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FORMATION OF DEPTH HOAR RESULTING FROM THERMAL OPTIMIZATION OF SNOW MICROSTRUCTURE

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ABSTRACT: Snow metamorphism can be described by the flows of heat and mass that result from gradients of temperature and water vapor concentration. The size of the forcing gradient necessary to induce the corresponding flow is a measure of the system efficiency. Kinetic metamorphism is a process of thermodynamic optimization in which the microstructure and material properties evolve to increase the heat transport efficiency. The optimal geometric configuration is a system of well-connected ice columns oriented in the direction of the heat flow. This morphology is commonly referred to as depth hoar and is a persistent weak layer that is frequently responsible for avalanches.

A series of laboratory experiments were conducted to examine the evolution of snow microstructure. Fresh snow was subjected to either constant temperature gradient or constant heat flux boundary conditions to produce an upward vertical heat flow through the sample. For all experiments, the effective thermal conductivity increased in the direction of heat flow, and the ratio of heat conduction to latent heat transfer increased with the formation of ice chains. By naturally improving the conductivity of the ice network and directing more thermal energy through the ice, the system became more efficient. This progression resulted from the system and its surroundings moving toward equilibrium in accordance with the second law of thermodynamics. Thus the formation of depth hoar, often responsible for destructive deep slab avalanches, is due to a natural optimization of the conduction pathways through the snow.

KEYWORDS: Entropy, Metamorphism, Microstructure, Optimization, Snow, Thermodynamics

1. INTRODUCTION

Deep slab avalanches are among the most dangerous and destructive events in nature. They occur when a thick cohesive snow slab overlies a layer of weak snow that often forms early in the season when the snowpack is thin. If the surface of the snowpack is subjected to cold temperatures for several weeks, the layer can metamorphose into vertical chains of depth hoar. This morphology is especially dangerous because while it can support the weight of a large slab, it is also brittle and poorly bonded parallel to the slope. If the layer becomes overloaded, the impending fracture can propagate large distances resulting in the release of the supported slab.

The environmental conditions required for kinetic metamorphism and the formation of depth hoar are well documented (Benson and Trabant 1973; Colbeck 1982; Kamata, Sokratov, and Sato 1999; Marbouty 1980; Miller and Adams 2009). However, the question of *why* the microstructure evolves into specific orientations when subjected to thermal forces has received little attention. The work presented here shows that the formation of depth hoar is a consequence of the system and its surroundings moving toward thermodynamic equilibrium in accordance with the second law of thermodynamics.

A system is said to be in global thermodynamic equilibrium if no changes are observed in the properties when it is isolated from its surroundings. If gradients in the state variables are present (i.e. temperature, pressure, concentration, energy, etc.), the second law of thermodynamics requires that the system move toward equilibrium by reducing and eliminating the gradients where ever possible. The natural formation of snow crystals in the atmosphere clearly illustrates this phenomenon.

Snow crystals form when thermal energy is removed from water vapor at a temperature below freezing and the vapor passes directly into the solid phase. The wide spacing and free motion of the water vapor molecules allows them to align with the hexagonal crystalline structure of ice. Bejan (2000) showed mathematically that the formation of a snowflake provides pathways for heat flow to facilitate equilibrium. A snowflake can be thought of as a conductive bridge between a warm nucleation site and the surrounding sub cooled water vapor. As the vapor condenses, it releases latent heat that is transferred to the snowflake. In this way, each snowflake is reducing

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temperature and vapor pressure differences in the air and moving the system toward equilibrium.

A similar process is believed to occur after the snow is deposited on the ground. Initially, the snowpack may be envisioned as a nearly isotropic porous material composed of ice and air. Temperature gradients that cause heat to flow through the snowpack reorganize the ice matrix into chains parallel to the gradient. Similar to the atmospheric snowflake formation, the snowpack ice matrix reorganizes to enhance the flow of heat in an effort to more quickly reach equilibrium. This process differs slightly from the formation of snowflakes in that the reorganization is uni-directionally aligned with the temperature gradient. With the formation of ice chains and an increase in effective conductivity, the system moves heat more efficiently than before the metamorphism and enhances the progression of the system and its surroundings toward equilibrium.

The two primary mechanisms for heat transfer in snow are conduction through the ice lattice and the movement of latent heat associated with vapor diffusion (Adams and Sato 1993; Lehning et al. 2002). Which of these modes dominates depends on the interconnectivity of the grains. Adams and Sato (1993) calculated that the majority of heat is moved via conduction when the ice grains were connected by well-developed necks. Conversely, diffusion driven heat transfer dominates when the necks are small relative to the grains. The effective thermal conductivity includes both of these modes of heat transfer.

