PARAMETERIZATION OF TURBULENT FLUXES IN BAROCLINIC ENVIRONMENT

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Short- and medium range numerical forecasts of mid- latitude cyclones are sometimes very dependent on the way of parameterizing subgrid physical processes. The study analysed the influence of the vertical diffusion in the global model ARPÈGE and the limited area model ALADIN with the use of first order closure parameterization of turbulent fluxes based on a computation of exchange coefficients (K- theory). The limitation of the Richardson number in Louis-type stability function showed a large importance in forecasting the rapidly developing cyclone of 20 December 1998 and the so called Christimas storms of 25-26 December1999. It is shown that turbulent fluxes may support cyclogenesis in areas with moderate or high baroclinicity, such as frontal zones. An analytical solution is proposed to simulate the turbulent exchange of momentum and enthalpy along a frontal surface. This is compared with results of a simpler scheme introducing limitation of the Richardson number with respect to the environmental baroclinicity. The schemes are tested on case studies with rapid cyclogenesis and in situations with numerical prediction of false cyclones. Calculations of temperature, moisture and energy budgets give a more detailed overview on global impacts of the proposed parameterizations. It is concluded that the application of schemes, which distinguish between barotropic and baroclinic atmosphere, may have positive influence on model results, concerning both forecasts of severe weather events and long range scores. Present experiences favour the rather simpler formulations based on a baroclinic regulation of the Richardson number for the operational praxis, whereas analytical expressions of slantwise turbulence require further development and testing.

Krátkodobé a strednodobé numerické predpovede cyklón miernych zemepisných šírok sú niekedy veľmi závislé od parametrizácie fyzikálnych procesov. Táto práca analyzovala vplyv vertikálnej difúzie v globálnom modeli ARPÈGE a v modeli na ohraničenej oblasti ALADIN s použitím parametrizácie turbulentných tokov s uzáverom prvého rádu založenej na počítaní koeficientov turbulentnej výmeny (K- teória). Limitovanie Richardsonovho čísla v Louisovej funkcii stability sa ukázalo byť veľmi dôležité pri predpovedi rýchlo sa prehlbujúcich cyklón 20. decembra 1998 a tzv. vianočných víchríc 25. a 26. 12. 1999. Ukazuje sa, že turbulentné toky môžu podporovať cyklogenézu v oblastiach s miernou alebo vysokou baroklinitou, ako sú frontálne zóny. Na simuláciu turbulentného prenosu hybnosti a entalpie pozdĺž frontálnej plochy je navrhnuté analytické riešenie. Toto je porovnané s výsledkami jednoduchšej schémy, ktorá zavádza limitovanie Richardsonovho čísla v závislosti od baroklinnosti prostredia. Schémy sú testované na prípadových štúdiách s intenzívnou cyklogenézou a v situáciách s numerickou predpoveďou falošných cyklón. Detailnejší pohľad na globálny vplyv navrhovaných parametrizácií poskytujú bilancie teploty, vlhkosti a energie. Konštatuje sa, že aplikácia schém, ktoré rozlišujú medzi barotropnou a baroklinnou atmosférou, môžu mať pozitívny vplyv na výsledky modelu vzhľadom na predpovede nebezpečných prejavov počasia, ako aj dlhodobú úspešnosť. Na základe súčasných skúseností je možné v operatívnej praxi uprednostniť skôr jednoduchšie formulácie založené na baroklinnej regulácii Richardsonovho čísla, zatiaľ čo analytické vyjadrenia turbulencie po naklonenej ploche vyžadujú ďalší vývoj a testovanie.

Key words: baroclinic, cyclogenesis, turbulent fluxes, parameterization

1. INTRODUCTION

Horizontal dimensions of turbulent eddies are usually below the grid scale of present operational global numerical models or limited area (LAM) models. However, turbulent transport of momentum, sensible heat and moisture, creates tendencies of meteorological parameters which are often comparable to tendencies provided by the model dynamics. For this reason, attempts to include the effect of turbulence can be found already in early generations of numerical models (Haltiner, 1980). During the last 40 years of numerical modelling a number of parameterization schemes were developed, which can be classified after their closure order or after the local/non-local character of the scheme (Stull, 1994). The local, first order closure scheme developed by Louis (1979) and later modified by Louis et al. (1982) was widely used by operational and research models and it is still a part of the physical parameterization of the ALADIN model (Radnóti et al., 1995) used also at SHMÚ (Derková et al., 2005). Turbulent fluxes in the Planetary Boundary

Layer (PBL) and in the free atmosphere are computed from exchange coefficients using the mixing length hypothesis and stability functions dependent on the Richardson number. The Louis scheme (similarly to many other turbulence schemes) expects only vertical turbulent transport of meteorological parameters, supposing that the horizontal gradients and exchange coefficients are usually small and can be neglected in models of 10 km and lower resolution. However, a realistic description of small scale phenomena (such as convection or flow over rugged topography) requires three-dimensional turbulent flux and tendency computation, currently available only in certain, rather mesoscale or research models (Lac, 2005 and Malardel, 2005).

Baroclinic zones and frontal boundaries are environments, where large horizontal wind shears, temperature and moisture gradients can be expected. Field campaigns, tower measurements and aerial observation showed increased turbulent kinetic energy (TKE) in the vicinity of frontal boundaries or low-level jets (Frank, 1994, Bond and Walter, 2002) indicating that these areas are favorable for enhanced turbulence even by statically stable stratification. Theoretical studies and numerical simulations of Cooper et al.(1992), Stoelinga (1996), Adamson(2001), Adamson et al., (2006) and Plant et al. (2006) showed the importance of the turbulent transport of momentum and heat for cyclogenesis particularly in areas of high baroclinicity. The motion of the air at frontal zones is typically slantwise, due to the presence of baroclinic or conditional symmetric instability (Holton, 2005, Bennets and Hoskins, 1979). Hence the traditional concept of physical diffusion which is based on parameters of vertical stability (as the Richardson number) must not realistically express the conditions for turbulence in strongly baroclinic environments. The aim of this study was to find a possible solution for parameterization of turbulent fluxes in the direction of upslope frontal motions. Another, computationally more simple scheme was based on the modification of the critical Richardson number with respect to the environmental baroclinicity.

The paper is divided into eight sections. The following section number two is a short discussion about the relationship between turbulence and large scale atmospheric processes as frontogenesis or cyclogenesis. Section number three introduces the turbulence scheme implemented in ARPÈGE/ALADIN models, particularly the modification of the Richardson number used in the Louis-type static stability function. An analytical solution for slantwise turbulent exchange in baroclinic zones is proposed in section number four. Section five is about a simpler and computationally more effective scheme, which makes the parameterization of turbulence more sensitive with respect to baroclinicity of the environment. Section six shows the results of presented schemes in selected case studies related to forecasts of rapid and false cyclogenesis, respecttively. Section seven compares the global effects of proposed parameterizations (e.g. temperature budgets). The last section discusses the importance of another approach to turbulent diffusion in baroclinic environment with proposals for future study.

