# Stability of the physics-dynamics interface

**Rafiq Hamdi and Piet Termonia**

Royal Meteorological Institute of Belgium

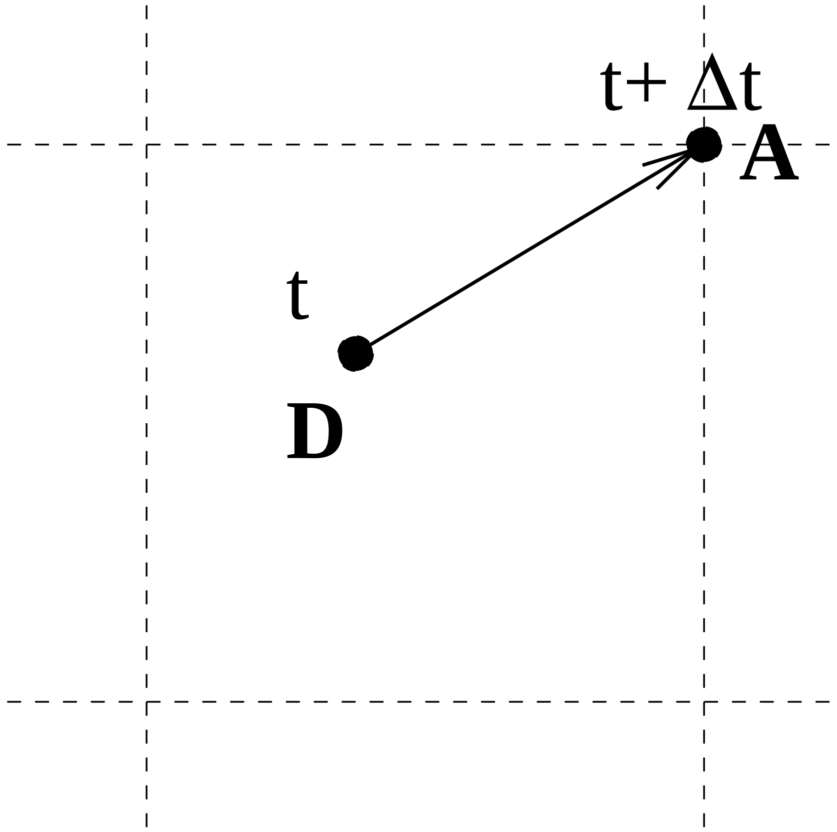
# Contents

- METHODOLOGY: drastically compressed version of my lectures at the TCWGPDI in Prague
- CHOICES: modular criteria for stability and accuracy $\leftrightarrow$ the organisation of the time step

# Room for coupling

- where on the semi-lagrangian trajectory?
- before or after the dynamics?
- parallel or sequential (= fractional)?

# Where on SL trajectory?



# Methodology

- simple system, but with EXACT solutions

# Methodology

- simple system, but with EXACT solutions
- Staniforth, Wood, and Côté, 2002

$$\frac{\partial F}{\partial t} + U\frac{\partial F}{\partial x} + i\omega F = -\beta F + R\,e^{i[kx+\Omega]}$$

# Methodology

- simple system, but with EXACT solutions
- Staniforth, Wood, and Côté, 2002

$$\frac{\partial F}{\partial t} + U\frac{\partial F}{\partial x} + i\omega F = -\beta F + R\, e^{i[kx+\Omega]}$$

- solution 1: free solution (homogeneous Eq.)

$$F(x,t) = F_k^{free} e^{-\beta t} e^{i[kx-(\omega+kU)t]}$$

# Methodology

- simple system, but with EXACT solutions
- Staniforth, Wood, and Côté, 2002

$$\frac{\partial F}{\partial t} + U\frac{\partial F}{\partial x} + i\omega F = -\beta F + R\, e^{i[kx+\Omega]}$$

- solution 2: forced regular solution

$$F(x,t) = \frac{R}{\beta + i(\omega + kU + \Omega)} e^{i[kx+\Omega t]}$$

$$\beta + i(\omega + kU + \Omega) \neq 0$$

# Methodology

- simple system, but with EXACT solutions
- Staniforth, Wood, and Côté, 2002

$$\frac{\partial F}{\partial t} + U\frac{\partial F}{\partial x} + i\omega F = -\beta F + R\, e^{i[kx+\Omega]}$$

