2TL scheme;

we have integrated for 2 hours, i.e. 480 timesteps for 3TL and 240 timesteps for 2TL schemes.

We use as a reference experiment the run with d_3 and 3TL SI scheme in the same setting as for the other experiments, but with a 3-times shorter timestep (Δt = 5 s).

We adopt the following criterion for stability: we consider an experiment as a stable one if the forecast for 2 hours is performed. This criterion is a compromise between an ideal stability criterion (the forecast may be run forever) and the time requirement for experiments we are able to accept.

We can see on the example of experiment with 3TL SI scheme and d_4 variable that this criterion may not give evidence of stability: this experiment may be considered as perfectly stable after 1 hour of forecast and it crashes shortly after (at 1:17:30).

Results

number of completed timesteps					
"d"-variable	3TL SI		2TL PC	2TL PC NE	
	—	`VESL`=0.1	—	—	`XIDT`=0.1
d_3	83	480	13	19	240
d_4	281	310	240	240	240

Table 3 : 3d experiments - PYREX

For all used time schemes the choice of d_4 gives more stable integration than the choice of d_3. For 3TL SI scheme with first order decentering, one can see on the fields plotted after 1 hour of integration (Fig. 1) that with d_3 there is a noise spread over a large mountainous area, while with d_4 there is almost no noise except for one small spot. This spot makes the model blow up after another 70 timesteps, while with d_3 the whole 480 timesteps are completed, but the resulting fields are extremely noisy (we conclude also here that the system with d_4 is more stable than with d_3).

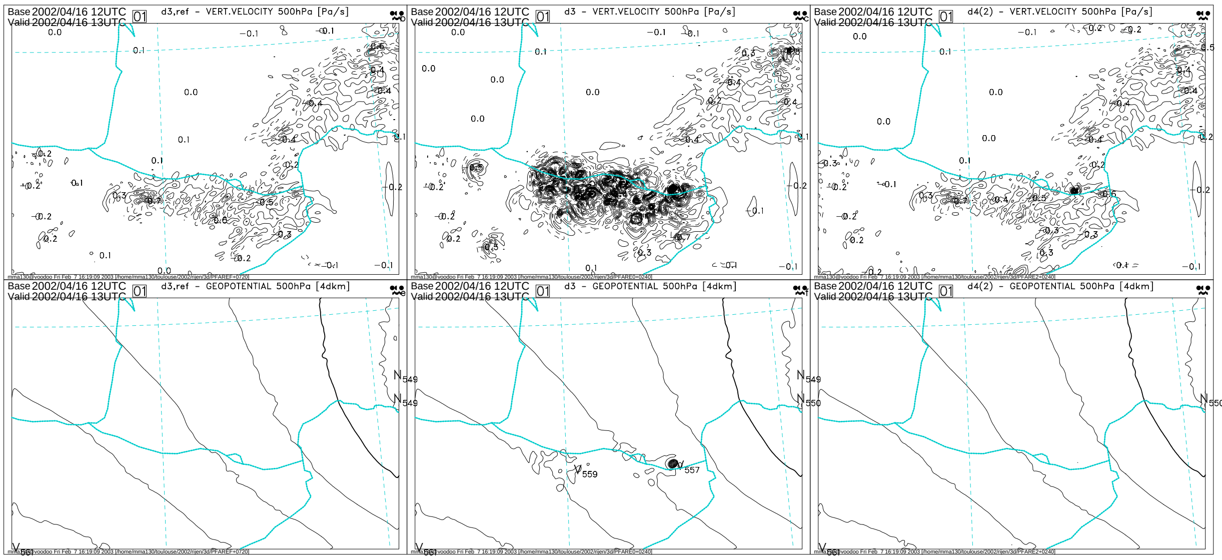

Figure 1 : Vertical velocity (up) and geopotential (down) at 500 *hPa*, using 3TL SI time scheme with decentering (ε = 0.1), forecast for 1 hour. From left to right: reference experiment with d_3, experiment with d_3 and experiment with d_4.

For 2TL PC scheme, the integration with d_3 becomes stable only after the introduction of a strong pseudo second-order decentering (ξ = 0.1). In the field of vertical velocity, we can see bigger noise on the top of the domain (50 *hPa*) in the case of extrapolating PC scheme in comparison to NE PC. This noise is removed completely by decentering. In other features, the fields are very similar for both variables; see fig. 2.

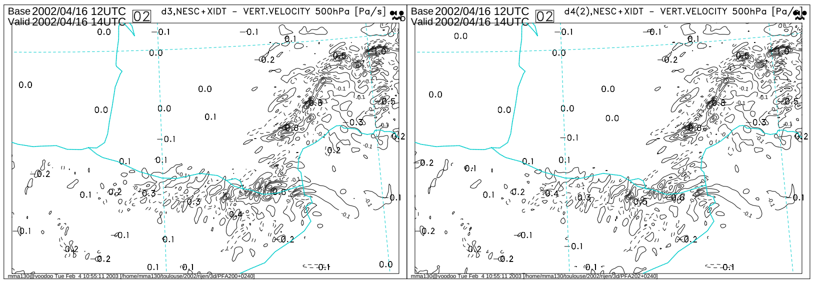

Figure 2 : Vertical velocity at 500 *hPa*, with 2TL PC NE and `XIDT`=0.1.
Left : experiment with d_3. Right : experiment with d_4.

4. CPU time & memory aspects

For all tested integration schemes, the usage of d_4 requires about 5 % extra memory and about 5 % extra CPU-time in comparison to the same integration run with d_3. A remarkable retardation appears at the level of setup. This retardation is caused by the conversion procedure from the initial historical file (where $g\,\partial w/\partial\eta$ is saved) to the model variable. The reason for this is that for d_4 in contrast to the other possible "*d*-variables", spectral transforms are needed for the computation of the term X that are very time-demanding. An optimization of the code for better vectorization capacity could improve the integration time & memory statistics.

5. Conclusions

From the set of carried out experiments we can conclude that the system with d_4 used as variable for vertical divergence shows enhanced stability in comparison with the system using d_3. The stabilizing effect is more pronounced in experiments with steeper orography. Better stability properties of d_4 have to be judged against more complicated model code and CPU time & memory requirements of the d_4 implementation.

References

[1] P. Bénard : Four slightly different variants for d_3, d_4 implementation, *Météo-France internal report*.

[2] R. Bubnová, G. Hello, P. Bénard and J.F. Geleyn, 1995 : Integration of the fully elastic equations cast in the hydrostatic pressure terrain-following coordinate in the framework of the ARPEGE/ALADIN NWP system, *Mon. Wea. Rev.*, **123**, 515-535.

[3] J. Masek : Evaluation of newly proposed NH prognostic variables II, *Report from stay in Prague*, February - March 2002.

[4] P. Smolíková : New strategies for NH temporal scheme, *Report from stay in Toulouse*, February - March 2001.

[5] P. Smolíková : New NH variable d_4, part I : implementation in the cycle CY25T1, *Report from stay in Toulouse*, February - March 2002.

[6] J.Vivoda : Analysis of stability of 2TL SI non-extrapolating predictor/corrector scheme in the limit of infinite time step, *ALADIN Newsletter* **21**, 2001.