# ESTIMATION OF TURBULENCE KINETIC ENERGY DISSIPATION RATE IN A BORA EVENT

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**Abstract:** During a bora event, the turbulence is strongly developed in the lee of the mountain. Wind speed at Senj was measured at 13 m above the ground with a 3D ultrasonic anemometer operating at 4 Hz sampling frequency. This data was used for evaluation of the turbulent kinetic energy (TKE) dissipation rate $\varepsilon$. The estimated value of $\varepsilon$ with the inertial dissipation method (IDM) for this event is 1.22 $m^2s^{-3}$ and is taken as the reliable one. An alternative method is based on the third-order streamwise velocity structure function (3SF) which requires a local turbulence isotropy. Using this approach a significantly lower value of $\varepsilon$ is obtained, not surpassing 0.70 $m^2s^{-3}$. Inspecting the stronger isotropy condition for the inertial subrange, i.e. the 4:3 ratio of vertical and transverse to streamwise velocity spectra, it is found that the present anemometer's sampling rate only approaches the inertial subrange. This partly explains the lower value of $\varepsilon$ obtained using the second method. Thus, the anemometer of a much higher sampling frequency is needed for further studies of the bora-related small-scale turbulence.

**Keywords:** bora wind, dissipation rate, inertial dissipation technique, structure function, local isotropy

## 1. INTRODUCTION

Bora (locally *bura*) is a downslope windstorm that blows at the eastern Adriatic coast from the northeast quadrant. It is formed in the lee when the relatively cold northeasterly flow impinges on the Dinaric Alps. It is most frequent during the winter season with duration from several hours to several days. It possesses a wide spectrum of average wind speeds, and due to gustiness the speed maxima may surpass 60 m/s (e.g. Belušić and Klaić, 2006). During a bora event, because of its dynamics and strong winds, the turbulence is strongly developed in the lee of the mountain. The focus here is on high frequency bora turbulence data which has been addressed in some details only recently (e.g. Belušić et al., 2006).

## 2. DATA AND METHODS

The 3D wind speed measurements were performed at Senj (44.99°N, 14.90°E, 2 m above MSL) at a height of 13 m above the ground with the WindMaster ultrasonic anemometer (Gill Instruments). The anemometer records the data with a sampling frequency of 4 Hz. The observed bora episode extends from 07 to 11 January 2006 (4-day time series). The mean wind direction during this event is 56.44°. The coordinate system is rotated in the mean direction and the horizontal wind is decomposed to along (streamwise) and cross (transverse) wind components. Figure 1a depicts the measured 4-day time series of the streamwise velocity component with 1 hr mean superimposed. The gray segment is one of the strongest 4-hour intervals in this event and is chosen for further analysis (zoomed, with 10 min mean superimposed in Figure 1b).

### 2.1. Inertial dissipation method

In order to evaluate the turbulent kinetic energy (TKE) dissipation rate $\varepsilon$, the inertial dissipation method (IDM) provided by the Kolmogorov's 1941 hypotheses can be employed. It requires that the Taylor's hypotheses on frozen turbulence (TH) be valid which is needed for transformation from time (frequency) to space (wavenumber) domain. The criterion for the validity of TH to is $\sigma_M < 0.5M$, where M is the mean wind speed and $\sigma_M$ is the standard deviation (e. g. Stull, 1988).

According to Kolmogorov, power spectrum follows the -5/3 law in the inertial subrange:

$$\log[S_u(k)] = -\frac{5}{3}\log k + \log(\alpha\varepsilon^{2/3}) \quad (1)$$

where the usual units are valid but neglected for simplicity in (1).

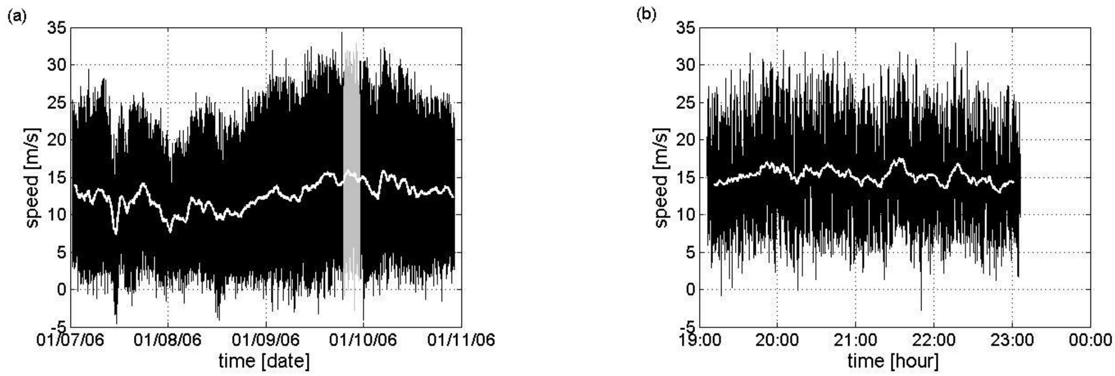


**Figure 1:** (a) 4-day raw time series (from 07 to 11 January 2006) of the streamwise-wind component measured at Senj, Croatia with its 1 hr mean superimposed. (b) 4-hour interval (gray area in Fig.1a) chosen for the analysis with the 10 min mean superimposed.

