D.3. Boundary filters

Principle

- performing a short forward integration
- adjusting a smooth function to the trajectory
- starting from the initial (or final) point of the filtered forecast

Computation of weights, using simple polynomial functions

$$h_n = \sum_{k=0}^{K} \left(\frac{n}{N}\right)^k / c(0,k) \ , \quad c(k,l) = \left(\sum_{n=0}^{N} n^{k+l}\right) / N^{k+l} \ + \ \delta_{k,l}\, \lambda_k \left(\frac{k!}{N^k}\right)^2$$

minimizing

$$J = \tfrac{1}{2} \sum_{n=0}^{N} \left|\bar{x}(n\Delta t) - x(n\Delta t)\right|^2 \ + \ \tfrac{1}{2} \sum_{p=0}^{P} \lambda_p \left|x^{(p)}(0)\right|^2 \ , \qquad \bar{x}(0) = \sum_{n=0}^{N} h_n x(n\Delta t)$$

K is the order of the polynomial function, $N\Delta t$ the length of the forward integration.

Computation of weights, using a more complicated scheme (spline-type functions)

$$h_n = (\mathcal{M}^{-1})_{0,n} \ / \ \sum_{p=0}^{N} (\mathcal{M}^{-1})_{0,p}$$

with $\mathcal{M}$ *[-2:N+2,-2:N+2]* filled with 0 apart from :

$$\mathcal{M}_{n,n} = 6 \ + \ \gamma_c \ , \ \mathcal{M}_{n,n+1} = \mathcal{M}_{n,n-1} = -4 \ , \ \mathcal{M}_{n,n+2} = \mathcal{M}_{n,n-2} = +1 \ \ , \ 0 \le n \le N$$

$$\mathcal{M}_{-1,-1} = \mathcal{M}_{N+1,N-1} = +1 \ , \ \mathcal{M}_{-1,0} = \mathcal{M}_{N+1,N} = -2 \ , \ \mathcal{M}_{-1,+1} = \mathcal{M}_{N+1,N+1} = +1$$

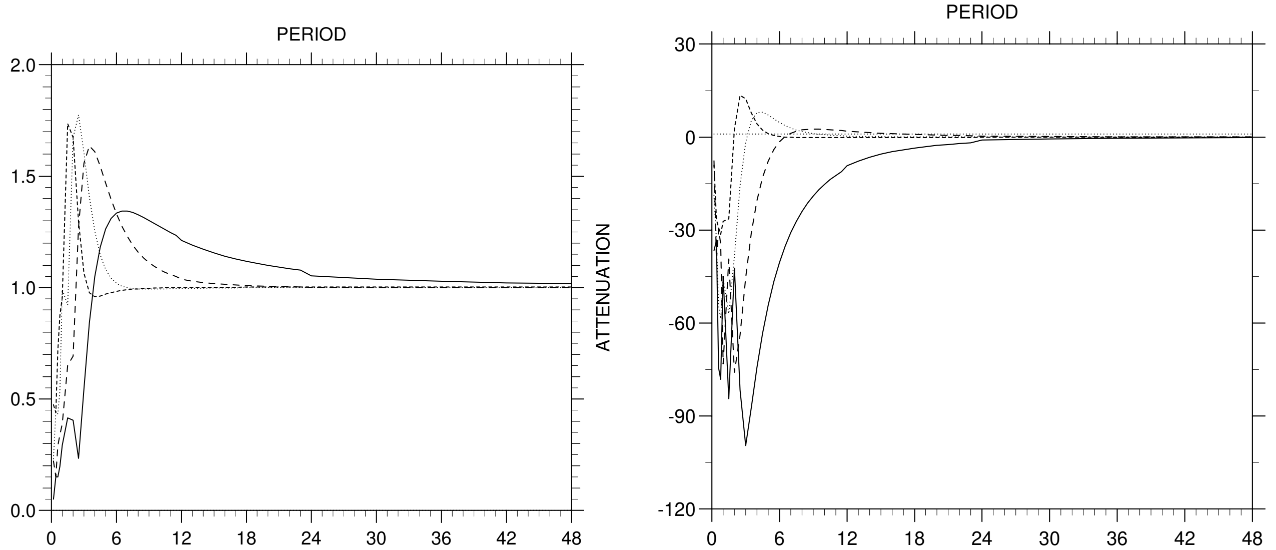
$$\mathcal{M}_{-2,-2} = \mathcal{M}_{N+2,N-2} = -\tfrac{1}{2} \ , \ \mathcal{M}_{-2,-1} = \mathcal{M}_{N+2,N-1} = +1 \ , \ \mathcal{M}_{-2,0} = \mathcal{M}_{N+2,N} = 0 \ ,$$

