Exploring some alternatives to improve the robustness of mass-based SI Spectral NH system

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Motivations

Allow very high-resolution (steep orography) and attractive time-steps for NWP, but still in the framework of the current constant-coefficient Semi-implicit approach.

Explored Avenues

1. Design of a modern Sound-proof NH approximate set of equations less stiff than fully-compressible (EE) system by exploiting Arakawa and Konor (2009) ⇒ Suppression of high-frequency vertically-propagating acoustic wave at their source ⇒ Potential benefit in term of stability.
2. design of a new prognostic variable for the EE system along the lines of the d_4 variable of Bénard *et al.* (2005), leading to a more stable constant-coefficient semi-implicit time scheme over steep slopes.

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Plan

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2. A MORE ROBUST VERTICAL MOMENTUM PROGNOSTIC VARIABLE

Switch to modern Sound-proof NH equations

Definition of pseudo-hydrostatic QE reference-state : $(\pi, \widetilde{\rho}, \widetilde{T})$

$$
\begin{aligned}
\widetilde{\rho} &= \frac{\pi}{RT}\left(\frac{p}{\pi}\right)^{R/C_p}, \\
\frac{\partial \pi}{\partial z} &= -g\widetilde{\rho} \\
\widetilde{T} &= \frac{\pi}{\widetilde{\rho} R}
\end{aligned}
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Determination of actual thermodynamic state: (p, ρ, T)

Defining the pressure departure as $\hat{q} = \log(p/\pi)$, it yields

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p &= \pi \exp[\hat{q}], \\
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QE approximation : Basic underlying idea

- Mass-continuity Eq. for $\widetilde{\rho}$:

$$\frac{\mathrm{D}\widetilde{\rho}}{\mathrm{D}t} = \underbrace{\left(\frac{\partial\widetilde{\rho}}{\partial\pi}\right)_{\theta,\hat{q}} \frac{\mathrm{D}\pi}{\mathrm{D}t}}_{\mathbf{H-compressibility}} + \underbrace{\left(\frac{\partial\widetilde{\rho}}{\partial\hat{q}}\right)_{\theta,\pi} \frac{\mathrm{D}\hat{q}}{\mathrm{D}t}}_{\mathbf{NH-compressibility}} = -\widetilde{\rho}\,\mathbb{D}_3$$

1. NH compressibility of the fluid is neglected in mass continuity Eq.
   $\Rightarrow$ Minimal condition for filtering vertically propagating acoustic waves,
2. Hydrostatic compressibility is maintained for good accuracy at large-scales.

$$\left(\frac{\partial\widetilde{\rho}}{\partial\hat{q}}\right)_{\theta,\pi} \frac{\mathrm{D}\hat{q}}{\mathrm{D}t} = \mathbb{D}_3 + \frac{C_v}{C_p}\left(\frac{\dot{\pi}}{\pi}\right) = 0$$

- $\hat{q}$ becomes a diagnostic variable.

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QE divergence constraint :

$$D - \frac{C_v}{C_p}\,\frac{1}{\pi}\int_0^{\pi} D\,\mathrm{d}\pi' \quad + \quad d_4 \;=\; 0$$

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$$d_4 = \underbrace{-\frac{g}{R\widetilde{T}}\frac{\pi}{m}\frac{\partial w}{\partial \eta}}_{d_3} \quad + \quad \underbrace{\overbrace{\frac{\pi}{R\widetilde{T}m}\frac{\partial V}{\partial \eta}\cdot\nabla\Phi}^{\mathrm{X_S}} \quad + \quad \frac{C_v}{C_p}\left[V\cdot\frac{\nabla\pi}{\pi} - \frac{1}{\pi}\int_0^{\eta}(V\cdot\nabla m)\,\mathrm{d}\eta'\right]}_{\mathrm{X}}$$

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$$\frac{\mathrm{D}\,d_4}{\mathrm{D}t} = -\frac{g}{R\widetilde{T}}\,\frac{\pi}{m}\left[\frac{\partial}{\partial \eta}\left(\frac{\mathrm{d}w}{\mathrm{d}t}\right) - \frac{\partial V}{\partial \eta}\cdot\nabla w\right] + (\mathsf{X} - d_4)\,(d_4 - \mathsf{X} + \mathsf{X_S}) + \dot{\mathsf{X}}$$

Symbolical description of QE adiabatic system

- Let us denote the state-vector $\mathcal{Z} = (\mathcal{X}, \hat{q})$.
  $\mathcal{X} = (V, d_4, \widetilde{T}, \text{Log}\pi_s)$ is the vector of prognostic variables.