- solution 3: forced resonant solution

$$F(x,t) = R\, t\, e^{i[kx+\Omega t]}$$

$$\beta = 0 \quad \text{and} \quad \omega + kU + \Omega = 0$$

# ALADIN/ARPEGE

| | computation | result |
|---|---|---|
| 1 | inv. FFT, inv. Legendre transformation | $F(t)$ |
| 2 | call physics (APLPAR) | $\Phi$ |
| 3 | update tendencies | $F_A^* = F(t) + \Delta t \Phi$ |
| 4 | compute departure(, middle) point (**D**, **M**) | |
| 5 | interpolate to **D**(, **M**) | $F_D^*$ |
| 6 | explicit part dynamics | $F^{exp}$ |
| 7 | FFT, Legendre transformation | |
| 8 | Helmholtz, Horizontal diffusion | $F_A^+$ |

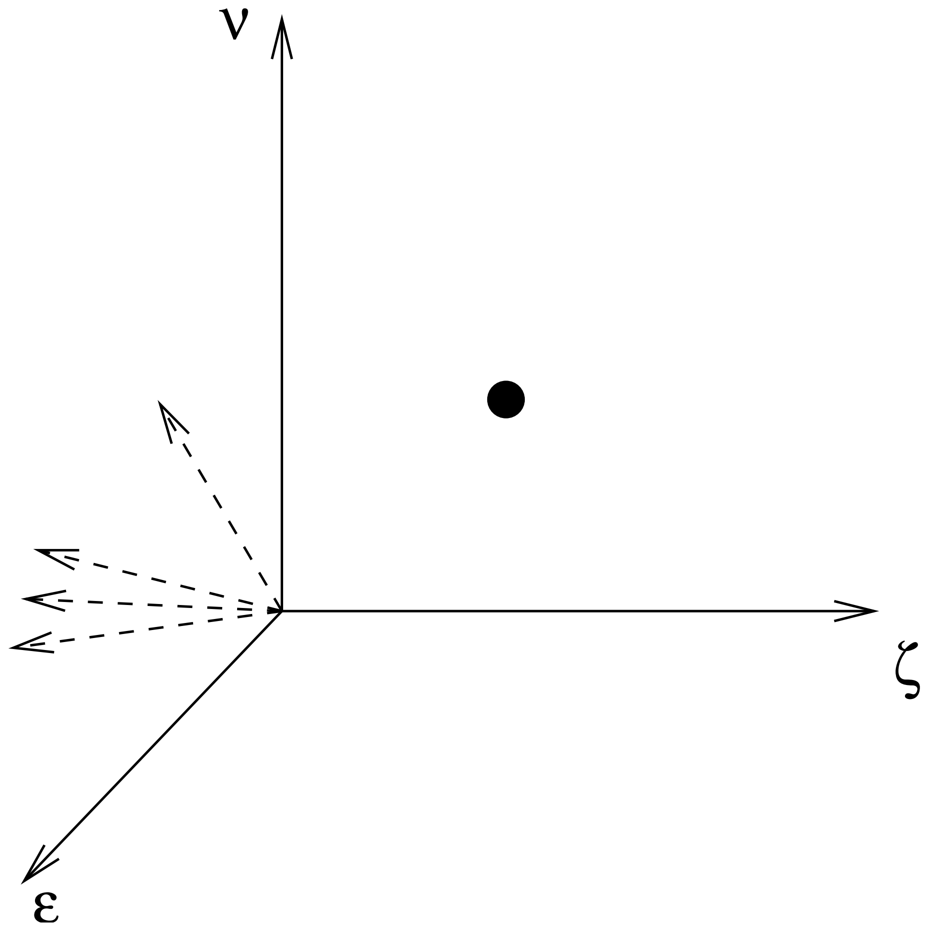
# ECMWF: SLAVEPP

| | computation | result |
|---|---|---|
| 1 | inv. FFT, inv. Legendre transformation | $F(t)$ |
| 2 | lin. terms, non-lin. $R(t)$ and $R(t - \Delta t)$ | $L^0_A, R^-_A, R^0_A$ |
| 3 | compute departure point (**D**) | |
| 4 | interpolate to **D** | $L^0$, $R^0 = (2R - R^-)$ |
| 5 | adiabatic explicit tendencies at arrival point ( **A**) | $\tilde{D}$ |
| 6 | interpolate diab. tendencies of rad., conv. and cl. at $t$ to **D** | $P^0$ |
| 7 | tendencies of parameterized processes | $P^+(F(t), \tilde{D}, fractional)$ |
| 8 | add tendencies of adiabatic and diabatic processes | $F^0_D - \frac{1}{2}L^0 + \Delta t(R^{\frac{1}{2}} + P^{\frac{1}{2}}$ |
| 9 | FFT, Legendre transformation | |
| 10 | Helmholtz, Horizontal diffusion | $F^+$ |