2. THE ROLE OF TURBULENCE IN LARGE SCALE ATMOSPHERIC PROCESSES

There are several ways, how to estimate the influence of turbulent fluxes on the formation and lifetime of atmospheric fronts or cyclones. Some textbooks (e.g. Bluestein, 1992) propose the method of quasigeostrophic diagnostics, which is, however, useful only for environments showing small baroclinicity. The method of assessing potential vorticity (PV) tendencies seems to be more plausible for studying the influence of diabatic processes on frontogenesis and cyclogenesis. Cooper (1992) defined the net PBL tendency of potential vorticity [G], later completed by Plant et al., 2006. The formula yields:

$$[G] = [G_E]_m + [G_B]_m + [G_E]_\theta + [G_B]_\theta \qquad (1)$$

$$= -\left(\frac{1}{\rho h^2}\right)\begin{Bmatrix} \frac{\Delta\theta}{\rho}\vec{k}\cdot\left(\vec{\nabla}\times\tau_S\right)+\frac{\tau_S}{\rho}\left(\vec{k}\times\nabla\theta_h\right)+ \\ \zeta_h H_S+\left(\Delta\vec{v}-h\nabla w\right)\cdot\left(\vec{k}\times\nabla H_S\right) \end{Bmatrix}$$

Here h is the depth of the boundary layer, ρ is density, $\Delta\theta$ and $\Delta\vec{v}$ are the vertical changes of potential temperature and wind vector through the PBL, $\nabla\theta$ and ζ_h are horizontal potential temperature gradient and absolute vorticity at the top of the PBL, H_S and τ_S are the heat flux and the stress at surface. The term ∇w means the averaged horizontal gradient of vertical velocity. The first two terms of the equation represent the PV tendency due to momentum exchange, where the first term means barotropic damping of vorticity and the effect of the second term corresponds to the baroclinicity of the atmosphere (its magnitude depends on the angle between the surface stress and thermal wind shear vectors). The work of Adamson et al. (2006) showed that the baroclinic contribution to the net PV tendency is very important for the evolution of mid-latitude cyclones. PV anomalies are generated close to warm and cold fronts near the surface, they are advected upwards and polewards along the warm conveyor belt and moved towards the centre of the cyclone (Fig. 1). On the other hand, it seems for the heat flux tendency that the barotropic third term is usually greater than the fourth, baroclinic term of the equation (Plant et al., 2006).

The method of Cooper is based on the assumption of linear vertical course of fluxes in the PBL – hence, the PV tendency considerably depends on the magnitude of the surface momentum and heat exchange. Locally, this may be not valid, for example in cases with none typical vertical change of meteorological parameters (as by temperature inversions or low level jets) where the vertical flux variation can be highly non-linear.

Except for the direct influence of turbulent fluxes there are also other possibilities of how vertical diffusion can provide an environment favorable for cyclogenesis. Such processes may occur far outside of the cyclone and act even with big time-delay. The event of 20 December 1998 was a typical case, where numerical simulations showed considerable sensitivity on turbulent fluxes more than 48 hours before the start of rapid cyclogenesis. Experiments were done with a reference setup of turbulent heat exchange (assigned USD0 further in the text) as it was used in the operational version of the ARPÈGE model and with parameterization allowing stable conditions for vertical diffusion at the top of the PBL (assigned USD7). The tuning of the scheme was provided with decreasing (USD0) or increasing (USD7) of the maximum allowed Richardson number in the computation of the eddy exchange coefficients (see more details in the next section).

Figure 1. ARPÈGE model 24 hour forecast of the mean sea level pressure (solid lines, in hPa) and of bulk potential vorticity tendency given by baroclinic component of the friction term [G_B] at model level 28 (nearly at the PBL top). Positive values of [G_B] are drown by dashed, negative by thinner solid lines. The isolines are contoured by: ± (1, 2, 5, 10, 15, 20, 30, 40, 60, 80, 100, 150, 200, 250, 500, 1000 and 2000) 10^{-1} PVU/day. The model integration was based on 16 December 1998 12 UTC.

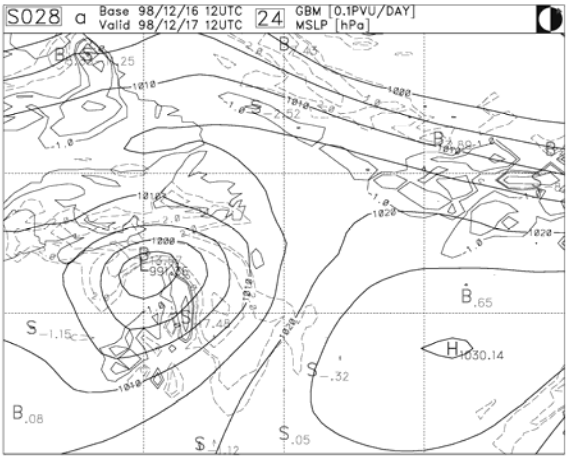
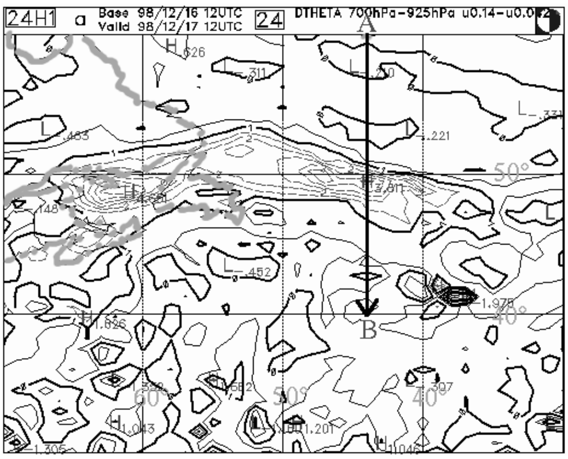


Figure 2. Static stability differences between USD7 and USD0 experiments after 24 h of ARPÈGE model integration based on 16 December 1998 12 UTC. Static stability is considered here simply as the difference between the 700 and 925 hPa level potential temperature. Areas with significant increase in stability for the USD7 experiment are marked by dashed lines (contoured by 0.5 K). The AB line shows the sense of the vertical cross-section in Figure 3.



Diagnostics (Simon, 2003) showed that more turbulent transport of sensible heat (USD0) reduces the static stability in certain, very stably stratified areas in the first 24 hours of computation (Fig. 2). This appears very important after 36 hours of integration, when the cyclone approaches the above mentioned areas. Strong upward motions are observed in the north-south cross-section through the eastern part of the cyclone (Fig. 3a for the USD7 experiment). Vertical motions are amplified in the USD0 experiment (Fig. 3b) what favors a deeper cyclone.

Figure 3. a) ARPÈGE model forecast of vertical velocity based on 16 December 1998 12 UTC and valid for 18 December 1998 00 UTC. The fields are viewed in vertical cross-section, which sense is marked in Figure 2. The experiment (assigned USD7) was running with setup allowing higher values of Richardson number, thus, supporting stable PBL environments. The white, solid lines denote the potential temperature (in K). Vertical velocity (Pa/s) is in shades of grey, contoured by black isolines. The maximum upward velocity is –2.2 Pa/s.

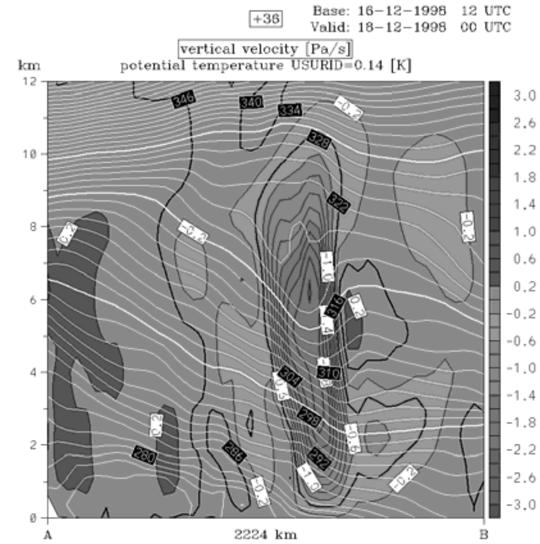
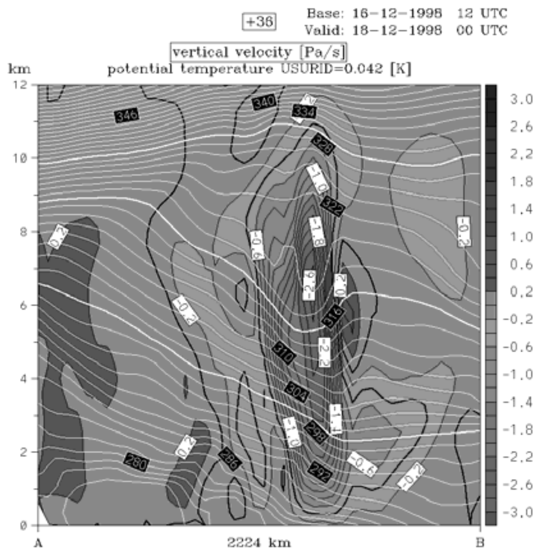
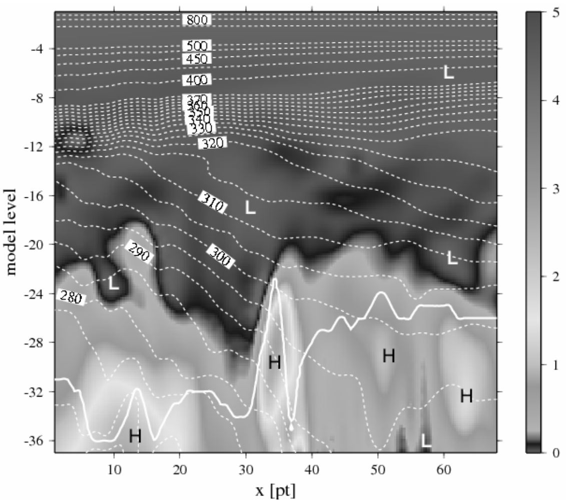


