9. Jozef VIVODA: "Application of the predictor-corrector method to non-hydrostatic dynamics"

Application of the Predictor-Corrector method to non-hydrostatic dynamics stability of Rossby-Haurwitz modes. Theoretical and practical aspects.

1. Introduction

The class of schemes that are iterative approximations to fully implicit schemes applied on the Euler-equations (EE) system was studied in Bénard (2003). Any scheme of that kind is referred to as Iterative Centred Implicit (ICI) scheme in further text. The Semi-implicit (SI) scheme and Predictor/Corrector (PC) scheme are special cases of ICI scheme, with one respectively two solutions of the semi-implicit solver.

The EE system permits the gravity, acoustic and Rossby-Haurwitz (RH) modes. The stability analysis of the gravity and acoustic modes has been performed in Bénard, (2003) with EE system cast in hydrostatic pressure based σ vertical coordinate of Laprise, (1992). The stability of small amplitude oscillations was studied in an isothermal atmosphere with temperature T_0 over flat terrain. It was found that the gravity and acoustic modes are conditionally stable even in the limit of infinite time step, when an appropriate set of prognostic variables is used. The two-time-level predictor/corrector scheme is stable, if T_0 and the SI reference temperature T^* satisfy condition : $1/2\ T^* < T_0 < 2T^*$. Nevertheless, the classical two-time-level (2TL) SI scheme is stable only if the background and the SI reference temperatures are equal, and the PC scheme becomes compulsory for realistic simulations which are naturally non-isothermal.

The Rossby-Haurwitz (RH) modes are usually treated explicitly in the models with the SI time stepping. With introduction of the two-time-level schemes with semi-Lagrangian (SL) advection treatment, the Coriolis force terms are treated semi-implicitly in some models (Temperton, 1997). The RH modes are partially implicit in a PC scheme as well. It is known that the implicit treatment of a mode slows it down. This is an acceptable behaviour for the modes that are not of meteorological interest (gravity modes in large scale hydrostatic models, or acoustic modes in non-hydrostatic models). However, RH modes are the main synoptic scale signals and their accurate representation is a necessary condition to consider the scheme to be applicable in a local area model.

Here, we analyse the free RH modes on a middle latitude β plane in the framework described previously. We are interested in the stability and accuracy properties of the RH modes in the limit of long time steps under the conditions for which the gravity and acoustic modes are stable.

2. The linear system with RH modes

The analysed linearized system of prognostic equation around isothermal, resting and isothermally balanced state yields :

$$\frac{\partial D}{\partial t} = -R\mathbf{G}\Delta T + RT_0(\mathbf{G}-1)\Delta P - RT_0\Delta q + f_0\xi - \beta U$$

$$\frac{\partial d}{\partial t} = -\frac{g^2}{RT_0}\cdot\mathbf{L}P$$

$$\frac{\partial T}{\partial t} = -\frac{RT_0}{Cv}\cdot(D+d) \qquad (1)$$

$$\frac{\partial P}{\partial t} = \mathbf{S}D - \frac{Cp}{Cv}\cdot(D+d)$$

$$\frac{\partial q}{\partial t} = -\mathbf{N}D$$

$$\frac{\partial \xi}{\partial t} = -f_0 D - \beta V$$

We have used the formulation with prognostic variables $q=\ln(\pi_s)$, $d=-gp/\pi RT\cdot\sigma\cdot\partial w/\partial\sigma$, and $P=(p-\pi)/\pi$. f_0 is the Coriolis parameter on a reference latitude and β is the South-North horizontal derivative of the Coriolis parameter on that reference latitude. This approximation is sufficient to describe the behaviour of RH modes on a limited size middle-latitude domain (Lindzen, 1967). The system is further referred to as *A* and is formulated using divergence and vorticity as prognostic variables (D resp. ξ). We consider the tangent plane, therefore a local Cartesian horizontal coordinate system (x,y) can be introduced and the arrays can be decomposed horizontally into bi-Fourier series. In such a framework, the horizontal wind components can be expressed in the terms of divergence and vorticity as :

$$U = \frac{ikD - is\xi}{-k^2 - s^2} \quad , \quad V = \frac{ikD + is\xi}{-k^2 - s^2} \qquad (2)$$

(k, s) are the horizontal wavenumbers defined along axes on horizontal plane.

