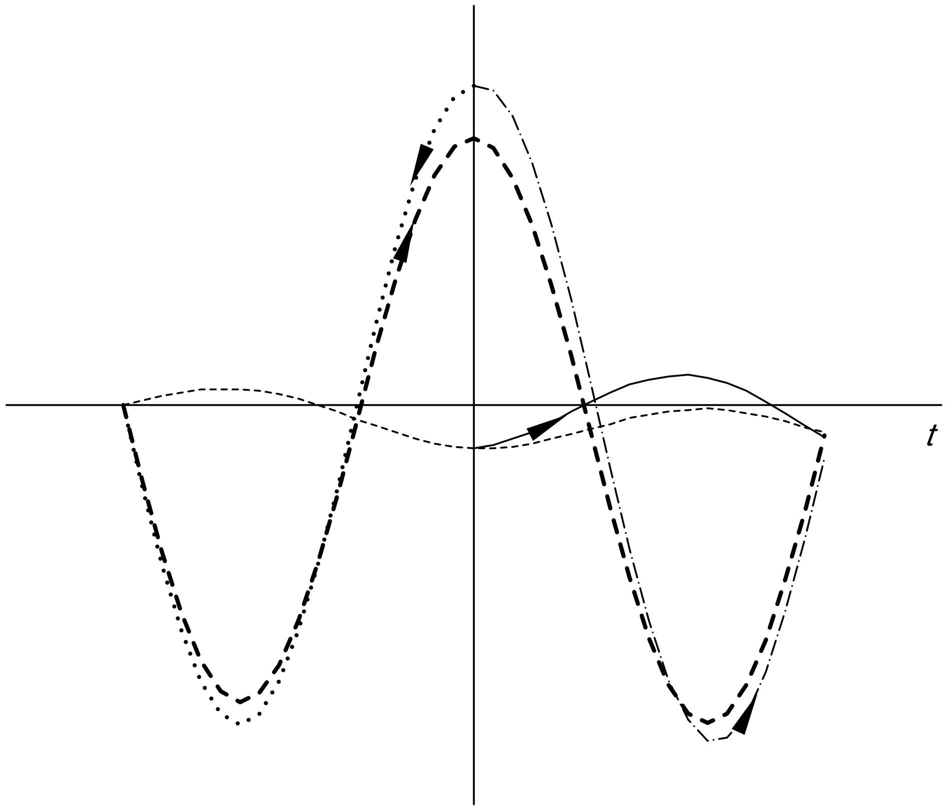
B.2. Comparaison of available schemes

Old scheme

➜ filtering once



LEGENDS :

1. Thick dotted line :
   adiabatic forecast
   from 0 to -T

2. Thick dashed line :
   full forecast
   from -T to +T

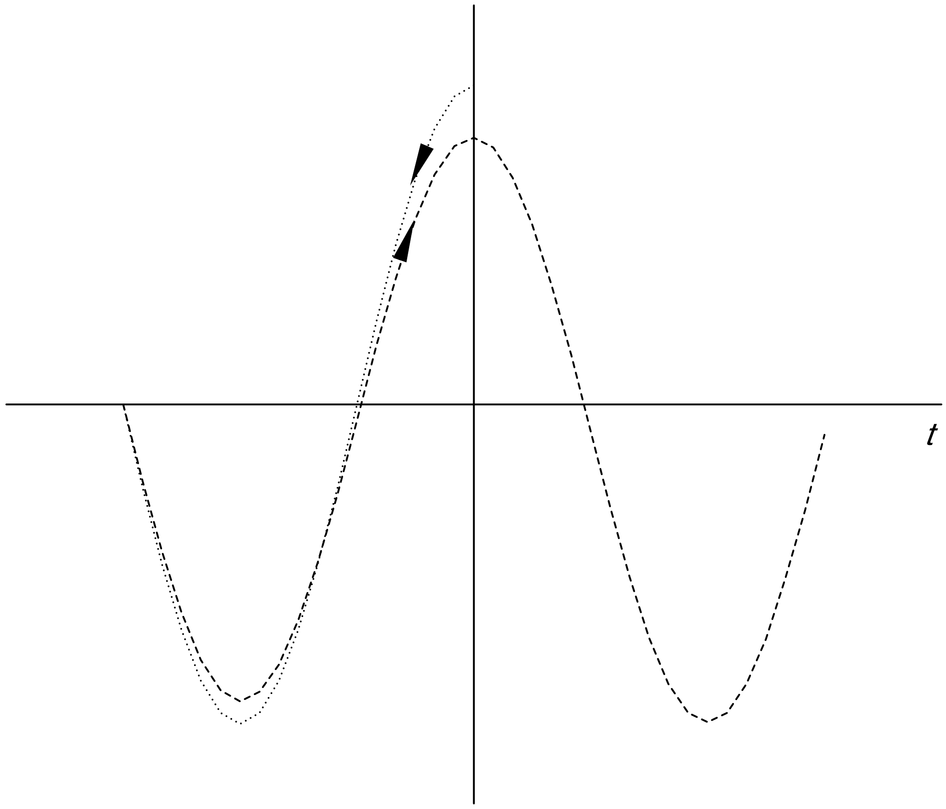
3. Thin dashed line :
   filtered forecast
   from -T to +T