Using TH, $\varepsilon$ can be evaluated from (Champagne et al., 1977):

$$\varepsilon = \frac{2\pi}{\overline{U}}\left[\frac{f^{5/3}S_u(f)}{\alpha}\right]^{3/2} \tag{2}$$

where $\overline{U}$ is the mean streamwise velocity component, $S_u(f)$ is the spectrum and $\alpha$ is the Kolmogorov constant for the velocity component.

According to Batchelor (1959) and Champagne (1978), a strong statement of local isotropy and the existence of inertial subrange are given by the 4:3 ratios of spectra of transverse ($v$) to streamwise ($u$) and vertical ($w$) to streamwise ($u$) component. Figure 2a shows that the ratio of spectra of $v$ to $u$ approximately reaches the theoretical value of 4/3 for $f > 0.78$ Hz, while $w$ to $u$ ratio (Fig.2b) reaches 4/3 only at the end of frequency band (which is the Nyquist frequency of 2 Hz). Therefore, it is obvious that the isotropic inertial subrange is not comprised within current frequency limits.

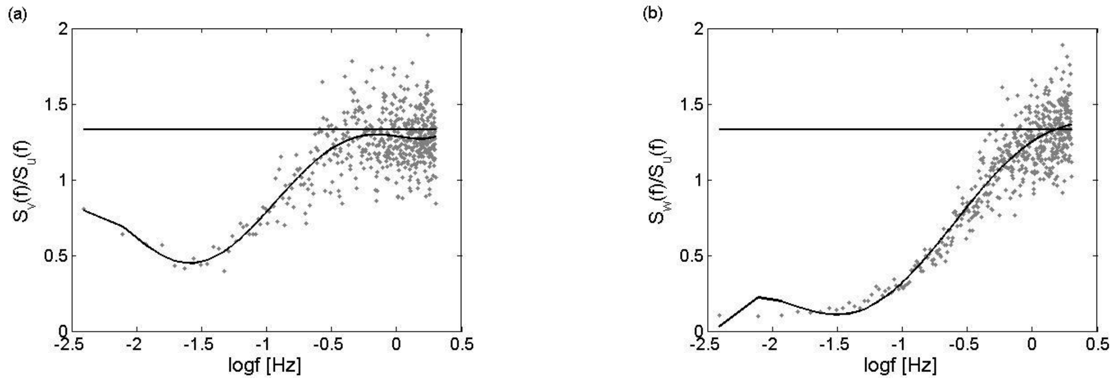


**Figure 2:** A plot of the ratio between: (a) The transverse $v$ and streamvise $u$ spectra; (b) The vertical $w$ and streamwise $u$ spectra showing the approach to the 4:3 ratio required by isotropy. The data are from Fig.1b.

However, several authors (e. g., Champagne, 1978; Mestayer, 1982) show that the -5/3 law for the streamwise component spectrum can often be extended outside of the inertial subrange. This information will be applied on the data from Fig.1b.

## 2.2. Kolmogorov's four-fifths law

An alternative method for evaluation $\varepsilon$, applicable only to the inertial subrange in a strict sense and requiring isotropy, is based on the third-order streamwise velocity structure function (3SF), i.e. the use of the Kolmogorov's four-fifths law (e.g. Piper and Lundquist, 2004) :

$$3SF = \overline{\Delta u^3} = \overline{[u(x+r) - u(x)]^3} = -\frac{4}{5}\varepsilon r \tag{3}$$

where $r$ represents the separation distance between the two measurements. The TH is used to convert (3) to the time domain. If $r = \overline{U}\tau$, where $\tau$ represents the time lag between the two measurements, then (3) can be rearranged to provide:

$$\varepsilon = \frac{5}{4}\left(\frac{1}{\overline{U}\tau}\right)\overline{\Delta u^3} \qquad (4)$$

The condition on both r or $\tau$ is that they must lie in the inertial subrange.

## 3. RESULTS

The standard deviation of the total wind speed for the chosen 4-hour interval is $\sigma_M$ = 0.35M, so the application of TH is justified. The Kolmogorov constant $\alpha$ = 0.53 (Champagne, 1978; Oncley et al., 1996; Piper and Lundquist 2004) is used in (2). The value of $\varepsilon$ obtained with the IDM for this interval whose $\overline{U}$ amounts to 15.1 m/s, equals 1.22 $m^2s^{-3}$. The log-log display of the spectrum is presented in Figure 3 showing the -5/3 behavior outside and nearby the inertial subrange.

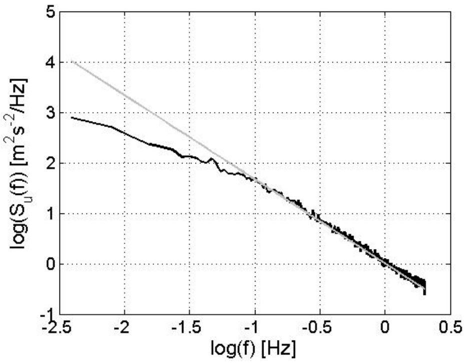


**Figure 3:** Log-log representation of the streamwise velocity power spectrum density (black curve) vs. frequency for the 4-hour bora interval studied here (from Fig. 1b). The gray line is the -5/3 slope.

