First steps towards the application of the Ensemble Transform Kalman Filter technique at the Hungarian Meteorological Service

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- Data assimilation
- Theoretical aspects of ETKF
- Realization of ETKF at HMS

Data assimilation

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How?

1. Optimal interpolation
2. Variational methods
3. Kalman filter

Kalman filter

Analysis:

$$x_a = x_f + P_f H^\top (H P_f H^\top + P_o)^{-1} (y - \mathscr{H}(x_f))$$

Kalman filter

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Denotion

P_f	error covariance matrix of forecast
P_a	error covariance matrix of analysis
P_o	error covariance matrix of observations
P_M	error covariance matrix of model error and linearization

Kalman filter

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M	linearized model ($i \rightarrow i+1$)
$\mathcal{H}$	observation operator
H	linearized of $\mathcal{H}$
$(\cdot)^\top$	transpose
i	time level

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Idea: *Kalman* (1960):

$$P_f^{i+1} = M P_a^i M^\top + P_M^i$$

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However: **not applicable in weather forecast**

- $P_f \in \mathbb{R}^{n \times n}$, $n \approx 10^7$
- $M \in \mathbb{R}^{n \times n}$ $\Rightarrow$ n **integrations**
- **need for tangent linear and adjoint models**

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$\Rightarrow$ **Extended KF**

$\Rightarrow$ **Ensemble KF**

$\Rightarrow$ **Reduced Rank KF**

$\Rightarrow$ **Ensemble Transform KF**

Ensemble Transform Kalman Filter

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Definition

Dispersions: $z_j := x_j - \overline{x} \quad (j = 1, ..., k)$

Their "vector": $Z := \frac{1}{\sqrt{k-1}}\left(z_1, z_2, \ldots, z_k\right)$

Error cov. matrix: $P \approx \frac{1}{k-1}\sum_{j=1}^{k}(x_j - \overline{x})(x_j - \overline{x})^\top = ZZ^\top$

(Each for f and a as well.)

Ensemble Transform Kalman Filter

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Question: transformation matrix T = ?

Ensemble Transform Kalman Filter

Assumptions

- **transformation:** $Z_a = Z_f T$
- **square-root:** $P_a = Z_a Z_a^\top$
- **from BLUE:** $P_a = (I - KH)P_f$

  $K = P_f H^\top (H P_f H^\top + P_o)^{-1}$ (Kalman gain)

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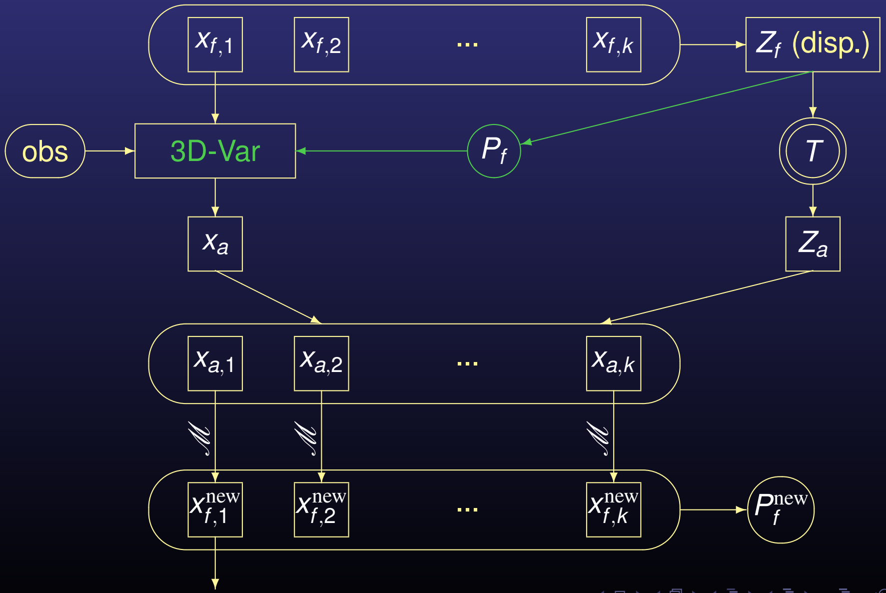
Bishop et al. (2001) $\Rightarrow$ $\boxed{T = C(\Gamma + I)^{-1/2}}$ **with**

$$Z_f^\top H^\top P_o^{-1} H Z_f = C \Gamma C^\top \quad \in \mathbb{R}^{k \times k}$$

(eigenvectors, eigenvalues)

Ensemble Transform Kalman Filter



Realization of ETKF

1. **Build matrix** $Z_f^\top H^\top P_o^{-1} H Z_f$
2. **Solve the eigenvalue problem** $Z_f^\top H^\top P_o^{-1} H Z_f = C \Gamma C^\top$
3. **Create transformation matrix** $T = C(\Gamma + I)^{-1/2}$
4. **Construct analysis dispersions with** $Z_a = Z_f T$
5. **Compute new analysis members with** $x_{a,j} = x_a + z_{a,j}$ $(j = 1, ..., k)$
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Sándor Kertész's idea

Matrix $Z_f^\top H^\top P_o^{-1} H Z_f = V^\top V$ **with** $V = \underbrace{P_0^{-1/2}}_{\sigma_{o,j}} \underbrace{HZ_f}_{?}$

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$\Rightarrow \quad Hz_{f,j} = \textbf{fg\_depar}_{\overline{x}_f} - \textbf{fg\_depar}_{x_{f,j}} \qquad j = 1, \dots, k$

OMSZ

Realization of ETKF

Hence:

$$V(i,j) = (P_0^{-1/2} H Z_f)(i,j) =$$

$$= \frac{1}{\sqrt{k-1}} \frac{1}{\textbf{obs\_error}(i)} \left[\textbf{fg\_depar}_{\overline{x}_f}(i) - \textbf{fg\_depar}_{x_{f,j}}(i) \right]$$

$$\Rightarrow \quad Z_f^\top H^\top P_o^{-1} H Z_f = V^\top V = \ \ldots$$

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Discussion

- **ETKF:** time-dependent P_f, ensemble system
- **Compare with EnKF:** less MIN but computation of T
- **HMS:** development is ready but no cycling
- **Next step:** cycling, diagnostics
- **Open problems:** sampling noise, coupling