In thermodynamics, the production of entropy is a measure of the system efficiency. According to Mahulikar and Herwig (2004), “a system will select the path or assemblage of paths out of available paths that minimizes the potential or maximizes the entropy at the fastest rate given the constraints”. This statement typically describes the progression of an isolated system toward equilibrium. In snow metamorphism, the system may be held away from equilibrium by the constraints at the boundaries. These include the global temperature gradient and the heat flux. If the gradient is constant, the potential (i.e. temperature gradient) cannot be minimized. In this case, the heat flux and entropy production increase with a rise in thermal conductivity resulting from the formation of ice chains. Conversely, if the heat flux is held constant, the system will seek to minimize the potential by reducing the temperature gradient. Both of these behaviors were observed in the research presented here.

According to the second law of thermodynamics, the entropy of the universe is always increasing and the entropy of an isolated system is maximized at equilibrium. As a system approaches equilibrium, the structure becomes more uniform and evenly distributed. When the system is held away from equilibrium the opposite may occur. The formation of ice chains represents a spontaneous production of order in the system. This is consistent with previous research by Bertalanffy (1952) who observed that “spontaneous order can appear in systems with energy flowing through them by their ability to build their order by dissipating potentials in their environments.” When the boundary conditions permit, the formation of ordered ice chains dissipates the potential across the snow bringing the system and surroundings closer to equilibrium.

By examining kinetic metamorphism of snow from the perspective of the second law of thermodynamics, we gain insight into the driving forces behind the evolution of the microstructure. The development of depth hoar that is often responsible for avalanche formation is not primarily the result of physical forces imposed on the snow. Rather, this mechanical evolution is a consequence of thermal forces that drive a change in the thermal conductivity to facilitate thermodynamic equilibrium.

2. METHODS

2.1. Experiments

Three experiments were conducted to examine the evolution of the system efficiency with the formation of depth hoar chains. Two experiments were closed to mass flow on all sides. The third experiment was open on one boundary and closed on all others. For all experiments, the boundaries were insulated except for two on opposite sides of the sample to allow uni-directional heat flow.

The snow used for the experiments was produced in a temperature controlled chamber at Montana State University. Snow was passed through a 6 mm sieve to remove ice particles that occasionally formed in the snow maker. The large sieve opening was chosen to prevent the snow from being broken into smaller particles which would unnecessarily increase the density. The box was filled with snow to a depth of 15 cm for experiment #1 and 12 cm for experiments #2 and 3. Figure 1 shows the dendritic new snow prior to sieving on a 1 mm grid.

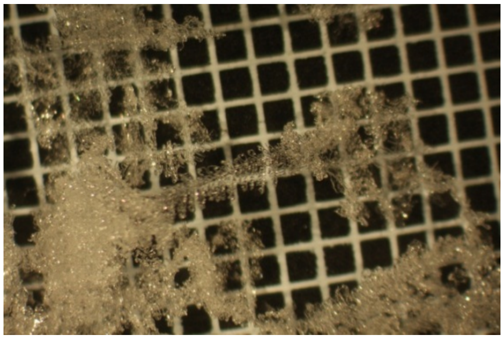

Figure 1: New snow on 1 mm grid

All metamorphism experiments were carried out in a temperature controlled environmental chamber. The air temperature of the 2 X 2 m working area could be controlled to ±0.2 K from 233 K to 273 K. The relative humidity in the room was measured to be 50% ±10% for all tests.

Figure 2 shows a profile view of the experimental setup used to metamorphose the snow. The apparatus consisted of a 0.5 X 0.5 X 0.4 m box constructed of a lumber frame with 5 cm thick R-10 polystyrene insulation sides. A piece of 20 gage sheet metal formed the bottom of the box to provide good thermal contact with the lower boundary. For the closed system experiments, the top of the sample was covered with a tight fitting 20 gage sheet metal lid.

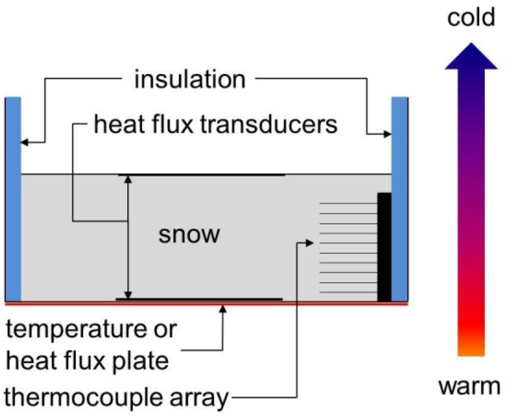

Figure 2: Profile view of experimental setup

Instrumentation was designed to record heat flux through the snow sample as well as temperatures at discrete locations throughout the snow depth. Heat flux measurements were made using 30 cm square solid state flat plate transducers. For the closed system experiments, sensors were located at the base and top of the snow, in the center of the sample area. For the open system experiment, one sensor was located at the snow base and the surface was open to the air. The snow extended approximately 10 cm laterally beyond the edges of the sensors to simulate an infinite snowpack. Temperature measurements were made in the air above the snow, at the snow base, and at one cm intervals through the depth using Type T thermocouples.