Figure 3. b) as in a), except for the USD0 experiment with lower threshold for the Richardson number which supports turbulent exchange. The maximum upward velocity is –2.6 Pa/s.



However, not all processes which are simulated by numerical models must be necessary observed in the real atmosphere (this concerns also the decrease of static stability in the USD0 experiment). The results are many times considerably dependent also on the setup of other parts of the physical parameterization (e.g. radiation or precipitation). Nevertheless, sensitivity studies and diagnostics of various parameters (e.g. Turbulent Kinetic Energy, TKE) show that turbulent processes are active in frontal baroclinic zones, even in environments with moderate or high static stability as it was in the case of the 20 December 1998 cyclone (Figure 4). An increase in turbulence can be expected also due to the occurrence of free or forced convection at a frontal boundary. This indicates that turbulent fluxes at baroclinic zones can have bigger impact on large scale synoptic processes than at non-frontal areas. This possibility is evaluated and discussed also in sections 4-8.

Figure 4. ALADIN/ALARO-3MT model 24 hour forecast of TKE. The base of the run is the same as in Figure 2. The coordinates of the northwest-southeast cross-section end-points are 55 N, 70 W and 35 N, 55 W. The dashed lines are potential temperature isolines drown by 5 K. Solid line represents the course of the diagnostic PBL height. The vertical axis is in model levels (-1 at the top of the model atmosphere and –37 at bottom). Note the increased PBL values of TKE (over 2 m^2 s^{-2}) at the baroclinic zone in the middle of the cross-section despite the statically stable environment.



3. VERTICAL DIFFUSION SCHEME WITH RICHARDSON NUMBER LIMITATION

Physical tendencies, which are results of the turbulence parameterization, are usually computed from the vertical divergence of turbulent fluxes (assigned J_ψ , where ψ is a general meteorological variable transported by turbulent fluxes). This relationship is sometimes denoted as "turbulent diffusion equation" and is frequently mentioned in textbooks (Tomlain and Damborská, 1999).

$$\frac{\partial \psi}{\partial t} = \frac{1}{\rho} \frac{\partial J_\psi}{\partial z} \tag{2}$$

The vertical diffusion scheme of the ARPÈGE/ALADIN model, used in this experiment, computes the turbulent fluxes with the aid of exchange coefficients K_ψ :

$$J_\psi = -\rho \overline{\psi' w'} = \rho K_\psi \frac{\partial \overline{\psi}}{\partial z} \tag{3}$$

Here K_ψ represents the rate of the vertical diffusion of the physical variable ψ (the m label is used for momentum, θ for enthalpy and moisture), w' are fluctuations of vertical velocity, ρ is density. Turbulent fluxes at surface are computed with bulk formula, which depend on surface drag coefficient and on the difference between the values of the diffused variable at the surface and at the closest model level to the surface, respectively (Gerard, 2001). The ARPÈGE/ALADIN parameterization is using an implicit time scheme, where the exchange coefficients are computed at time $t - \Delta t$ but the fluxes act at future time $t + \Delta t$, where Δt is the time step used for physical processes. The physical tendencies are computed at so called "full" model levels in eta- system of vertical coordinates, whereas the fluxes are defined at "half" levels of the Lorenz-type staggered grid (Simmons and Burridge, 1981). The "full" and "half" levels vary so that each "full" model level is situated between two "half" levels.

Exchange coefficients are parameterized via mixing length and stability function.

$$K_\psi = l_m l_\psi \left| \frac{\partial \overline{v}}{\partial z} \right| F\left(Ri'\right) \tag{4}$$

The mixing length l_ψ is characterized by modified function of Blackadar (1962), taking into account the usual distribution and vertical variation of the exchange coefficients (O´Brien, 1970). The original stability function $F\left(Ri'\right)$ (Louis, 1979 and Louis et al., 1982) was primarily dependent on the Richardson number (Ri). Thus, turbulent exchange decreased with the growth of the static stability and decrease of wind shear. However, several problems appeared with this scheme in the operational praxis in situations with very stably stratified PBL (typically at winter time in regions with polar night), when turbulent fluxes were almost zero by very high Richardson numbers. Hence, a modified Richardson number Ri' was introduced to the stability function. For stable stratification $\left(Ri > 0\right)$ and for enthalpy and moisture exchange the function yields:

$$F\left(Ri'\right) = \frac{1}{1 + 3b\,Ri' \sqrt{1 + d\,Ri'}} , \tag{5}$$

where b , d are constants.

The modified Richardson number is expressed as follows:

$$Ri' = \frac{Ri}{\left(1 + \alpha \dfrac{Ri}{Ri_{cr}}\right)^{1/\alpha}} \tag{6}$$

Here, Ri_{cr} is called as a critical Richardson number. A critical Richardson number is usually defined as a limit value, beyond which turbulence decays (mostly at Ri close to 1).

The actual role of the critical Richardson number in this physical parameterization is to provide a limitation to very high Richardson numbers, thus, keeping a minimum vertical diffusion needed to compensate the effect of radiation. The vertical course of Ri_{cr} is set by function (Bellus 2000, Geleyn 2001) so that its value decreases with height.

The function α provides different way of limitation of Richardson number for momentum exchange and for the enthalpy (moisture) fluxes:

$$\alpha = \frac{3Ri + Ri_d}{Ri + Ri_d} \tag{7}$$

For momentum exchange, α is equal to 1, while for enthalpy and moisture α varies between 1 and 3, dependent on the Richardson number and on the setup of the Ri_d parameter. The course of Ri_d is also defined by vertically changing function. The scheme is regulated by the overturned value of the Ri_d number at the top of the atmosphere (denoted USURID in the ARPÈGE/ALADIN code). Higher Ri_d numbers (23.8 in the USD0 experiment) will give values of the α function closer to 1. This makes the Richardson number more suppressed (for instance, by $Ri = 5$ the scheme would return $Ri' = 0.66$ at 1500 m height that is considered as the top of the PBL), on the other hand, this means also bigger fluxes of moisture and enthalpy in situations with high static stability or small wind shears.

Smaller Ri_d numbers (7.14 in the experiment USD7) mean less limitation to the Richardson number (by $Ri = 5$ the Richardson number used in the Louis stability function is equal to 0.98). Hence, the conditions for turbulence will be more stable and the vertical exchange of moisture and enthalpy is less enhanced.

The parameterization of the Richardson number modifications formed after several experiments in case studies that showed a key role of this parameter by forecasting intense cyclogenesis (Simon, 1999, Geleyn et al., 2001). This is a consequence of static stability decrease at the top of the PBL resulting from an increase of sensible heat fluxes as already shown in section 2.