- Prognostic Eqs. :

$$\frac{\partial \mathcal{X}}{\partial t} = \mathcal{A}(\mathcal{X}) + \mathcal{M}(\mathcal{Z}) \qquad \text{(Eulerian form)}$$

$$\frac{\mathrm{D}\mathcal{X}}{\mathrm{D}t} = \mathcal{M}(\mathcal{Z}), \qquad \text{(Lagrangian form)}$$

- QE constraint:

$$\mathcal{D}(\mathcal{X}) = 0$$

- $(\mathcal{L}^*, \mathcal{C}^*)$ respectively denote the linear counterpart operators of $(\mathcal{M}, \mathcal{D})$ around a constant-coefficient SI-background $\mathcal{Z}^*$.

3-TL SI Eulerian time discretization

For $\nu \in [0, N_{\text{iterhelm}} - 1]$ (inner-loop)

- Time-discrete prognostic Eqs. :

$$\frac{\mathcal{X}^{+(\nu)} - \mathcal{X}^{-}}{2\Delta t} = \mathcal{A}\left(\mathcal{X}^{0}\right) + \mathcal{M}\left(\mathcal{Z}^{0}\right) - \mathcal{L}^{*}.\mathcal{Z}^{0} + \frac{\mathcal{L}^{*}.\mathcal{Z}^{+(\nu)} + \mathcal{L}^{*}.\mathcal{Z}^{-}}{2}$$

- Newton-like iterative treatment of QE constraint :

$$\mathcal{C}^{*}.\mathcal{X}^{+(\nu)} = \mathcal{C}^{*}.\mathcal{X}^{+(\nu-1)} - \mathcal{D}\left[\mathcal{X}^{+(\nu-1)}\right]$$

Extension to 2-TL ICI SL time-discretization

For $i \in [1, N_{\text{siter}}]$ (outer-loop)
For $\nu \in [0, N_{\text{iterhelm}} - 1]$ (inner-loop)

- Time-discrete prognostic Eqs. :

$$\frac{\mathcal{X}_F^{+(i,\nu)} - \mathcal{X}_{O_{(i-1)}}^0}{\Delta t} = \frac{\mathcal{M}\left[\mathcal{Z}^{+(i-1)}\right]_F + \mathcal{M}\left[\mathcal{Z}^0\right]_{O_{(i-1)}}}{2} + \frac{\mathcal{L}^*.\mathcal{Z}_F^{+(i,\nu)} - \mathcal{L}^*.\mathcal{Z}_F^{+(i-1)}}{2}$$

- QE constraint iterative treatment:

$$\mathcal{C}^*.\mathcal{X}_F^{+(i,\nu)} = \mathcal{C}^*.\mathcal{X}_F^{+(i,\nu-1)} - \mathcal{D}\left[\mathcal{X}^{+(i,\nu-1)}\right]_F$$

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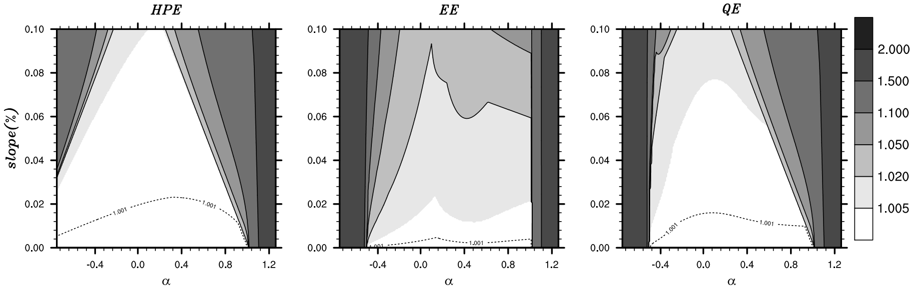
Linear stability analysis with orography