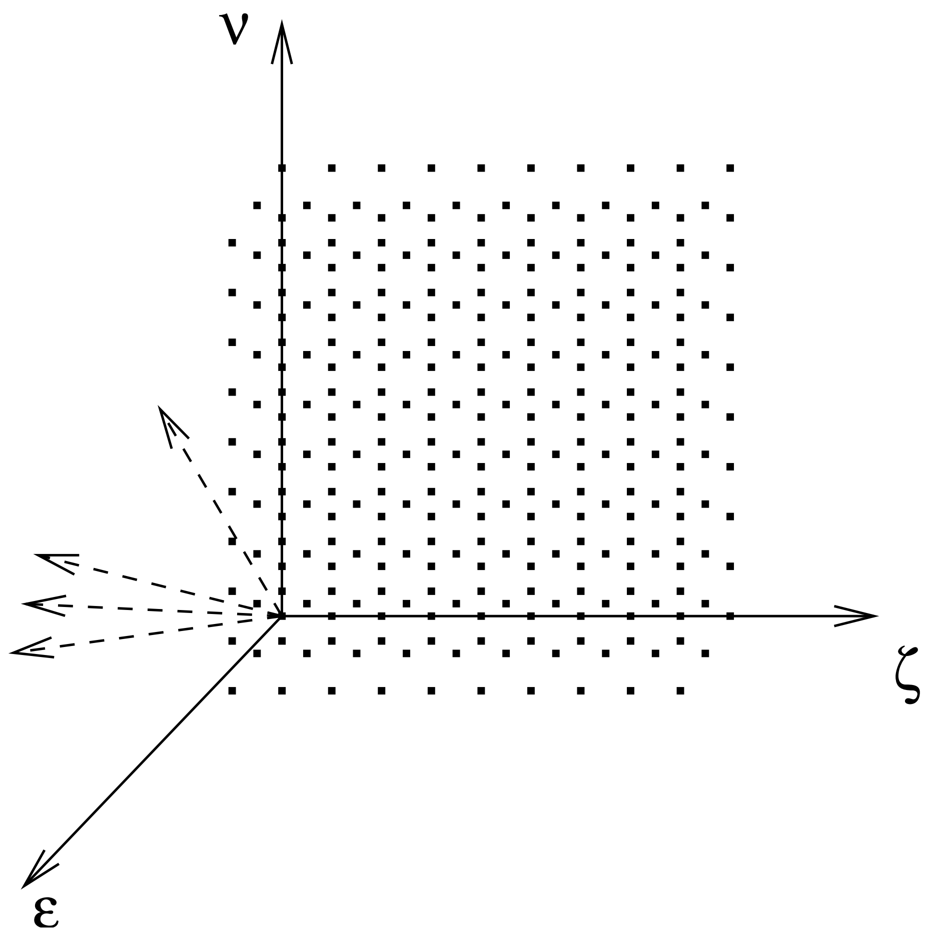
# Algorithmics: PARALLEL

| | | |
|---|---|---|
| GP | $\Phi\,(lev.\,I)$ | $\frac{G_\alpha - F_A^0}{\Delta t} = (1-\epsilon_\alpha)\xi_\alpha\phi_\alpha[F_A^0, G_\alpha]\,, \quad \alpha = 1,\ldots,M$ |
| | Interface | $F_A^* = F_A^0 + \Delta t\sum_{\alpha=1}^M \frac{G_\alpha - F_A^0}{\Delta t}$ |
| | Expl. Dyn. | $F_A^{exp} = \left(1 - \frac{i\omega}{2}\Delta t\right) e^{-ikU\Delta t} F_A^* - \frac{i}{2}(\omega - \omega^*)\Delta t F^{(0)}$ |
| | $\Phi\,(lev.\,II)$ | $\frac{G_\alpha^{exp} - F_A^{exp}}{\Delta t} = (1-\epsilon_\alpha)(1-\xi_\alpha)(1-\nu_\alpha)\phi^E[e^{-i\lambda_\alpha kU\Delta t}(F_A^0; F_A^{exp}; G_\alpha^{exp}$ <br> $\alpha = 1,\ldots,M$ |
| | Interface | $G_A^{exp} = F_A^{exp} + \Delta t\sum_{\alpha=1}^M \frac{G_\alpha^{exp} - F_A^{exp}}{\Delta t}$ |
| SP | Impl. Dyn. | $F_A^{dyn} = \left[1 + \frac{i\omega^*}{2}\Delta t - \Delta t\sum_{\alpha=1}^M (1-\epsilon_\alpha)(1-\xi_\alpha)\nu_\alpha\phi_\alpha^I\cdot\right]^{-1} G_A^{exp}$ |
| | Hor. Diff. | |
| | Ders. | $\partial_x^p F_A = (ik)^p F$ |
| GP | $\Phi\,(lev.\,III)$ | $\frac{G_\alpha^{TT} - F_A^{dyn}}{\Delta t} = \epsilon_\alpha(1-\zeta_\alpha)\phi_\alpha^E[F_A^0; F_A^{dyn}; G_\alpha^{TT}]\,, \quad \alpha = 1,\ldots,M$ |