4. Full line :
   final forecast
   starting from init. analysis

5. Dashed-dotted line :
   final forecast
   starting from analysis

(high frequency perturbation
+ low frequency forcing)

or, simpler,



1. Dotted line :
   adiabatic forecast
   from 0 to -T
   starting from the analysis

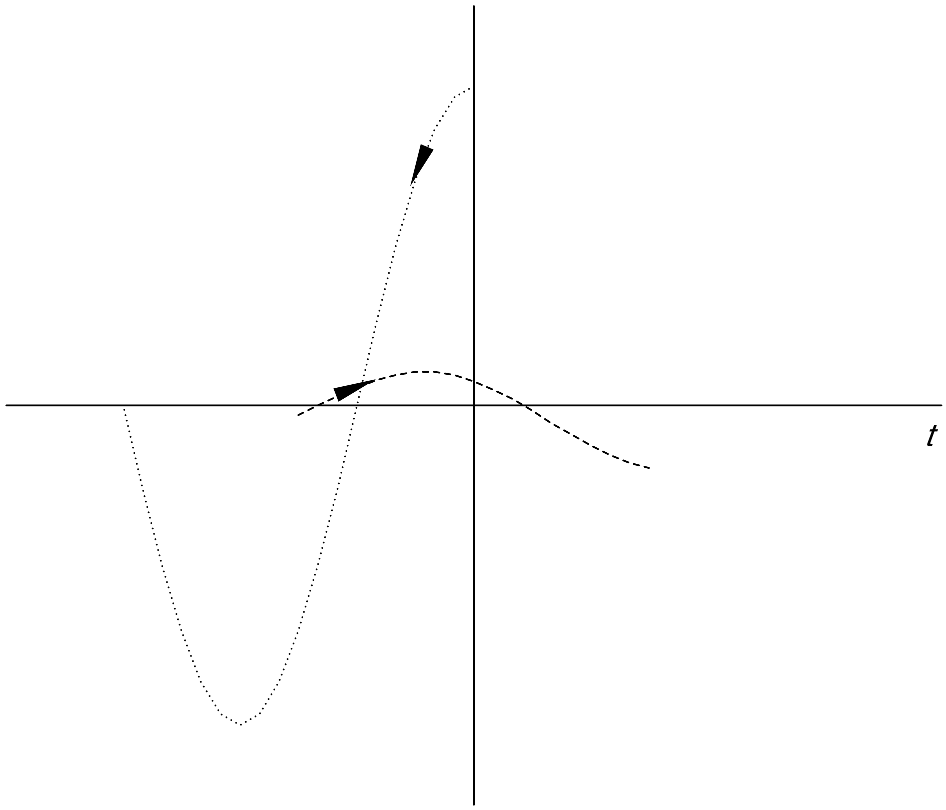
2. Dashed line :
   full forecast
   from -T to +T
   starting from the end point of the adiabatic one
   filtered

Cost of initialialization : T∗ Ca + 2T∗Cf

Ca : relative cost of the adaibatic model; Cf : relative cost of the full model

New scheme

➜ filtering <u>twice</u>



EGENDS :

1. Dotted line :
   adiabatic forecast
   from 0 to -T'
   starting from the analysis
   <u>filtered</u>

2. Dashed line :
   full forecast
   from -T' to +T'
   starting from the middle point of the filtered adiabatic forecast
   <u>filtered</u>

Cost of initialialization : T'∗ Ca + T'∗Cf

Formulation of the filtered state

Old scheme :

forward forecast :

$$\bar{y}(0) = H(\omega_0)\cdot x(0) + f_1 \cdot \left[\frac{H(\omega_0)\cdot \exp(i\omega_0 T) - H(\omega_1)\cdot \exp(i\omega_1 T)}{i(\omega_0 - \omega_1)} \right]$$

New scheme :

backward forecast :

$$\bar{x}\,(-T'/2) = H(\omega_0)\cdot x\,(-T'/2)$$

forward forecast :

$$\bar{y}(0) = H(\omega_0)^{\mathbf{2}}\cdot x(0) + f_1 \cdot \left[\frac{H(\omega_0)\cdot \exp(i\omega_0 T'/2) - H(\omega_1)\cdot \exp(i\omega_1 T'/2)}{i(\omega_0 - \omega_1)} \right]$$

Advantages of the new scheme

Filtering is applied twice, hence :

- ♥ The squared response is to be considered, which results in more damping (similar impact as increasing the time-span).
- ♥ A shorter length of the forward integration is allowed ($T' \approx T$ in the previous examples).
- ♥ The computational cost is reduced, by at least 1/3 .
- ♥ The DFI bias is reduced, as is the time-span of the filter.
- ♥ with the same efficiency in filtering high frequencies !
- ♥ Starting from a filtered state allows larger timesteps in the forward integration.

Incremental initialization

$$I = G + \overline{(A - G)} \approx G + \overline{A} - \overline{G}\,, \quad \text{instead of} \quad I = \overline{A}$$

(I : initialized state, A : analysis, G : first guess),

assuming that the first guess is noise-free and the analysis increment (A-G) is small so that nonlinear terms may be neglected.

⇒ further reduction of the problem, since the DFI bias applies only on analysis increments