Ocean circulation: quiz 1

This quiz aims at testing your basic knowledge of geophysical fluid dynamics and the associated mathematical background. It will be highly useful for a good understanding of Chapter 2 and Chapter 3 of the lecture. Please give a mathematical answer together with its interpretation in plain words.

1. **Gradient.** Given a physical scalar such as temperature $T(x, y, z, t)$, define its gradient vector. What is its physical meaning?

2. **Divergence.** Given the velocity vector $\mathbf{v}(x, y, z, t)$, define its divergence. What is its physical meaning?

3. **Vorticity.** Given the velocity vector $\mathbf{v}(x, y, z, t)$, define its vorticity. What is its physical meaning?

4. **Material (Lagrangian) versus local (Eulerian) derivative.** Given temperature $T(x, y, z, t)$, give the notation and physical meaning of its Lagrangian and Eulerian temporal derivatives.

5. **Chain rule of derivation.** Given a function $f(x, y, z, t)$, give the chain rule for its temporal derivation $\frac{df}{dt}$ as a function of its partial derivatives $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, $\frac{\partial f}{\partial z}$ and $\frac{\partial f}{\partial t}$. Deduce the mathematical relation between Lagrangian and Eulerian accelerations. Interpret their difference.

6. **Taylor series expansion.** Given a function $f(x, y)$, express its first order Taylor expansion and interpret its meaning. Deduce a simplified expression for density $\rho(T, S)$ as a function of salinity and temperature, with respect to a reference state $\rho_0(T_0, S_0)$.

7. **Conservation laws.** Which one of these quantities does not dispose of a law of conservation?

   - Mass.
   - Heat.
   - Pressure.
   - Momentum.

8. **Mixing.** Let two incompressible water masses of volume V_i $(i = 1, 2)$, temperature T_i and salinity S_i mix entirely. Deduce from conservation of mass (for salt) and heat the temperature T and salinity S of the resulting water mass.

9. **Reynolds averaging.** Given a variable x, define its Reynolds average and perturbation. Express a heat transport uT as a function of the Reynolds average and perturbation of velocity u and temperature T. How do you interpret each term?

10. **Reynolds number.** Recall the expression for the Reynolds number. What terms of the momentum equation (Newton's 2nd law) does it compare? What is its typical value for large-scale oceanic motion, and how do you interpret it?

11. **Rossby number.** Recall the expression for the Rossby number. What terms of the momentum equation does it compare? What is its typical value for large-scale oceanic motion, and how do you interpret it?

12. **Hydrostatic balance.** Recall the hydrostatic relation. What is its physical interpretation? How well is it verified in the ocean?

13. **Geostrophic balance.** Recall the geostrophic balance. What is its physical interpretation? Where does it hold in the ocean?

14. **Ekman balance.** Recall the Ekman balance. What is its physical interpretation? Where does it hold in the ocean?