To obtain, the SI linear system *B*, used in this study, we set f_0=0 and β=0, and substitute T_0 by the SI reference temperature T* in the system [1].

3. The two-time-level non-extrapolating ICI scheme

We analyse the stability and accuracy properties of the two-time-level non-extrapolating ICI scheme. The general ICI scheme written in vector formalism is :

$$\frac{X[t+\Delta t(n)] - X[t]}{\Delta t} = A\cdot\frac{X[t+\Delta t(n-1)] + X[t]}{2} + B\cdot\frac{X[t+\Delta t(n)] - X[t+\Delta t(n-1)]}{2} \qquad (3)$$

with state vector $X=(D, d, P, q, T, \xi)$. The n index denotes the order of iteration ($n\in(1,...,NSITER$; n=0 SI scheme, n=1 PC scheme). When the process converges the last term on the rhs will converge to zero. The state at the beginning X[t+Δt(0)] is unknown and [3] cannot be used to figure it out. The non-extrapolating two-time-level SI scheme is used to calculate it :

$$\frac{X[t+\Delta t(0)] - X[t]}{\Delta t} = A\cdot X[t] + B\cdot\frac{X[t+\Delta t(0)] - X[t]}{2} \qquad (4)$$

and therefore the particular ICI scheme is referred to as non-extrapolating ICI scheme.

4. The stability and phase speed error of RH modes in the two-time-level non-extrapolating ICI scheme

Eigenfunctions of the system on the tangent plane are expressed in the form :

$$X(x,\sigma) = \hat{X} e^{i(kx+sy)} \sigma^{i\nu - \frac{1}{2}} \tag{5}$$

with ν being the dimensionless vertical wavenumber. The associated vertical wavelength for a given ν in an isothermal atmosphere is $l_z = 2\pi/\nu\, H$.

Assuming that the amplitude of the mode is time dependent and :

$$\hat{X}[t+\Delta t) = \lambda \cdot \hat{X}[t]$$

we can apply the classical Von-Neumann analysis of stability and look for complex amplification factors λ. The stability is defined by the magnitude of λ and the relative phase speed error r_p can be computed as $r_p = \arg(\lambda) / \omega_a \Delta t$. The analytical frequency ω_a is computed from the dispersion relation of the system [1].

The stability was computed for the horizontal spectrum of continental size domain (the zonal horizontal wavelength varies from 36 km to 3600 km). The meridional wavelength was fixed to 3600 km. The time step length used in analysis was 600 s. It is the settings of the case study in the following section.

The temperature of the isothermal atmosphere is T_0 = 300 K (vertical height scale H = 8.8 km). The SI scheme is determined by the reference temperature T^*= 300 K and the acoustic reference temperature T_a^* = 50 K (for detailed explanation see Bénard, 2004).

We analyse three vertical modes to assess the behaviour of the RH modes in the whole spectrum. The mode with ν=0, that represents the external mode of the model with top in infinity. To sample whole spectra we further choose the modes with the vertical wavelengths 10 km and 2 km (ν=5.5 and ν=27.5) . They are referred to as "slightly internal" and "internal" mode in further text.

The stability of predictor and corrector of the two-time-level PC scheme is shown in Figure 1. The dependence of stability on the horizontal wavelength (given in km) is shown. The modes are stable for all horizontal scales (from 36 km to 3600 km).

