8. Chistopher SMITH :"Stability analysis and precision aspects of the boundary condition formulation in the non-hydrostatic dynamics and exploration of the alternatives for discrete formulation of the vertical acceleration equation both in Eulerian and semi-Lagrangian time marching schemes"

Research Objectives

The purpose of this research project is to investigate deficiencies in the non-hydrostatic (NH) version of ALADIN, specifically with regard to the upper and lower boundary conditions. In numerical weather prediction (NWP) it is standard practice to separate "dynamics" processes from "physics" processes. The same methodology is adopted in NH-ALADIN. For this reason it is necessary that NH-ALADIN should have a "dynamical core" which is capable of solving the inviscid, adiabatic equations of non-hydrostatic fluid flow. For these equations the correct boundary condition at a solid boundary is the so-called free-slip condition. This condition requires that there be no flow in the direction normal to the boundary, and that there be no lateral stresses in the fluid near the boundary. Since an inviscid fluid is incapable of supporting lateral stresses, this boundary condition does not provide additional information to the specification of the problem; rather it is a necessary condition, which must always be satisfied by an inviscid system.

An important tool for the development of NH-ALADIN is the two-dimensional (2D), vertical slice version of the model. This tool allows model results to be compared against well-known analytical results, making it invaluable for assessing whether the model captures true non-hydrostatic behaviour. Experiments with this 2D model indicate that NH-ALADIN sometimes generates spurious behaviour over orography, particularly when a semi-Lagrangian (SL) time-stepping scheme is used. However, this does not indicate any superiority in the Eulerian time scheme. All these tests involve computing steady-state solutions, for which an Eulerian scheme has a clear advantage. This is because an Eulerian scheme uses spatial coordinates which are independent of time, whereas a semi-Lagrangian scheme makes use of Lagrangian, time-varying coordinates. For realistic applications, where the flow is far from steady-state, this advantage is likely to be absent. For reasons of robustness and stability when long time-steps are used, the semi-Lagrangian scheme is much to be preferred over the Eulerian scheme. The problems suffered by the SL scheme in the idealised 2D test cases therefore require a solution.

Research Conclusions

Further numerical investigations, again with the 2D model, showed that the discretisation of the vertical momentum equation was strongly implicated in the spurious model behaviour. There is now strong evidence to suggest that the spurious behaviour is closely related to the fact that vertical divergence is used as the prognostic quantity describing vertical motion. We may list some weaknesses associated with this choice of prognostic variable:

1. The numerical treatment of vertical motion differs markedly from that of horizontal motion. For horizontal motion the prognostic quantities are the components of horizontal wind. Whereas vertical motion is represented by vertical divergence, not vertical wind. This difference is perhaps undesirable for very high resolution modelling.
2. Under certain flow conditions the numerical scheme for solving the vertical motion equation may be unstable. This is due to the fact that a significant amount of downstream data may be used in the computation of an advection increment.

3. Whether or not the numerical scheme for vertical motion suffers the above instability, it has bad error propagation characteristics. An error introduced near the lower boundary is immediately propagated to all the higher grid levels at the next time-step.
4. The prognostic equation for vertical divergence contains more source terms than that for vertical wind. All source terms must be discretised using finite differences in the vertical. Currently, the finite difference representations in NH-ALADIN are first order accurate only. Therefore using vertical divergence, rather than vertical wind, as a prognostic quantity necessarily reduces the overall accuracy of the model.
5. It is a non-trivial matter to introduce the free-slip lower boundary condition as a necessary constraint on the discrete NH-ALADIN model. This is due to the fact that the semi-implicit stage of the scheme does not allow for horizontal variation in the coefficients of the Helmholtz equation. However, the situation is further complicated if vertical divergence is the prognostic variable. This is because the free-slip boundary condition is most naturally expressed in terms of wind components, not their derivatives.

Proposed Solution

The above conclusions indicate that a change should be made in NH-ALADIN: to use vertical wind, rather than vertical divergence, as a prognostic variable. The strongest case for this change is made by the last item in the above list. Failure to respect the free-slip lower boundary condition, as a necessary constraint on the discretised equations, results in the spurious behaviour seen in the 2D tests. When vertical wind is used as a prognostic variable it is possible to apply a pragmatic remedy to the lower boundary condition problem. By simply over-writing the vertical wind data on the lower boundary, using the boundary condition, it is possible to obtain correct solutions over a reasonably wide range of flow situations.

Future Work

The proposed implementation of the free-slip constraint at the lower boundary requires further testing, in a three-dimensional NWP context. There also remain issues to be addressed relating to the stability properties of the semi-implicit scheme for the NH-ALADIN model. Vertical divergence is used to formulate the semi-implicit stage, regardless of whether vertical wind or vertical divergence is the prognostic quantity. The robustness of the model may possibly be enhanced if the semi-implicit stage is always formulated using the chosen prognostic quantity.

Conclusion

It has been demonstrated that NH-ALADIN does not satisfy a necessary constraint, imposed by the free-slip lower boundary condition. As a consequence of this the model often exhibits spurious behaviour when simulating flow over orography. A complete solution to this problem is not currently available, due to restrictions imposed by the spectral formulation on the kind of implicit equations which may be solved in the semi-implicit stage. A pragmatic compromise solution has been proposed. This solution requires that the prognostic quantity representing vertical motion should be vertical wind, rather than vertical divergence. Additional arguments against using vertical divergence as a prognostic quantity have also been put forward.