4. PARAMETERIZATION OF TURBULENT TRANSPORT ALONG SLANTED SURFACE

The motion of the parcels at frontal boundaries is typically slantwise, following the surfaces of constant wet-bulb potential temperature (moist case) or the isentropic surfaces (dry case). In regions with symmetric instability the motion follows the absolute momentum surfaces. Hence, the turbulent exchange of physical properties probably does not result exclusively in the vertical direction. Large horizontal wind shears across frontal boundaries suggest the possibility of horizontal turbulent exchange of momentum which can be provided along the frontal surface (Ferrari et al., 2004). The problem of the parameterization of slantwise turbulence has two parts: a) representation of turbulent fluxes on slanted surface b) computation of tendencies in a model, which physical parameterization is principally vertical – this is mainly due to the parallel way of integration on more processors (Radnóti, 2002).

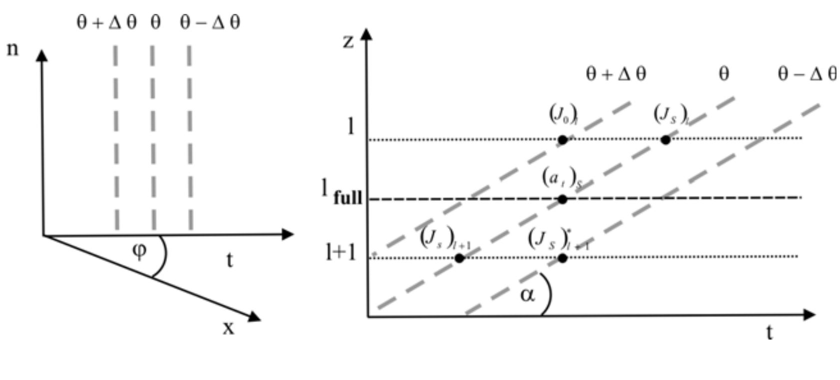
The easiest way of the slantwise flux computation is to assume that the turbulent exchange is provided along the isentropes in the direction opposite to the potential temperature gradient. This way we introduce a t, n natural coordinate system (Fig. 5). First order closure will be applied on velocity fluctuations u' at isentropic surface, which can be defined in theta vertical coordinate system as follows:

$$u' = (du)_\theta = \left(\frac{\partial u}{\partial x}\right)_\theta dx = \left\{\frac{\partial u}{\partial z} + \frac{\left(\frac{\partial u}{\partial x}\right)_H}{\left(\frac{\partial z}{\partial x}\right)_\theta}\right\}(dz)_\theta \tag{8}$$

Figure 5. Scheme for the computation of slantwise turbulent fluxes.

Left: The t, n natural coordinate system is turned in the direction opposite to potential temperature gradient (isolines of potential temperature are marked by thick, dashed lines). The φ angle denotes the rotation of the natural coordinate system with respect to the zonal x, y Cartesian coordinate system (y –axis not shown).

Right: Vertical cross-section along the t- axis. Dotted lines represent the "half"model levels, thin dashed line is the "full" model level (in case of no orography). Thick dots are grid-points. The α angle denotes the elevation of the frontal zone.



After introducing the mixing length for momentum l_z instead of $(dz)_\theta$ the formula for the slantwise momentum flux J_S yields:

$$J_S = -\rho \overline{u'u'} = -\rho l_z^2 \left\{ \frac{\partial u}{\partial z} + \left(\frac{\partial u}{\partial x} \right)_H \frac{1}{tg\,\alpha} \right\}^2 \approx J_0 + J' \qquad (9)$$

Here J_0 means the original vertical turbulent flux of the t-component of momentum and J' is purely the contribution of the slantwise turbulent exchange. The position of the J_S flux is expected to the right of the J_0 flux, as the turbulent exchange is directed upslope, along the frontal surface. The α angle represents the elevation angle of the frontal surface. Tangens function of α can be computed from vertical and horizontal gradients of potential temperature:

$$tg\,\alpha = \left(\frac{\partial z}{\partial x} \right)_\theta = -\frac{\left(\frac{\partial \theta}{\partial x} \right)_H}{\left(\frac{\partial \theta}{\partial z} \right)} \qquad (10)$$

It is assumed that the parcel has neutral stability during its transport along the isentropic surface, hence, there is no need for a stability function. It is obvious that the slantwise flux computation with constant l_z has sense only in baroclinic fields, because by α approaching zero the second term would rise to infinity.

The tendency (acceleration) resulting from momentum exchange on slope is added only in the direction of the t- axis of the natural coordinate system:

$$a_t = (a_t)_0 + (a_t)_S \text{ and } a_n = (a_n)_0\,, \qquad (11)$$

where $(a_t)_0$ is the acceleration provided by vertical exchange of momentum and $(a_t)_S$ is the result of the momentum exchange along the slanted surface. In the original physical parameterization the acceleration is computed as vertical divergence of the vertical momentum flux. For momentum exchange on isentropic surfaces the acceleration is the result of the divergence of slantwise turbulent flux along the isentropes. However, the current way of the model integration does not support direct computation of differences between neighboring grid-points. Thus, the slantwise flux divergence along the path Δs on a slanted surface has to be expressed via vertical differences $(\Delta z = z_l - z_{l+1})$.

$$(a_t)_S = \left(\frac{\partial u}{\partial t} \right)_S \approx \frac{\Delta (J_S)}{\Delta s} \approx \frac{(J_S)_l - (J_S)_{l+1}}{\Delta z} \sin\alpha \qquad (12)$$

The term $(J_S)_{l+1}$ cannot be expressed directly, because it is defined by vertical and horizontal gradients at neighboring grid-point to the left of the present vertical column, which are not available for the computation. Hence, the term is estimated from the known slantwise turbulent flux $(J_S)^*$ at $l+1$ level with use of second order wind derivatives (J_{zx}), which express the horizontal change of vertical and horizontal wind shear.

$$(J_S)_{l+1} = (J_S)^*_{l+1} + (J_{zx})_{l+1} \qquad (13)$$

However, only vertical differences of horizontal wind shear were available in this parameterization (spectral computation of second order horizontal derivatives is difficult and computationally expensive). Thus, the simplified correction (the α angle is expected to be horizontally constant) on slantwise flux at level $l+1$ yields: (14)

$$(J_{zx})_{l+1} = -\rho l_z^2 \left\{ -2 \left[\left(\frac{\partial u}{\partial z} \right)_{l+1} + \frac{1}{tg\,\alpha} \left(\frac{\partial u}{\partial x} \right)_{l+1} \right] \ldots \right.$$
$$\left. \ldots \frac{d}{dz} \left(\frac{\partial u}{\partial x} \right)_{l+1} \frac{\Delta z}{tg\,\alpha} + \left[\frac{d}{dz} \left(\frac{\partial u}{\partial x} \right)_{l+1} \frac{\Delta z}{tg\,\alpha} \right]^2 \right\}$$

The correction of the slantwise flux must be projected to both flux terms in the turbulent diffusion equation. This is achieved with a so called residual term, which is added to original slantwise flux representation. Thus, the final form of the slantwise tendency computation can be expressed as:

$$(a_t)_S = \frac{(J_S + J_{res})_l - (J_S + J_{res})_{l+1}}{z_l - z_{l+1}} \sin\alpha\,, \qquad (15)$$

where the residual term is a sum of second order derivative corrections J_{zx} at current level:

$$(J_{res})_l = \sum_{i=1}^{l} (J_{zx})_i = (J_{zx})_1 + (J_{zx})_2 + \cdots + (J_{zx})_l \qquad (16)$$