| | Interface | $G_A^{TT} = F_A^{dyn} + \Delta t\sum_{\alpha=1}^M \frac{G_\alpha^{TT} - F_A^{dyn}}{\Delta t}$ |
| | Impl. $\Phi$ | $F_A^+ = \left[1 - \Delta t\sum_{\alpha=1}^M \epsilon_\alpha\zeta_\alpha\phi^I\right]^{-1} G_A^{TT}$ |

# The choice?



# The choice?



# Actually, what is stability?

- Wish list: I have my physics wich I have thoroughly validated in dynamics $X$ with time step organisation $x$ and I want to plug it in dynamics $Y$. It is stable in $X$ with time step organisation $x$ so ...
- it is a priori not clear it will be stable in another $\Phi - D$ coupling
- need for more severe criteria, recall: stable, unconditionally stable, ...

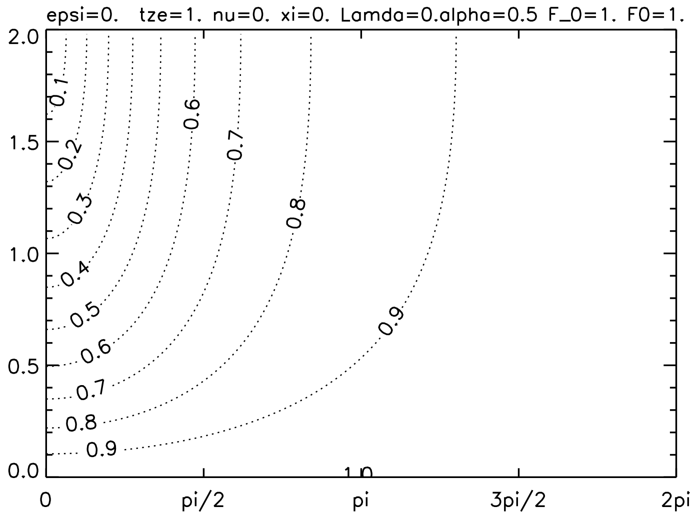
# 3 steps toward more unconditionality

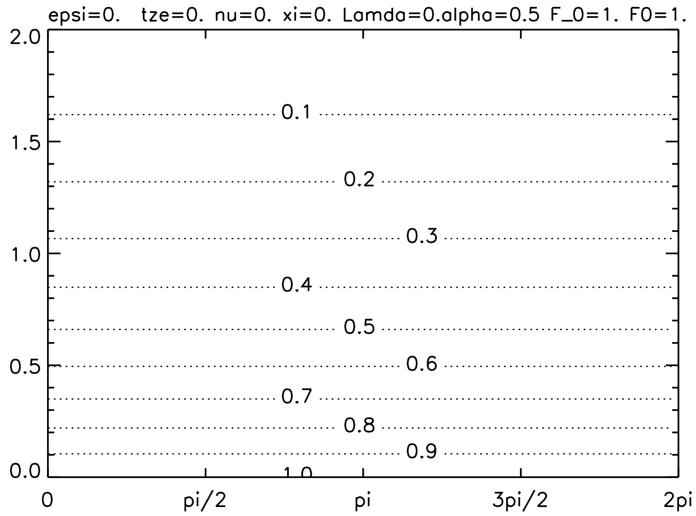
- absolute stability $\left|\frac{F^+}{F^0}\right| \rightarrow \left|\frac{F^+}{F^{exact}}\right|$,
- restriction to schemes where stability is independent of the dynamics (read $\tilde{\omega}^*$) compare to the exact solution, i.e.
- we want $\Phi$ to be stable modulo the stability of $D$, modular stability:

$$\left|\frac{F^+}{F^{semi\ exact}}\right|$$

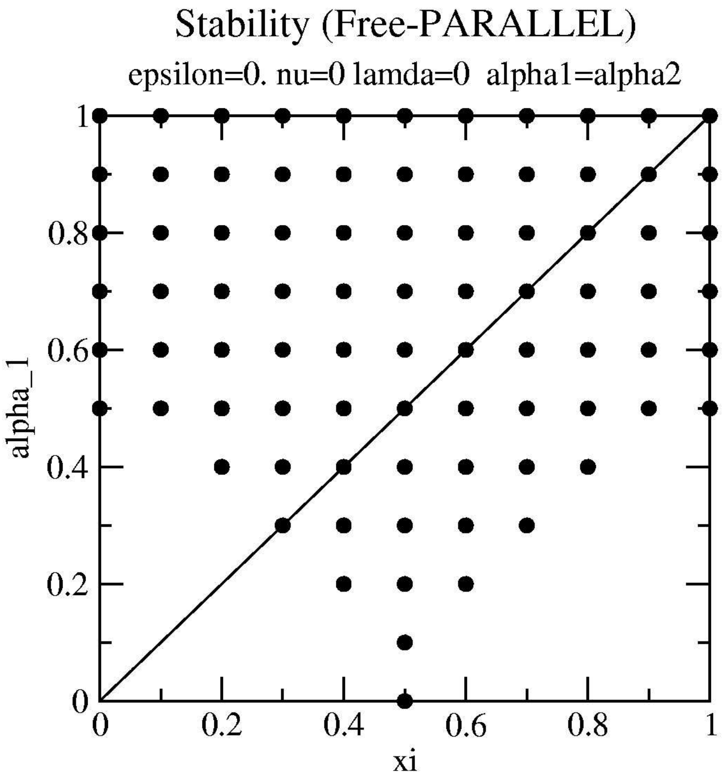
$$F^{semi\ exact} \equiv e^{-\beta\Delta t} \otimes \frac{\left(1 - \frac{i}{2}\tilde{\omega}\right) e^{-i\tilde{U}} - \frac{i}{2}\left(\tilde{\omega} - \tilde{\omega}^*\right)}{1 + \frac{i}{2}\tilde{\omega}^*}$$