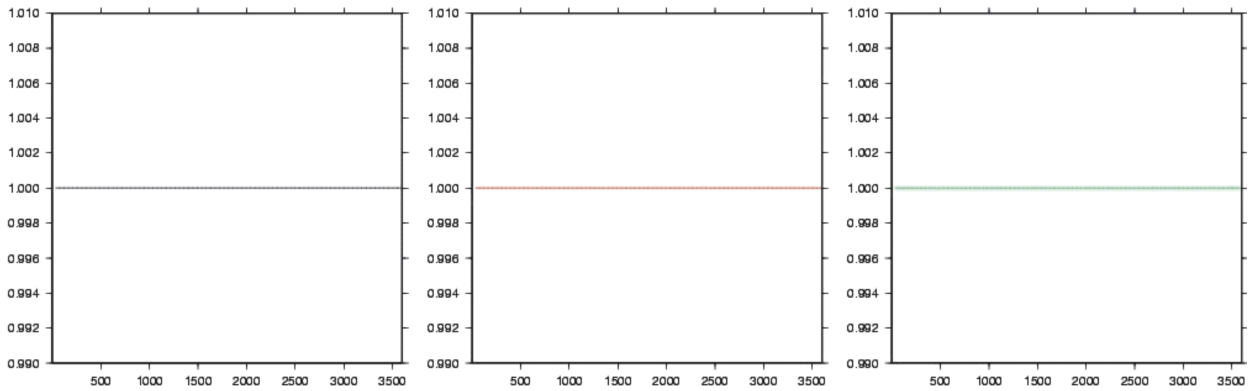

Figure 1 : The stability of external, slightly internal and internal RH modes as a function of zonal horizontal wavelength given in km. Meridional horizontal wavelength was fixed to 3600 km. The modes are stable for the whole horizontal spectrum.

The relative phase-speed error is shown in Figure 2. Solid curve represents the predictor and dashed curve the corrector. The external, slightly internal and internal modes are plotted as the left, middle and the right plot. The result is opposite to what could have been naively expected. The PC scheme accelerates the RH modes. However, the acceleration is very small about 0.1 %. The horizontal wavelength of accelerated modes depends on the vertical mode. The magnitude of acceleration is negligible comparing to the effects caused by other, mainly physical processes.

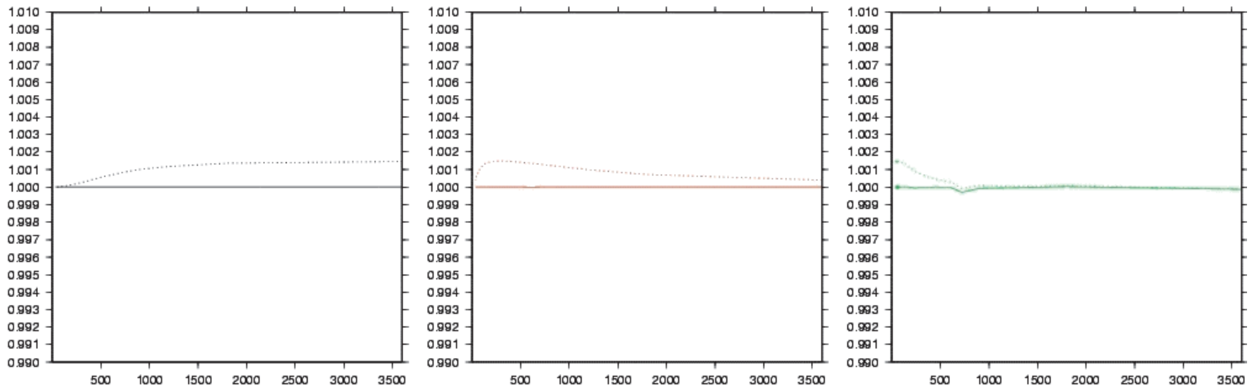

Figure 2 : The relative phase speed error of external, slightly internal and internal RH modes as a function of zonal horizontal wavelength given in km. Meridional horizontal wavelength was fixed to 3600 km. The PC scheme slightly accelerates the modes. The magnitude of acceleration is about 0.1 % and it is too small to have any practical impact on mesoscale model simulations.