It is straightforward that vertical difference of the residual terms will express the desired correction on slantwise turbulent flux at level $l+1$. The flux is after transformed from natural to Cartesian coordinate system (rotated by angle φ), where the x, y components of the momentum flux are:

$$J_x = (J_x)_0 + \left[(J_x)_0 \cos\varphi + (J_y)_0 \sin\varphi + \right.$$
$$\left. + J' + J_{res} \right] \cos\varphi \sin\alpha \qquad (17)$$

$$J_y = (J_y)_0 + \left[(J_x)_0 \cos\varphi + (J_y)_0 \sin\varphi + \right.$$
$$\left. + J' + J_{res} \right] \sin\varphi \sin\alpha \qquad (18)$$

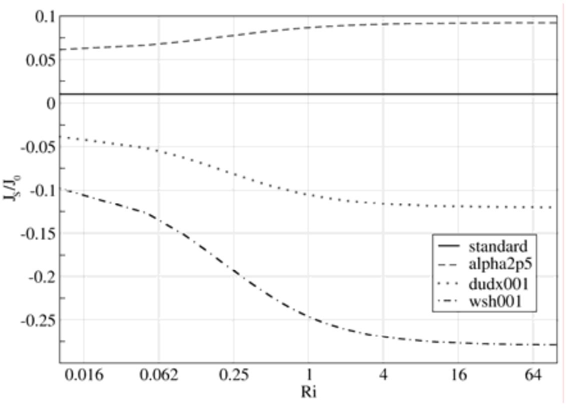
It is approximated that both φ and α angles are invariants to this transformation. It is also possible to adjust sensible heat and moisture fluxes J_θ expecting similar rates of the changes as by momentum fluxes J_m.

$$J_\theta = J_{0/\theta}\left(1+\frac{J_S \sin\alpha}{J_{0/m}}\right) \tag{19}$$

The above presented scheme is computationally stable by typical magnitudes of velocity and potential temperature gradients in the atmosphere. The increase of the momentum and heat fluxes in selected case studies was nearly 2-3 percent with respect to original values. Analytical evaluation shows that the contribution of the slantwise flux increases with higher Richardson numbers (Fig. 6). This influence is visible above all by smaller vertical and higher horizontal wind shear. The elevation of the frontal surface has also impact on the magnitude of the slantwise turbulence. However, the scheme seems reasonable only for elevations typical for synoptic scale baroclinic zones (below 5 degrees). Otherwise, by situations with small static stability, the turbulent flux would be overestimated.

Spatial distribution of the diagnosed slantwise flux (Fig. 7) shows that local maxima can be found in baroclinic zones in northern and southern parts of the cyclone. However, highest positive and negative values lay in northern part of the domain, along the zone of high pressure gradient. The distribution of the slantwise flux is qualitatively different from the field of baroclinic PV tendency (Fig. 1), although there are some similarities (local maxima in cyclonic areas). There is no visible relationship with the areas showing sensitivity on decreased static stability in Figure 2.

Figure 6. Dependence of the slantwise (J_S) and vertical turbulent momentum flux (J_0) ratio from Richardson number. The solid line denotes the simulation during conditions in typical baroclinic atmosphere (frontal surface elevation angle $\alpha = 0.58^\circ$, horizontal wind shear $\partial u/\partial x = 1.10^{-4}s^{-1}$, vertical wind shear $\partial u/\partial z = 5.10^{-3}s^{-1}$). The dashed line corresponds to the influence of increased angle $\alpha = 2.5^\circ$. The dotted line represents conditions with high horizontal wind shear ($\partial u/\partial x = 1.10^{-3}s^{-1}$) and dash-dotted line corresponds to lower vertical wind shear ($\partial u/\partial z = 1.10^{-3}s^{-1}$).



5. RICHARDSON NUMBER LIMITATION WITH SIMPLE BAROCLINIC PARAMETERIZATION

Case studies focusing on rapid cyclogenesis showed that areas sensitive on the parameterization of turbulent fluxes are closely related to well defined baroclinic zones in the western part of the Atlantic ocean. The magnitude of the horizontal temperature gradient in these areas exceeds 2. 10^{-5} K/m (Fig. 8), while typical mean global values are below 1. 10^{-5} K/m.

Figure 7. Slantwise momentum flux distribution in 24 hour ARPÈGE forecast valid for 17 December 1998 12 UTC. Thin dashed lines show positive, solid lines negative flux distribution. The isolines are contoured for values: ± (10, 25, 50, 100, 250, 500, 750, and 1000) 10^{-3} kg m^{-1} s^{-2}. The mean sea level pressure is shown by thicker solid lines (by 5 hPa).

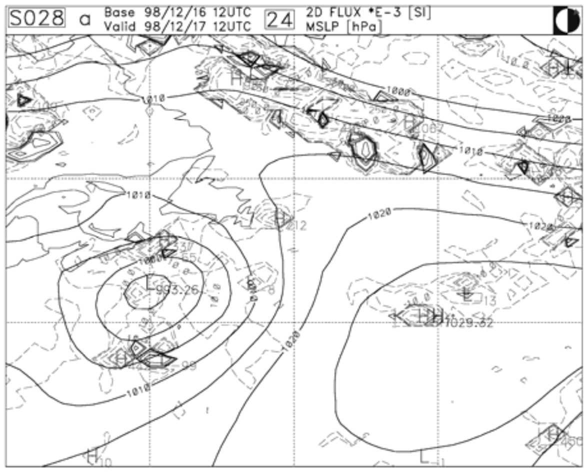


Figure 8. ARPÈGE model 18 h forecast of the mean sea level pressure (solid lines) and magnitude of the temperature gradients on pressure levels $|\nabla_p T|$ (dashed lines) valid to model level 28. The forecast is based on 16 December 1998 12 UTC. Only values of $|\nabla_p T|$ bigger than 2 10^{-5} K m^{-1} are contoured by 0.5 10^{-5} K m^{-1} starting from 2 10^{-5} K m^{-1} isoline. Note the areas of increased temperature gradients easterly from Newfoundland. This zone corresponds to sensitivity areas shown in Figure 2.

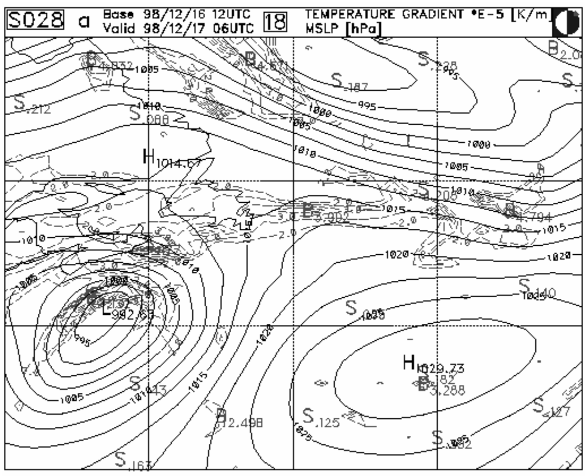


Figure 9. a) Vertical course of the global average temperature gradient (solid line, the unit is 1 10^{-5} K m^{-1}) diagnosed from ARPÈGE model 84 hour forecast and the optimal course of the critical temperature gradient β_T (dashed line, in 10^{-5} K m^{-1}).