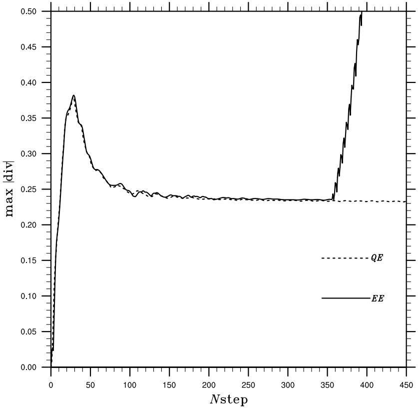
• Time-discrete space-continuous linear stability analysis of 3-TL SI scheme with a uniform sloped orography (without advection). Settings: $\Delta x = 2000\,\text{m}$, $\Delta t = 200\,\text{s}$.

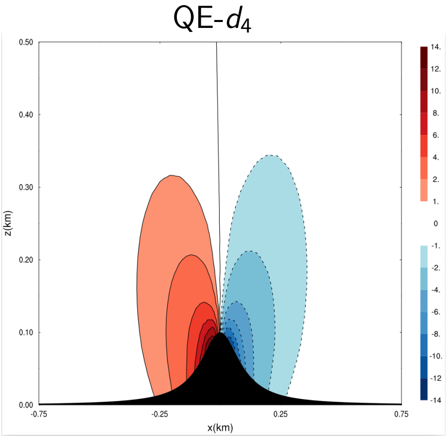


• Amplification factor as function of the slope, and the non-linear thermal residual factor :

$$\alpha = (T - T^*)/T^*$$

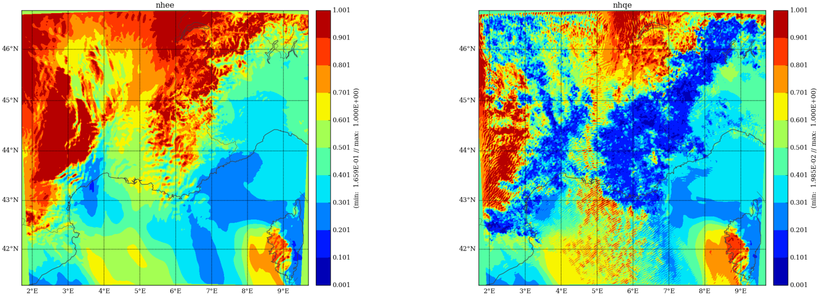
- **Potential flow 2D test-case** : Basic-state is defined by: $U = 15\,\mathrm{m/s}$, $N = 0.02\,\mathrm{s}^{-1}$, maximum height and half-width of the agnesi-mountain are both equal to 100 m $\Rightarrow$ maximum slope of 33°. Settings : $\Delta x = 10\,\mathrm{m}$, and $\Delta\eta$ is chosen in such a way that $\Delta z \approx 15\,\mathrm{m}$, $\Delta t = 0.25\,\mathrm{s}$, SITR=350 K, SITRA = 35 K, and $N_{\mathrm{iterhelm}} = 1$.





Status

- QE code in available in cycle 46 (95% of QE code).
- Validation using "mitraillette-test" are now in progress.



- Obviously, there is still some bugs to deal with !!!

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Asymmetric rigid BBC time treatment

- Non-homogeneous rigid BBC specification $\Rightarrow$ a distinct BBC treatment between the explicit grid-point part and the implicit spectral part.

Rigid BBC in NL model $\mathcal{M}$:

$$
\begin{aligned}
w_S &= \frac{1}{g}\left(V_S \cdot \nabla \Phi_S\right), \\
\dot{w}_S &= \frac{1}{g}\left[\dot{V}_S \cdot \nabla \Phi_S + V_S \cdot \nabla\left(V_S \cdot \nabla \Phi_S\right)\right]
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Rigid BBC linear model $\mathcal{L}^*$:

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w_S &= 0 \\
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- Need for an extra Free-slip BBC : $V_S = V_{NLEV}$

A potential cause of poor stability over steep slopes

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- Need for an extra Free-slip BBC : $V_S = V_{\mathrm{NLEV}}$