5. Lothar storm case study

To test the accuracy of the model for synoptic scales the Lothar storm case was chosen. It is the first of the two storms from December 1999 that passed over Europe during Christmas. The meso-cyclone formed over Atlantic Ocean and it moved rapidly towards Europe. During the 26th of December 1999 the storm hit France, Germany, Switzerland and Italy and caused severe damages. Operational forecast of the global model ARPEGE provided reasonable guidance for Lothar storm. When ALADIN is coupled with ARPEGE prediction using quadratic coupling or 3 hour frequency coupling, the prediction of Lothar storm is sufficiently accurate to be considered successful in the view of current LAM performance. In Figure 3, we can see the 26th December 1999 analysis of ARPEGE at 12, 18 and 24 UTC.

The Lothar storm is possible to predict using the dry adiabatic model version. However, such prediction is not possible to verify against the model analysis. Therefore we used the diabatic version of model ALADIN. We run the prediction using initial condition from 12 UTC on the 25th of December 1999. The reference run was obtained with hydrostatic model version using the two-time-level SI SL scheme with `LSETTLS` extrapolation (Figure 4). The integration domain was the ALADIN/LACE domain, the horizontal mesh size is 12.2 km and Δt =600 s. The position of the cyclone is shifted slightly to the east and is 1 hPa shallower (972 hPa in the centre) than in the analysis (left plot in Figure 3).

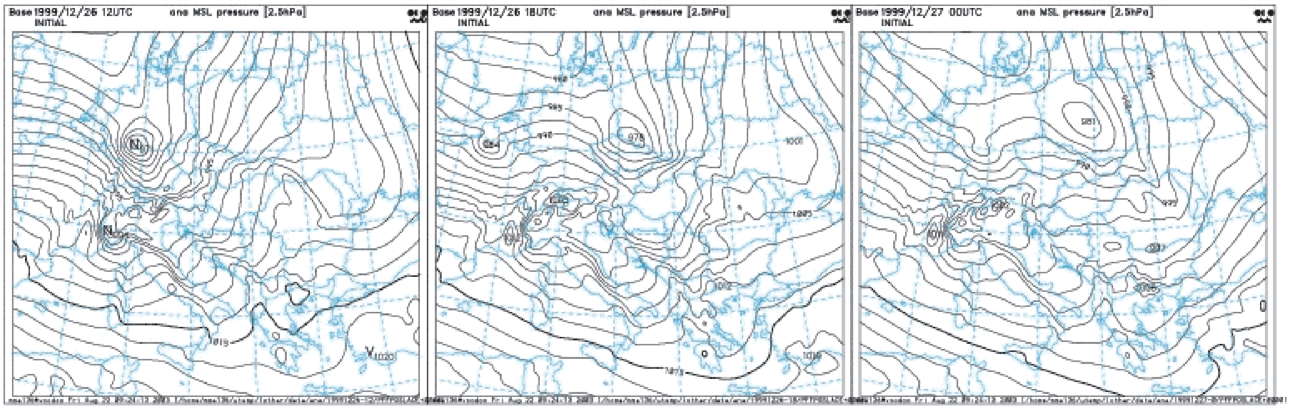

Figure 3 : The 3d-var analysis of model ARPEGE, of the Lothar storm case from the 26th of December 1999. The mean-sea-level pressure is plotted with contour interval 2.5 hPa. The storm moved eastward over France, Germany and finally during the 27th of December 1999 it weakened over southwest Poland.

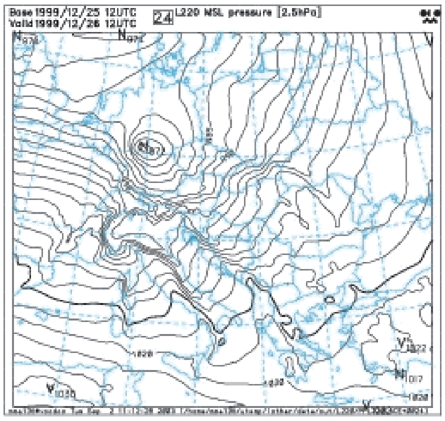

Figure 4 : The 24 h reference prediction of Lothar storm from 25th of December 1999 12 UTC. The prediction obtained with hydrostatic model version with 2TL SI SL LSETTLS scheme. The results obtained with the 2TL SI SL non-extrapolating scheme

To assess the impact of ICI scheme itself, we integrated the Lothar case using the two time-level non-extrapolating ICI scheme with hydrostatic model version. The method of extrapolation used to compute nonlinear residuals and the wind for trajectories was changed. The first order accuracy algorithm in time was used during predictor, while LSETTLS approach is second order accuracy. The second order accuracy is restored during subsequent iterations.