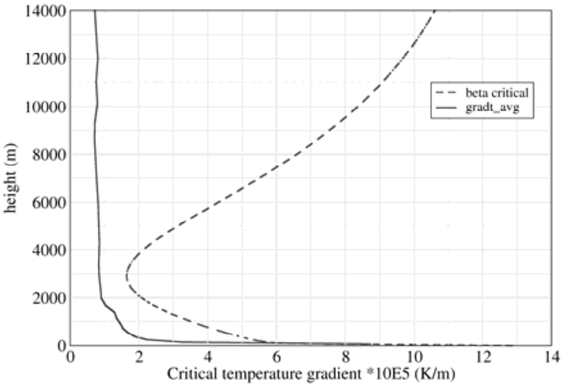
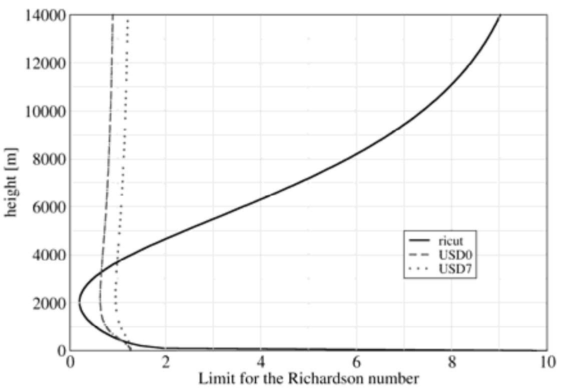


Figure 9. b) Function of the limit for Richardson number in baroclinic environment Ri_L (solid line). The course of modified Richardson number Ri' is denoted by dashed line for the USD0 experiment and by dotted line for the USD7 case (both using only the scheme described in section 3 without any baroclinic parameterization). The original Ri number is vertically constant and was equal to 5 for both experiments.



If the adjustment of the Richardson number limitation given by equation (6) and (7) would concentrate on areas of fronts and frontal waves, this kind of parameterization could support cross-frontal circulation and cyclogenesis without global decrease of static stability in the PBL. Proposed correction of the reference ARPÈGE/ ALADIN scheme is done in two steps. The first step is the diagnostics of baroclinic zones, where the magnitude of the horizontal temperature gradient computed in pressure coordinate system must be bigger than threshold β_T which changes with height z.

$$\beta_T = \beta_0 - d\left(\frac{\kappa z}{1+\dfrac{\kappa z}{\lambda}}\right)\left(a+\frac{2}{\exp\left\{\dfrac{z-H}{b}\right\}+\exp\left\{\dfrac{H-z}{c}\right\}}\right) \tag{20}$$

The setup of parameters (a , b , c , d , H , β_0 , κ , λ) is chosen in such a way that the threshold for baroclinicity decreases in the PBL with height reaching its minimum value at 3 km above the surface (Fig. 9a). Thus, support of turbulence is preferred in areas with well expressed baroclinic zones in the mid-troposphere but avoids areas, where temperature gradients are amplified due to orography or upper frontal zones.

The limitation of the Richardson number yields:

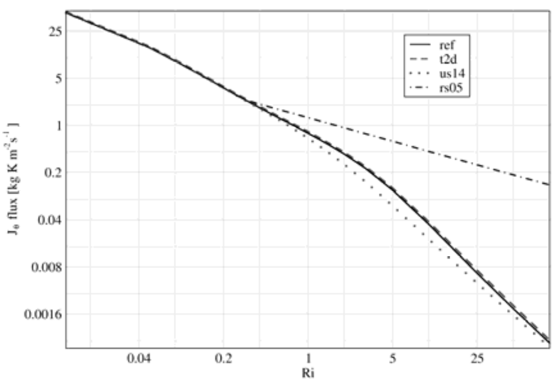
$$Ri' = \min\left(Ri_L, Ri\right) \qquad \text{if } \left|\nabla_p T\right| \geq \beta_T \tag{21}$$

The Ri_L parameter is here as a function, which further restricts the application of the scheme – preferring lower stability at levels at the top of the PBL (Figure 9b). The formula for Ri_L is similar to (20):

$$Ri_L = Ri_0 - d'\left(\frac{\kappa' z}{1+\dfrac{\kappa' z}{\lambda'}}\right)\left(a'+\frac{2}{\exp\left\{\dfrac{z-H'}{b'}\right\}+\exp\left\{\dfrac{H'-z}{c'}\right\}}\right) \tag{22}$$

The presented parameterization of turbulent fluxes is rather empirical – the setup of both β_T and Ri_L functions was chosen by experiments on case studies with rapid cyclogenesis. The impact of the scheme in baroclinic areas is much bigger than by "analytical" type of parameterization of section 4 or by the reference scheme of section 3 (Fig. 10). Thus, the new way of limitation allows a significantly higher rate of vertical diffusion if the Richardson number exceeds the local Ri_L threshold.

Figure 10. Dependence of the turbulent sensible heat flux (in kg K m^{-2} s^{-1}) on Richardson number. The solid line denotes the reference conditions (Richardson number modifications without baroclinic parameterization, setup of Ri_d number as by USD0 experiment). The dashed line is related to experiment with addition of the slantwise turbulent flux. The dotted line is the course of the flux with more stable setup of Ri_d as by USD7 experiment. The dash-dotted line shows the impact of the baroclinic Richardson number limitation corresponding to β_T and Ri_L setup shown in Figure 9.



6. CASE STUDIES

Operational forecasting of cyclones encounters two main problems. First is the prediction of very rapid cyclogenesis, which is usually a consequence of significant baroclinic instability and very strong flow in mid- and upper tropospheric levels. This development is almost always accompanied by severe weather and the cyclones are sometimes called “bombs” due to the “explosive” deepening of pressure in their centre (Bluestein, 1993). Typical examples were the event of 20 December 1998 cyclogenesis (Fig. 11) and the so called “Christimas storms” of 25 and 27 December 1999 which caused severe weather over the large territory of France and Germany (Kurz, 2001). Fortunately, such events are rare but it is very important to forecast them.

The second problem is forecasting of unrealistically deep mesoscale cyclones which do not occur in the real atmosphere. The unofficial name of these features is “Arpègead” or “Aladinad”, because the problem was first recognised in the ARPÈGE/ALADIN models (Tardy, 2003). However, it is still uncertain if the problem is specific for these numerical models. It was shown that the occurrence of false cyclogenesis is linked with unrealistically strong PV anomalies in the lower and middle troposphere, which are most probably the result of diabatic processes (Simon and Váňa, 2003). False cyclogenesis can be successfully suppressed with the introduction of semi-Lagrangean horizontal diffusion - SLHD (Váňa et al., 2007). It was speculated that false cyclogenesis rises due to absence of horizontal turbulent fluxes, which can be replaced by SLHD. However, real physical reasons of this phenomenon remain unknown. A typical example is the case of false Adriatic cyclone of 20 July 2001 (Vakula, 2002).

During the tests 13 cases were evaluated on forecasts of rapid (or severe weather) cyclogenesis and 6 cases were related to generation of false mesoscale cyclones. The parameterization of slantwise turbulence described in section 4 showed a rather neutral or slightly negative impact in cases with rapid cyclogenesis and in comparison to the reference ARPÈGE parameterization (section 3). It is interesting that in the 20 December 1998 case the scheme rather increased the forecast of pressure both in cyclonic and anticyclonic regions (Fig. 12). However, the forecast of the cyclone over La Manche Channel was not succesfull (as well as for the reference forecast, which is not shown). On the other hand, baroclinic limitation of Richardson number (section 5) helped to forecast the position and depth of the cyclone with very good precision (Fig. 13). This scheme showed substantial improvements of forecasts also in the cases of the 1999 “Christimas storms” and of the cyclone “Kyrill” on 19 January 2007.

The 20 July 2001 false cyclogenesis is one of the most resistant cases. Except for the use of the SLHD scheme (Váňa, 2003) the experiments with changes of physical parameterization or introduction of different initial conditions were not able to completely solve this problem and give a correct forecast. Tests with slantwise turbulence parameterization in ALADIN model show both positive and negative slantwise fluxes in the vicinity of the false cyclone (Fig. 14). Nevertheless, the impact on mean sea level pressure was not very big, the cyclone became less deep approximately by 2 hPa against the reference forecast (999 hPa) which was not using the slantwise flux parameterization. On the other hand, the baroclinic Richardson limitation makes the false cyclone even a bit deeper (Fig. 15). Both parameterizations have a rather neutral impact on other forecasts of false cyclogenesis, with the exception of the 3 May 2002 case, where both schemes successfully suppressed the generation of a false cyclone over the Genoa Bay.