$$\mathcal{M}_{-2,+1} = \mathcal{M}_{N+2,N+1} = -1 \ , \ \mathcal{M}_{-2,+2} = \mathcal{M}_{N+2,N+2} = +\tfrac{1}{2}$$

$$\sum_{p=-2}^{N+2} \mathcal{M}_{n,p} \cdot x(p\Delta t) \ \approx \ \gamma_c \cdot x(n\Delta t) + (\Delta t)^4 \cdot x^{(4)}(n\Delta t) \ \ , \ \ \gamma_c = \left(2\pi\Delta t / T_c\right)^4$$

$$\sum_{p=-2}^{N+2} \mathcal{M}_{-1,p} \cdot x(p\Delta t) \ \approx \ (\Delta t)^2 \cdot x^{(2)}(0) \ \ , \ \ \sum_{p=-2}^{N+2} \mathcal{M}_{-2,p} \cdot x(p\Delta t) \ \approx \ (\Delta t)^3 \cdot x^{(3)}(0) \ , \ \ \text{idem at } N\Delta t$$

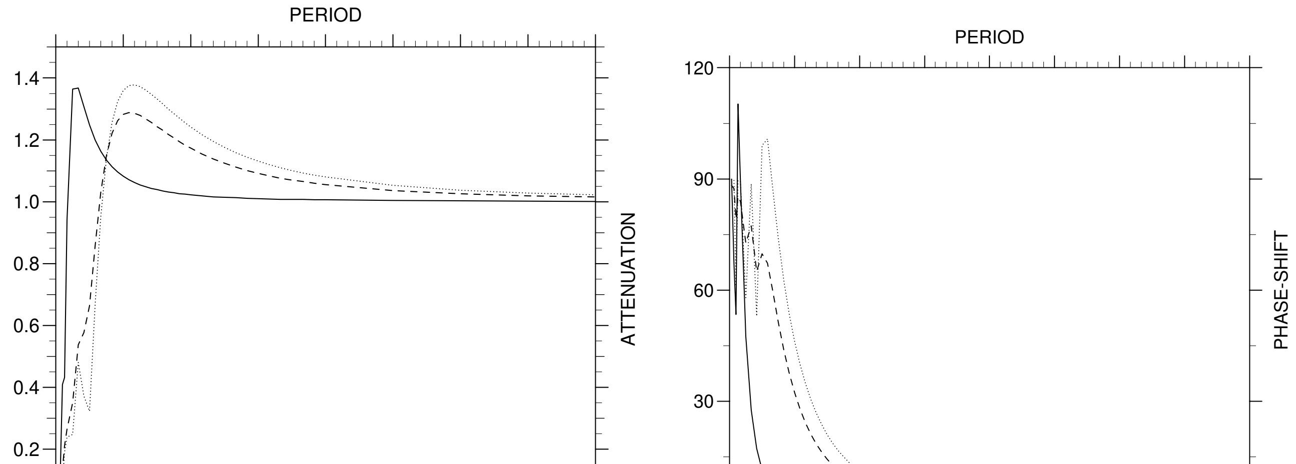
Example of effective responses (polynomial fit)



Full line : order 1 - Long-dashed line : order 2 - Dotted line : order 3 - Short-dashed line : order 4

Time-span : 4 h ; Time-step : 600 s (24 steps)

Example of effective responses (splin fit)



Full line : Tc= 6 h ; Time-span =1 h
Dashed line : Tc = 6 h ; Time-span =4 h
Dotted line : Tc =12 h ; Time-span =4 h
Time-step : 180s