The physics was called once per time step, during predictor. The physical tendencies were considered to be valid in departure point of SL trajectories at time instant *t*.

In the left column of Figure 5, the 24 h prediction is showed computed with ICI scheme using different number of iterations (equivalent to n=0 and 1 as defined in [3]). The top left plot is calculated using two-time-level SI non-extrapolating scheme described by [4]. As it was mentioned already, it is a first order accuracy scheme, but the obtained results are reasonable in the case of Lothar storm. The cyclone is well positioned and the minimum mean-sea-level pressure (MSLP) is underestimated by 1 hPa comparing to analysis. Further iterations of the SI solver using ICI scheme did not bring any significant change in storm prediction. The differences between the SI non-extrapolating scheme and the PC scheme predictions are showed on top right plot of Figure 5 The maximum difference is 2.1 hPa. The PC scheme prediction itself is displayed in the bottom left plot and the differences gained by further iteration of ICI scheme are showed on the bottom right plot. It is apparent that the scheme is converging since the differences between successive iterations of the ICI scheme converge towards zero.

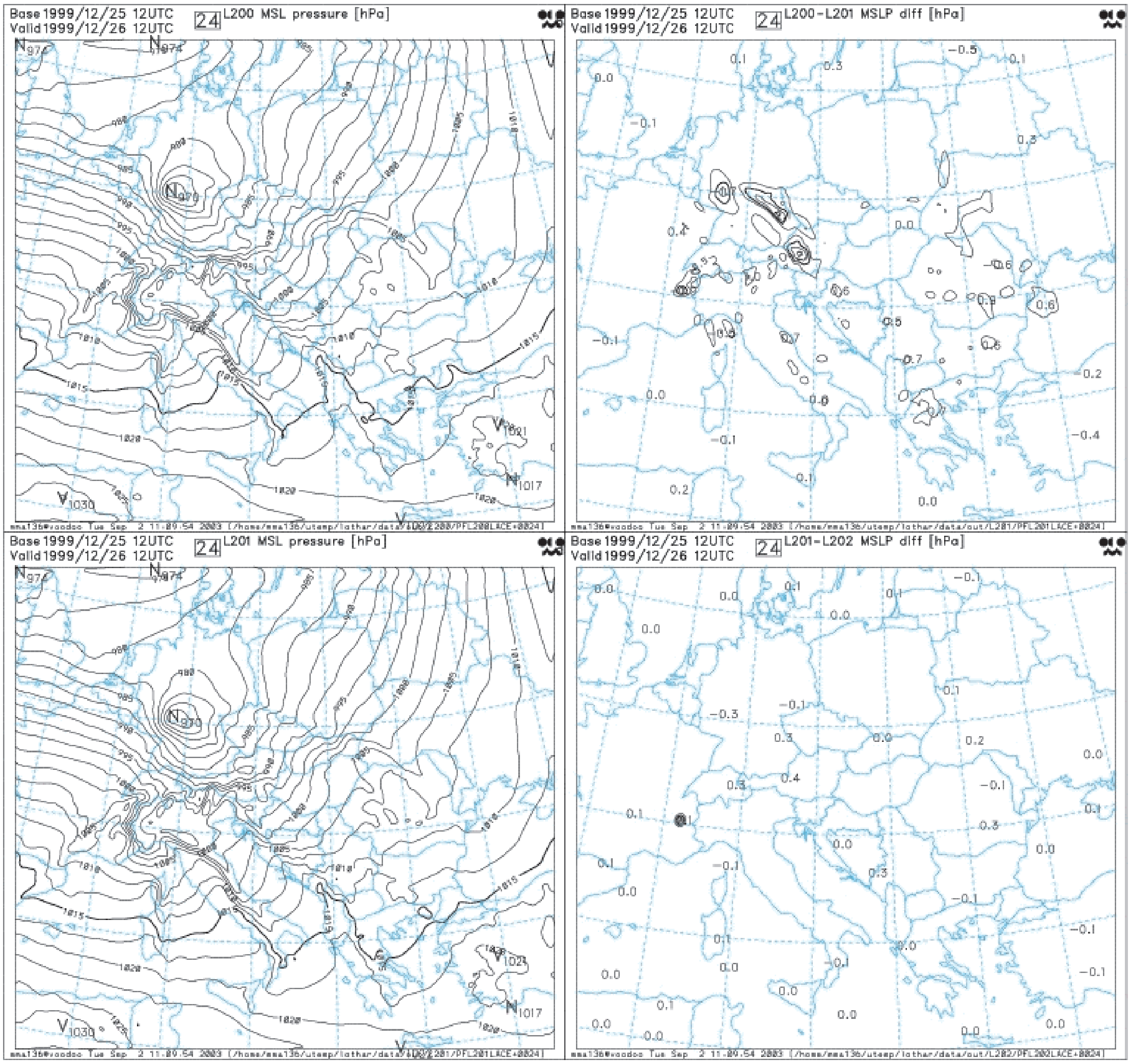