Figure 11. ECMWF analysis of mean sea level pressure (in hPa) valid for 20 December 1998 00 UTC. Note the deep cyclone over La Manche channel that was the reason for the severe weather over a big part of France on 20 December 1998.

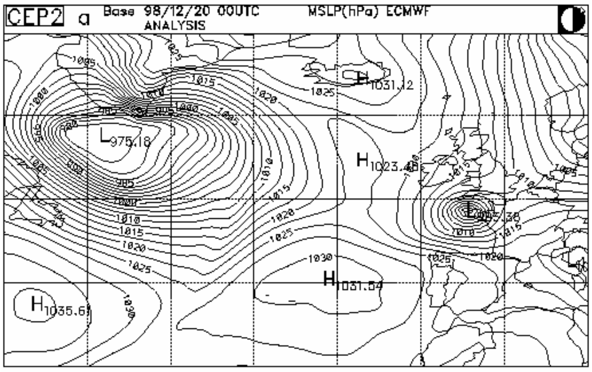


Figure 12. ARPÈGE model 84 h forecast of mean sea level pressure (in hPa) valid to 20 December 1998 00 UTC. The experiment was using the parameterization of slantwise turbulent flux for momentum and sensible heat.

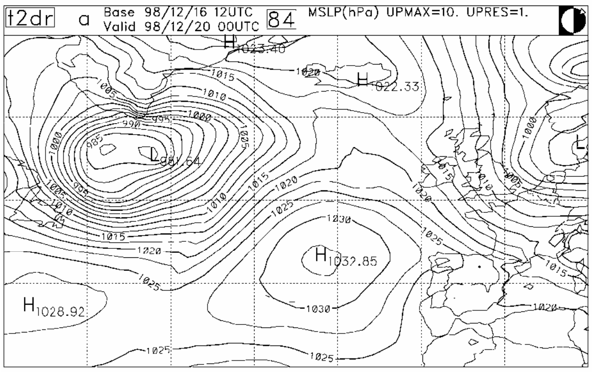
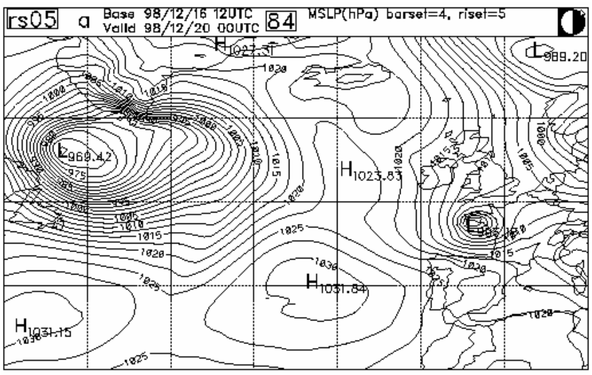


Figure 13. The same as Figure 12 except for the experiment with baroclinic Richardson number limitation.



7. COMPARISON OF BUDGETS AND TENDENCIES

Diagnostics on Domains in a Horizontal plain (DDH) is a method to compute temperature, moisture and energy budgets with the possibility of comparing different dynamical and physical fluxes. The method was developed at Mètèo France and it is used primarily in the global ARPÈGE model (Piriou, 2001). DDH is very suitable for objective evaluation of new physical parameterizations. This concerns also experiments on cyclogenesis, because forecasts on mean sea level pressure do not always show, if the physical processes supported by the scheme were realistic or not. Moreover, the DDH diagnostics gives more light on complicated feedback and interacttions between the physical parameterization and model dynamics.

For the standard ARPÈGE/ALADIN scheme (section 3), a difference in the temperature budget between the reference (USD0) and the stable setup of the modified Richardson number (USD7) shows change of polarity in temperature tendency around 800 hPa level (Fig. 16). Warming at low levels and cooling in the mid-troposphere means a drop of static stability which is mainly a consequence of increased turbulent fluxes supported by a decrease of the maximum allowed Richardson number.

Figure 14. ALADIN model 24 h forecast of mean sea level pressure (in hPa, solid lines) based on 20 July 2001 00 UTC and of slantwise turbulent flux (dashed lines are positive, solid are negative values) for the experiment with parameterization of slantwise turbulent exchange. The flux isolines are contoured for values: ± (50, 100, 150, 200, 250, 300, 400, 500, 600, 800, 1000, 1500, 2000 and 4000) 10^{-3} kg m^{-1} s^{-2}.

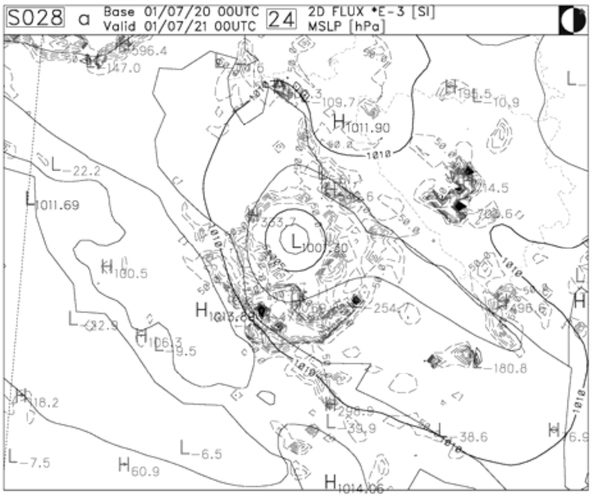


Figure 15. ALADIN model 24 h forecast of mean sea level pressure (in hPa) based on 20 July 2001 00 UTC with the experiment with baroclinic Richardson number limitation.

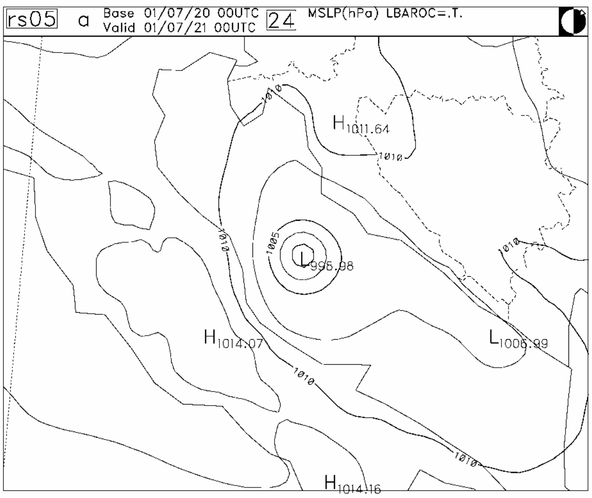


Figure 16. Differences in global tendencies of temperature (in K) between the USD0 and USD7 experiments during a 96 hour forecast period of the ARPÈGE model. The solid line shows the temperature tendency related to dynamic terms (dyn), contribution of precipitation fluxes (prec) is emphasized by diamonds, solar radiation (ray_sol) fluxes by rectangles. The tendency from thermal radiation (ray_ther) is drowned by dashed line with triangles, solid line with crosses denotes the so called residual terms (residu). The overall tendency (tend) is marked by a solid line with stars, the tendency from turbulence fluxes (tur) is shown by solid line with triangles.