The smallest $\tau$ allowed in our data for the use in 3SF is 0.25 s. Further analysis assumes $\tau$ to be in the range of 0.25 to 1.75 s. According to the Kolmogorov's 4/5 law given in (3), the linear dependence of 3SF on $\tau$ is expected within the inertial subrange. That is exactly what Figure 4a indicates, although the isotropic range is not reached here. Figure 4b shows the behavior of $\varepsilon$ calculated from (4) with the increasing time lag. The mean value of this estimation equals 0.70 $m^2s^{-3}$ and is taken as the estimation of $\varepsilon$ with this method.

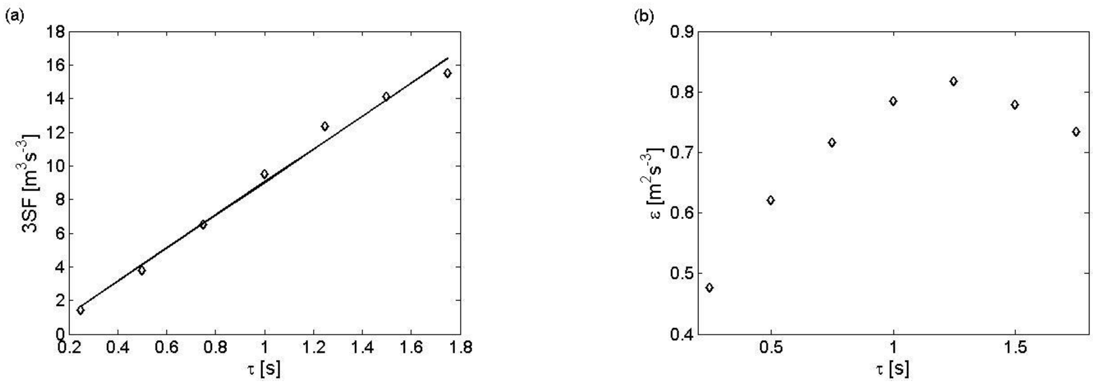


**Figure 4:** (a) The dependence of third order structure function on time lag $\tau$. The line denotes the linear fit.
(b) Dependence of the TKE dissipation rate $\varepsilon$ on time lag $\tau$. The data are from Fig.1b.

Both methods presented above are applied to additional five 4-hour intervals with different values of $\overline{U}$ extending from approximately 9 to 14 m/s. The goal was to gain insight into the dependence of $\varepsilon$ on $\overline{U}$. Figure 5 shows preliminary results of this analysis.

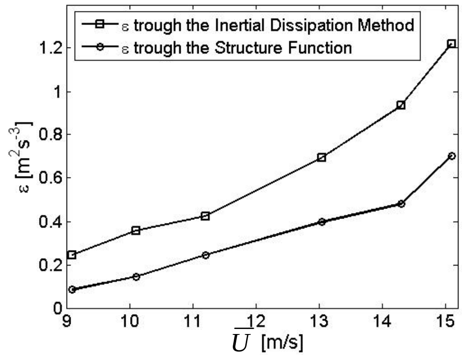


**Figure 5:** The dependence of the TKE dissipation rate $\varepsilon$ on mean streamwise velocity component $\overline{U}$ . The squares are obtained with the inertial dissipation method and the circles with the third order structure function.

The mean value of the ratio between $\varepsilon$ calculated with (2) and (4) for each $\overline{U}$ equals 2.08. This preliminary result indicates that the 3SF underestimates $\varepsilon$ by about 50 %.

## 4. CONCLUSION

Estimation of the TKE dissipation rate, $\varepsilon$ , is presented for a typical bora case in Senj, Croatia, using 4 Hz wind data. Although the isotropic inertial subrange was not comprised, it is seen that the -5/3 slope extends sufficiently towards lower frequencies that are roughly comparable to our sampling frequency. Therefore, the estimation of $\varepsilon$ using inertial dissipation method, IDM, which for the data from Fig.1b equals 1.22 $m^2s^{-3}$, can be considered reliable. Another method based on third-order streamwise velocity structure function, 3SF, is applicable only in the well-defined inertial subrange (i.e. where the requirement of turbulence isotropy is met). This partly explains the lower value of $\varepsilon$ (0.70 $m^2s^{-3}$) obtained using this method. Even outside of the inertial subrange, the linear dependence of 3SF on the time lag $\tau$ is present.

Preliminary results of the dependence of $\varepsilon$ on $\overline{U}$ show the similar behavior for both methods deployed, Figure 5. This suggests that the 3SF underestimates $\varepsilon$ by about 50 % compared to IDM. As expected, $\varepsilon$ increases with increasing $\overline{U}$ . A more detailed exploration of this issue is needed.

Measurements with the higher sampling frequency that would enable the study of the inertial subrange, is needed for more conclusive results on the value of $\varepsilon$ for the bora wind turbulence.

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