Figure 5 : The 24 h prediction from 25th of December 1999 12 UTC obtained with the 2TL ICI SL non-extrapolating scheme with n=0 (top left plot) and n=1 (bottom left plot). The differences between the runs with different number of iterations of ICI scheme are plotted in the right column. The differences between the SI scheme and the PC scheme runs are on top right plot, and the differences between the PC scheme and ICI scheme with n=2 are on the bottom right plot.

We have integrated the Lothar case using the non-hydrostatic model version as well. We used the prognostic variable *d*4 (namelist variable `NVDVAR=4`) and we set the SI acoustic temperature to 50K and keep the value of SI gravity temperature 350 K (namelist variables `SITRA=50` K and `SITR=350` K, for detailed explanation see Bénard, 2004). We used the 2TL ICI non-extrapolating scheme with SL advection. The results are showed on Figure 6. Comparing to the equivalent hydrostatic run, there is a further underestimation of the minimum MSLP by 1 hPa, but this difference disappears after the first iteration of ICI scheme. The results obtained with the non-hydrostatic model version are almost identical to the hydrostatic results. This proves that the tested scheme applied on ALADIN's non-hydrostatic dynamics efficiently stabilises all the possible sources of instabilities for the "hydrostatic" time step length (orographically induced waves, SHB temperature and pressure instabilities), while keeping the quality of the simulation of the large scales as in the hydrostatic model. We can deduce from Figure 6, that the PC scheme is fully sufficient to integrate the model for resolution around 10 km, because there is no significant change in the results when using two or more iterations of the SI solver.