# Stability independent of $D$

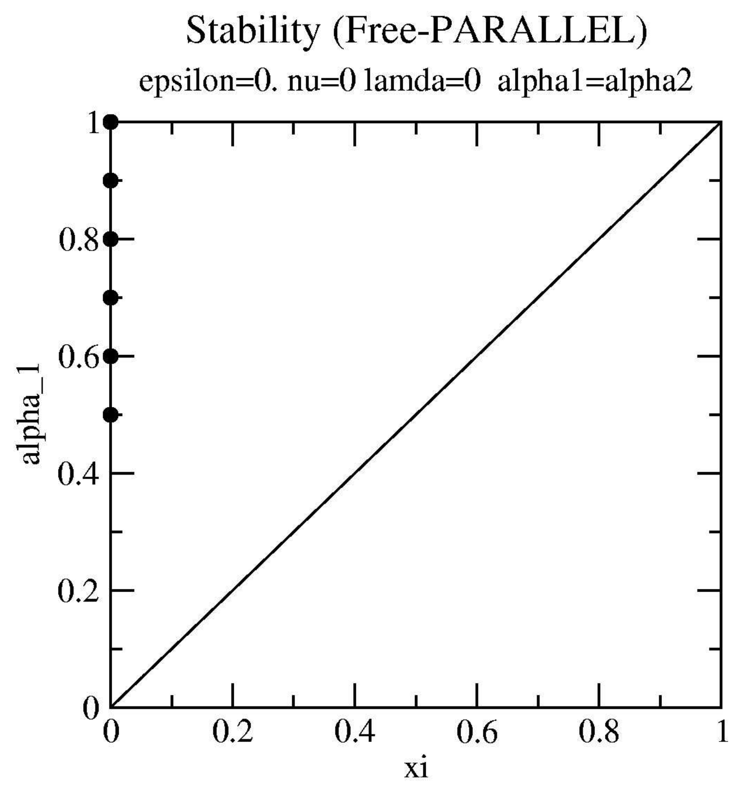


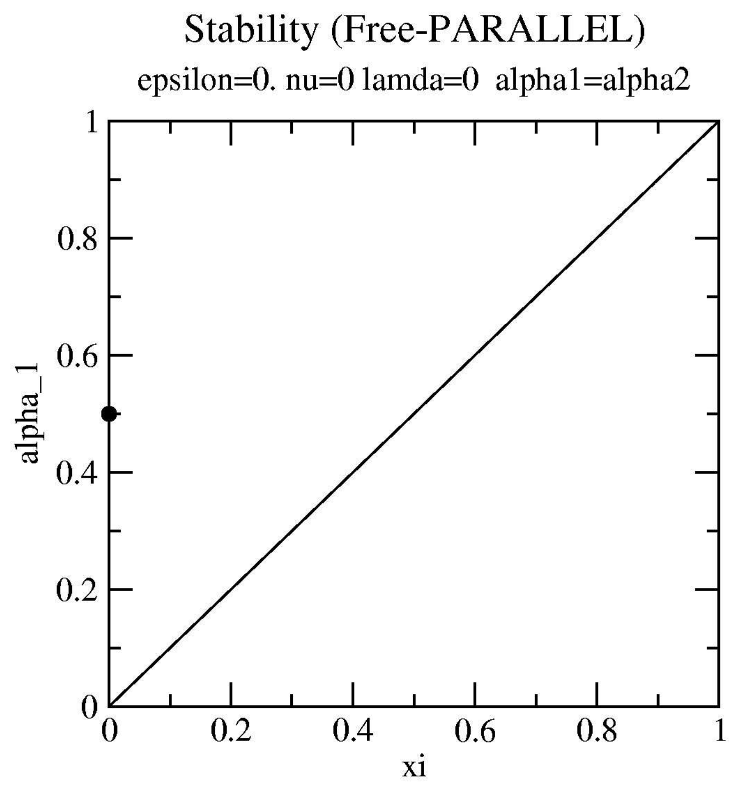
# Absolute stability



# Independent of $D$



# Relative stability



# ... in English

- there are two levels of where we get (de)stablization:
  - “in” the interface, i.e. where we couple to the model
  - below the interface, due to the amount of (split-) implicitness
- moving the interface later in the time step organisation has a stablizing effect

# Accuracy

is take the coefficient of the second-order term in the expansion in $\Delta t$.
For instance in the case $\epsilon = 0, \nu = 0$:

$$\xi P_{\alpha_1}^{exp} \longrightarrow D^{exp} \longrightarrow (1-\xi) P_{\alpha_1}^{imp} \longrightarrow D^{imp}$$

yields

$$\frac{1}{2}\beta F^0 \left[ \left( \beta - 2\beta\xi + 2\beta\xi^2\alpha_1 \right) + i \left( \omega^* - \xi\omega \right) \right] \Delta t^2$$

- $\xi = 1 \ \alpha_1 = \frac{1}{2}$: ARPEGE/ALADIN
- $\xi = \frac{1}{2} \ \alpha_1 = 0$: ECMWF compromise (Wedi paper)

# So ...

- we don’t want to jump to conclusions ... BUT

# So ...

- we don’t want to jump to conclusions ... BUT
- it seems as if ARPEGE/ALADIN uses the wrong choice ...?

# So ...

- we don’t want to jump to conclusions ... BUT
- it seems as if ARPEGE/ALADIN uses the wrong choice ...?
- HOWEVER this knife cuts at 2 sides:

# So ...

- we don't want to jump to conclusions ... BUT
- it seems as if ARPEGE/ALADIN uses the wrong choice ...?
- HOWEVER this knife cuts at 2 sides:
- this probably means that ARPEGE/ALADIN physics satisfies more severe "unconditionally" stability criteria!

# THANKS

to the organisers for this opportunity

to start paying back my debt to science...