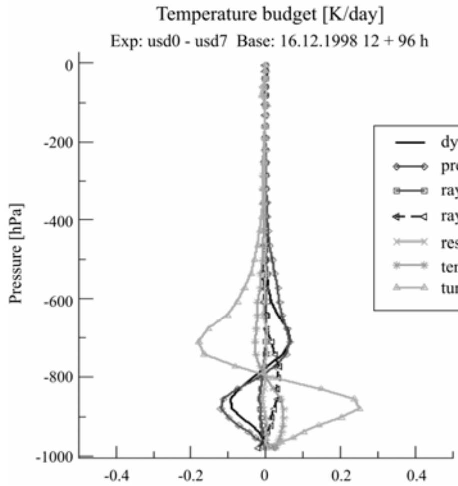
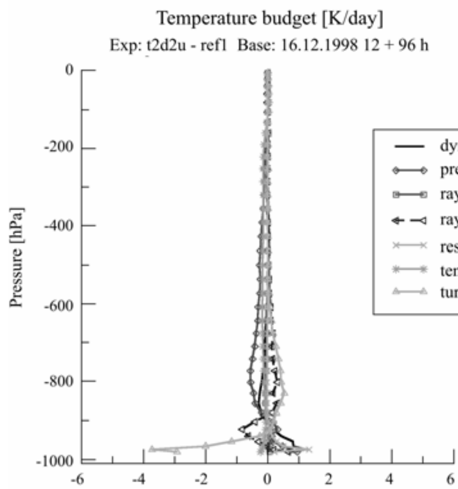


Figure 17. The same as 16 except for the slantwise turbulence parameterization



The comparison of the temperature budgets between the experiment with slantwise turbulence parameterization and the reference output not using this scheme shows a decrease of temperature at low levels, but an increase in static stability (Fig. 17). The influence of the scheme is rather neutral in the mid- and upper tropospheric levels. The strong impact of the turbulent fluxes at low levels is compensated for by dynamical tendencies (as advection terms) or precipitation flux. It can be shown that the magnitude of the turbulent flux is very dependent on the computation of the residual term (see equation 16). The simple addition of the slantwise turbulent flux shows a much smaller global impact (Fig. 18). The tendency from turbulent fluxes in this experiment is positive nearly until 925 hPa level and negative above. It is, however, also largely compensated by dynamical and precipitation fluxes. Thus, there is no systematic decrease of static stability as by experiments with suppressing of the Richardson number.

Baroclinic limitation of Richardson number shows a decrease of static stability around 750 hPa level (Fig. 19).

Figure 18. The same as in 17 but with exclusion of the residual term in equation (15)

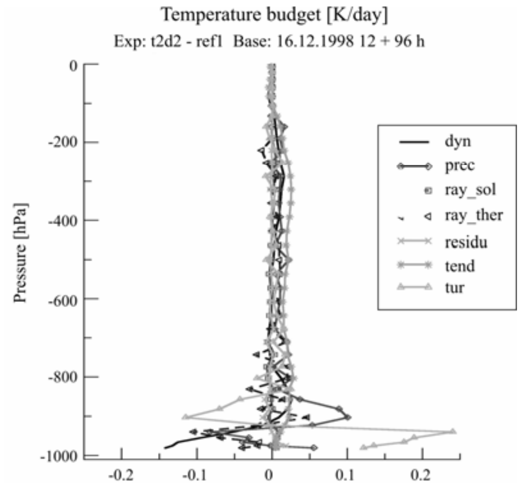
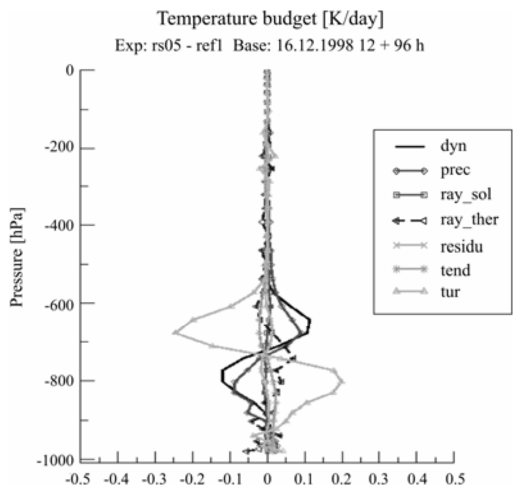


Figure 19. The same as 16 except for the baroclinic Richardson number limitation



Nevertheless, the global temperature tendencies are smaller as for the original limitation of the Richardson number, which is probably due to better selectiveness of the scheme related to environmental baroclinicity. Moreover, the impact is smaller at surface and the tendencies are faster decreasing in the upper troposphere (lesser probability of tropopause violation).

Moisture tendency is most significant by the baroclinic limitation of the Richardson number showing a systematic (but not big) decrease of moisture at tropopause. The increase of the kinetic energy budget is observed by all evaluated schemes, although the way of contribution of turbulent fluxes is very different. For example, a decrease of the Richardson number and higher exchange of momentum leads to kinetic energy damping in the PBL as predicted by theory (see e.g. Bluestein, 1992). Nevertheless, this is compensated for with baroclinic (dynamical) terms in kinetic energy tendency equation, what leads to small but positive global tendency of kinetic energy. This underlines how difficult is to evaluate and generalize the influence of turbulent diffusion on large scale atmospheric processes.

8. DISCUSSION

The presented study evaluated and compared three possible methods of parameterization of turbulent fluxes which have influence on forecasting cyclogenesis. It was shown that the contribution of turbulence to cyclogenesis is most effective in baroclinic zones. This motivated the construction of a scheme, which simulates turbulent transport of momentum along slanted (frontal) surface. Although the scheme is successfully working in both global and limited area model, it seems that at used horizontal resolutions (> 10 km) the magnitude and the influence of slantwise turbulent exchange is in most of the cases much smaller in comparison with vertical diffusion. This is supported also by analytical comparisons of vertical and slantwise turbulent fluxes. It looks also rather improbable that slantwise (or horizontal) distribution of momentum would have a key role in damping mesoscale cyclogenesis. Some recent experiments indicate that the problem of false cyclones is related to insufficient vertical diffusion of momentum and heat at the interface between unstable surface and very stable PBL layers aloft. Successful dynamical diffusions, like SLHD, are probably much more effective and several times overlapping the impact of physical horizontal diffusion valid for current spatial resolution. Slantwise turbulent fluxes are also not related with decrease of static stability at frontal boundaries and cannot produce results, which were achieved by limitation of the Richardson number.

Analytical studies suggest that the importance of horizontal (slantwise) turbulent exchange will rise with an increase of the model resolution and with the possibility to simulate meso- and microscale frontal boundaries (like gustfronts), which are characterized by high horizontal wind shears and temperature gradients. A more objective look on turbulence at baroclinic zones could be done with the aid of two- or three- dimensional Large Eddie Simulations (LES). However, this would need the running of a non-hydrostatic model with spatial resolution of around 100 m and on a large domain (of at least 1000 km wide) to obtain realistic simulation of frontal circulation. Hence, a similar experiment would be computationally very costly and demanding at the present time.

The aim of a simple and computationally effective baroclinic limitation of the Richardson number was rather to find a reasonable compromise between the realistic PBL simulation and the possibilities to forecast extreme cyclogenetic events. A decrease of static stability tied to baroclinic zones is less affecting the forecasts of temperature inversions in anticyclones or in non-frontal areas of cyclones. This idea is supported also by long range scores which show neutral or slightly positive impact on certain surface or upper air fields (e.g. geopotential). On the other hand, the parameterization of slantwise turbulent fluxes should be treated as preliminary. Further development and testing are needed to make it more realistic and feasible for operational praxis (e.g. determination of the so called residual term with higher precision).

Results presented in this paper advise that modification of turbulent fluxes alone give neither ultimate nor realistic solutions to forecast cyclogenesis. Regulation of the stratiform precipitation scheme and related latent heat release has at least comparable impact on low level PV distribution and formation of cyclones. Hence, correct setting of physical parameterization with respect to cyclogenesis will be difficult also in newer generations of numerical models as ALADIN/ALARO-3MT (Brožková, 2006) and by different parameterizations of turbulent fluxes: TKE scheme in ARPÈGE and AROME models and the so called pseudo TKE method in ALADIN/ALARO-3MT. Because the latter scheme uses the calculation of first order closure K- coefficients as a first guess for stationary solutions of the TKE prognostic equation, the development of Richardson number modification will likely continue. Formulations dependent on environmental baroclinicity probably show a way, how to find compromises and increase the efficiency in some other physical parameterizations as well. Examples are specification of the mixing length or of the PBL height, which could be a topic for future studies.

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