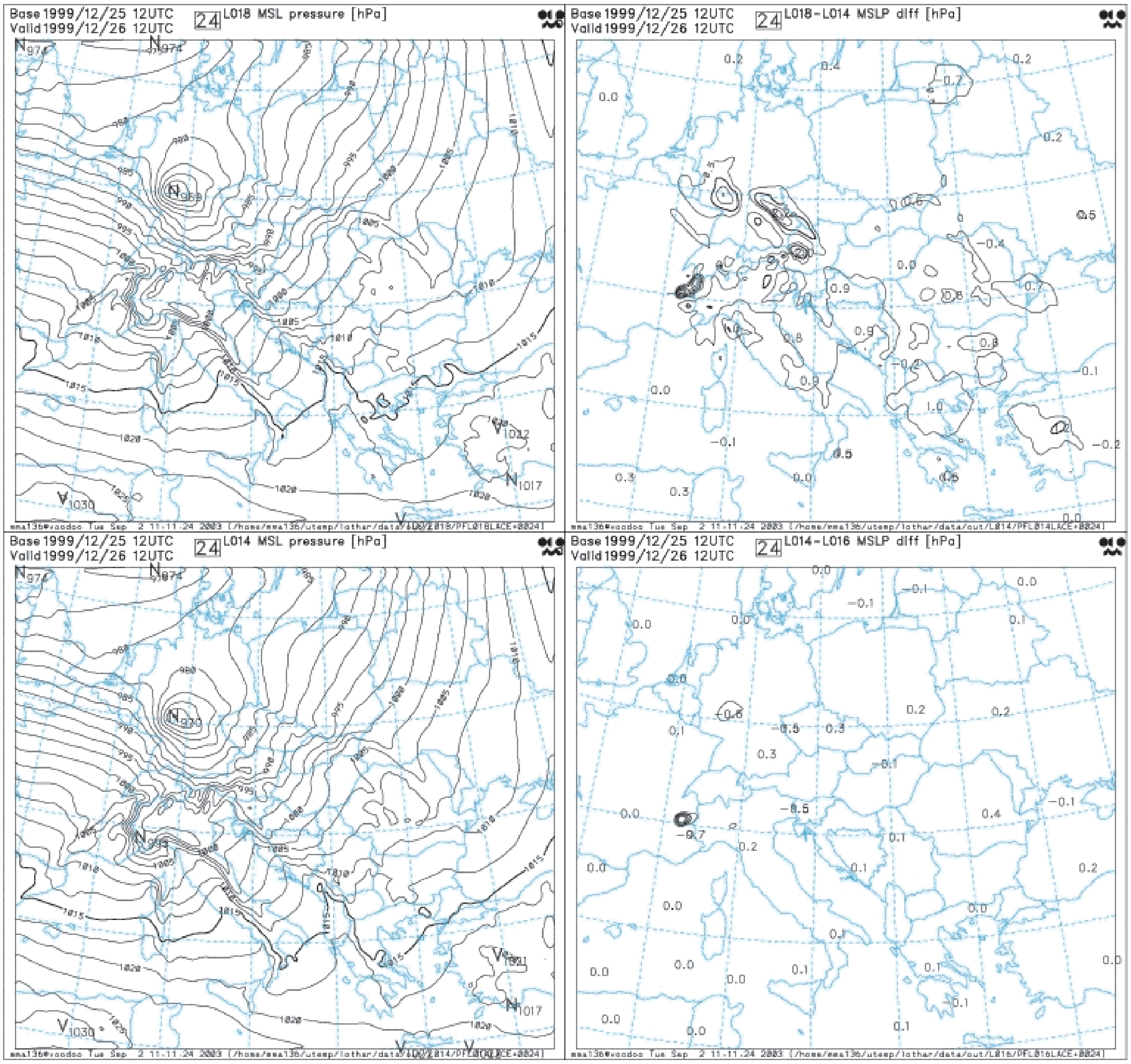

Figure 6 : The same as on Figure 5, but obtained with the non-hydrostatic version of ALADIN.

The aim of this study was to show that the 2TL ICI scheme doesn't deteriorate results obtained with the standard SI scheme when applied to a synoptic scale case. The linear analysis indicated that the RH modes are slightly accelerated by two-time-level PC scheme. The frequencies of the RH modes are 0.1 % greater than their analytical value. However, there are still nonlinear residuals not included in the analysis and a case study with the full nonlinear model has to be carried out to draw a final conclusions. We showed that the acceleration and remaining nonlinear residuals had no impact on accuracy of short-range diabatic model integration.

The results obtained with 2TL ICI SL non-extrapolating scheme with either hydrostatic or non-hydrostatic version are almost equivalent to the reference results from the hydrostatic model integrated using the 2TL SI SL LSETTLS scheme at the 10 km scale. This confirmed that the partially implicit treatment of slow modes in ICI scheme doesn't deteriorate them.

The simulations using the non-hydrostatic model version further proved that the methods used to stabilise acoustic modes and the orographically induced instabilities are neutral to slow modes. The Lothar case study proved that for resolutions around 10 km the 2TL SL PC non-extrapolating scheme without additional filtering is sufficient to integrate NH model ALADIN formulated with the prognostic variables $d4$, P and q. No significant improvement can be gained by additional iterations of the SI solver.

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