Imposing homogeneous rigid BBC

Let us consider the following variable :

$$\mathbb{W} = \frac{(\partial_\eta \Phi)}{g}\left[\dot{\eta} + \frac{(\partial_t \Phi)}{(\partial_\eta \Phi)}\right] = w - \frac{1}{g}\,(V \cdot \nabla \Phi)$$

⇒ Rigid BBC becomes :

$$\begin{aligned} \mathbb{W}_\mathrm{S} &= 0 \\ \dot{\mathbb{W}}_\mathrm{S} &= 0 \end{aligned}$$

⇒ At the top, material condition leads to

$$\mathbb{W}_\mathrm{T} = \frac{\partial_t \Phi_T}{g}$$

Use of a new vertical divergence variable

Let us define

$$d_5 = -\frac{g\rho}{m}\left(\partial_\eta \mathbb{W}\right) \underbrace{-\frac{\rho}{m} V \cdot \nabla \left(\partial_\eta \Phi\right)}_{\mathsf{X}}$$

$$\Rightarrow \qquad \mathbb{D}_3 = D + d_5$$

- Prognostic Eq. for d_5 :

$$\frac{\mathrm{D}\, d_5}{\mathrm{D}t} = -\frac{g\rho}{m}\left[\frac{\partial}{\partial\eta}\left(\frac{\mathrm{D}\mathbb{W}}{\mathrm{D}t}\right) - \frac{\partial V}{\partial\eta} \cdot \nabla\mathbb{W}\right] + \left(\mathsf{X} - d_5\right) d_5 + \dot{\mathsf{X}}$$

A more robust prognostic variable over steep slopes

- Governing equation of $\mathbb{W}$:

$$\frac{\mathrm{D}\mathbb{W}}{\mathrm{D}t} = \frac{\mathrm{D}w}{\mathrm{D}t} - \frac{1}{g}\frac{\mathrm{D}[V\cdot\nabla\Phi]}{\mathrm{D}t}$$

Eulerian explicit approach

$$\frac{\mathrm{D}\mathbb{W}}{\mathrm{D}t} = \mathcal{M}_w\left(\mathcal{X}\right) - \left[\dot{V} - (V\cdot\nabla)V - \dot{\eta}\left(\partial_\eta V\right)\right]\cdot\frac{\nabla\Phi}{g}$$

$$-V\cdot\nabla\left[w - \dot{\eta}\frac{(\partial_\eta\Phi)}{g}\right] - \dot{\eta}\partial_\eta\left[V\cdot\frac{\nabla\Phi}{g}\right]$$

⋆ Require extra spectral transforms to compute $\nabla[V\cdot\nabla\Phi]$ and $\nabla[\dot{\eta}(\partial_\eta\Phi)]$

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ICI Semi-Lagrangian approach (in the spirit of LGWADV option)

$$\frac{\mathrm{D}\mathbb{W}}{\mathrm{D}t} = \frac{1}{2}\left\{\mathcal{M}_w\left(\mathcal{X}\right) - \frac{2[V.\nabla\Phi]}{g\Delta t}\right\}_F^{+(i-1)} + \frac{1}{2}\left\{\mathcal{M}_w\left(\mathcal{X}\right) + \frac{2[V.\nabla\Phi]}{g\Delta t}\right\}_{O_{(i-1)}}^{0}$$

⋆ May require some extra SL interpolations for $[V\cdot\nabla\Phi]$.

A more robust prognostic variable over step slopes

• Fully-discrete linear stability analysis of 2-TL ICI ($N_{\text{siter}} = 1$) with a prescribed sinusoidal orography (without advection).

• Amplification factor as function of horizontal wave Courant number for three different slopes : 15°, 25°, and 45°. For current d_4 (left panel)

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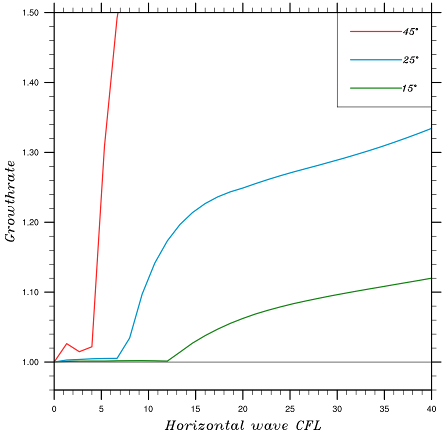

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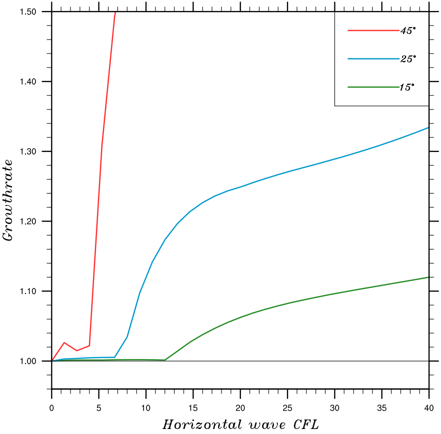
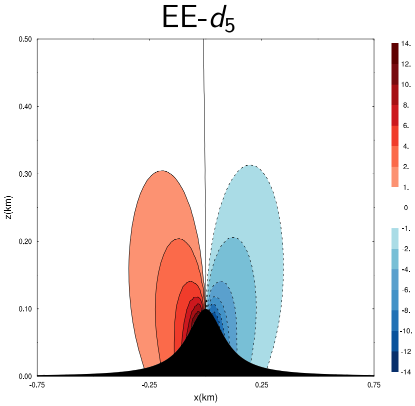
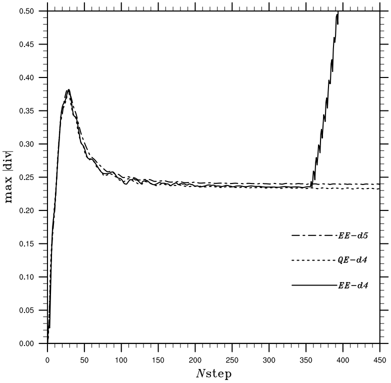


- **Potential flow 2D test-case** : re-run for EE system with d_5





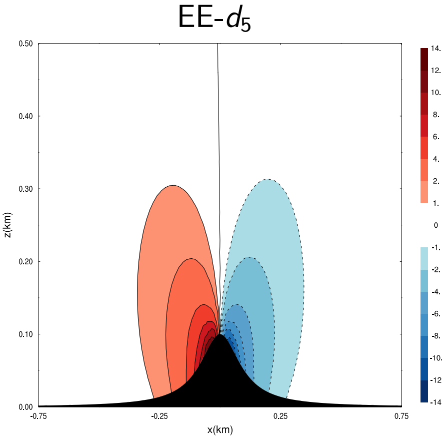
Summary

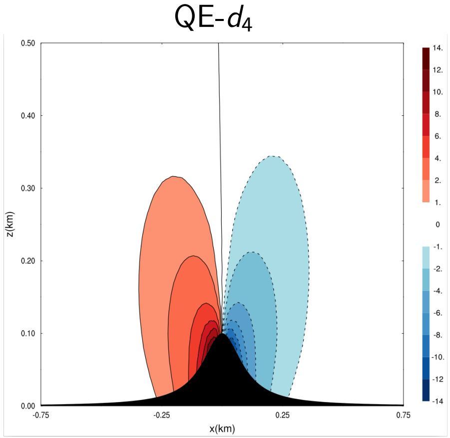
- Can we still improve the stability of the constant-coefficient SI spectral NH system ?
  $\Rightarrow$ Yes, we can !
- Switch to QE system may provide a substantial gain in stability.
- New variable d_5 is very promising for EE system, and can also be extended to QE system.
- There is always a price to pay, nothing is given for free : extra spectral transforms, extra SL interpolations.

Perspectives

- Validation of QE code will be pursued.
- Coding of this new variable d_5 should be envisaged.

Thanks for your attention !!!


EE-d_5
0.50
0.40
0.30
0.20
0.10
0.00
z(km)
-0.75
-0.25
0.25
0.75
x(km)
14.
12.
10.
8.
6.
4.
2.
1.
0
-1.
-2.
-4.
-6.
-8.
-10.
-12
-14


QE-d_4
0.50
0.40
0.30
0.20
0.10
0.00
z(km)
-0.75
-0.25
0.25
0.75
x(km)
14.
12.
10.
8.
6.
4.
2.
1.
0
-1.
-2.
-4.
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-8.
-10.
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