Heat flow through the snow was induced by two different sets of boundary conditions. For experiments #1 and #2, the room air temperature was held steady at approximately 258 K. Experiment #3 was performed at a constant room temperature of 255 K. This resulted in a nearly constant temperature at the top surface of the snow for all experiments. For one of the closed system experiments and for the open system, the sample base was subjected to a constant temperature boundary condition by placing the snow box on a temperature controlled metal plate. Silicon oil was circulated from a heater through copper pipe attached to the backside of the plate. The heater maintained the fluid and plate at the desired temperature, typically 272 K.

To examine a constant heat flux boundary condition, a silicon rubber fiberglass heater pad was attached to a 5 cm thick polystyrene insulation panel and placed under the snow box for the second closed system experiment. The magnitude of heat flux entering the snow was controlled by varying the voltage supplied to the pad using an AC voltage regulator. The boundary condition that exists at the ground during the natural formation of depth hoar is likely a combination constant temperature and constant heat flux.

Using the heat flux and temperature measurements, the effective thermal conductivity was calculated using Fourier's law of heat conduction. The conductivity (k) was found by dividing the heat flux $(\boldsymbol{q})$ by the temperature gradient (∇T) measured over the 9 cm range of the embedded thermocouple array: $k = {}^{\boldsymbol{q}}/_{\nabla T}$. The bottom and top measurements on the array were 1 cm and 10 cm above the snow base, respectively. The snow surface was located 2 or 5 cm above the top thermocouple depending on the experiment.

The entropy production was calculated from an entropy balance over the snow sample. Similar to energy and mass, entropy enters and leaves the system through the boundaries. In simplified terms, the entropy crossing a boundary (dS) is equal to the heat flux (dq) divided by the temperature of the boundary (T): dS=dq/T. Since entropy is not a conserved quantity (i.e. it is produced inside the system by irreversible processes) the outward flux is greater than the input. Thus the net outflow of entropy is equal to the entropy production: $dS_{out}=dS_{in}+dS_{prod}$. The more efficient a system is at conducting heat, the less entropy it will produce for a given heat flow at a given temperature.

3. RESULTS AND DISCUSSION

3.1. *Experiment #1: closed system; Constant heat flux into snow base*

A snow sample with a density of 200 kg/m^3 was subjected to a heat flux of 11 W/m^2 (±1 W/m$^{2)}$ for approximately 25 days. Figure 3 shows the final morphology of a system of depth hoar chains on a 1 mm grid. Figures 4 to 7 show the evolution of the temperatures, temperature gradient, effective thermal conductivity, and entropy production, respectively.

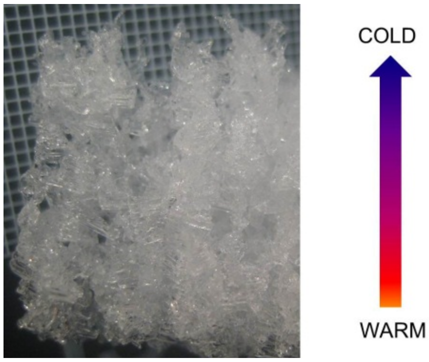

Figure 3: Experiment #1: Vertical chains of depth hoar on 1 mm grid

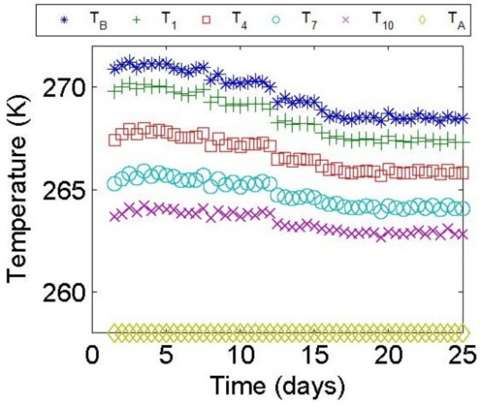

Figure 4: Experiment #1: Temperature (T_B - Base; T_1 – 1 cm; T_4 – 4 cm; T_7 – 7 cm; T_{10} – 10 cm; T_A - Air) Distances are measured from base of snow sample.

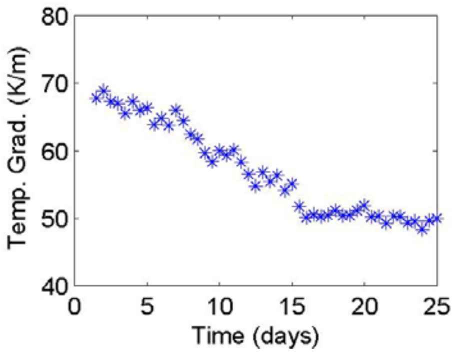

Figure 5: Experiment #1: Temperature gradient

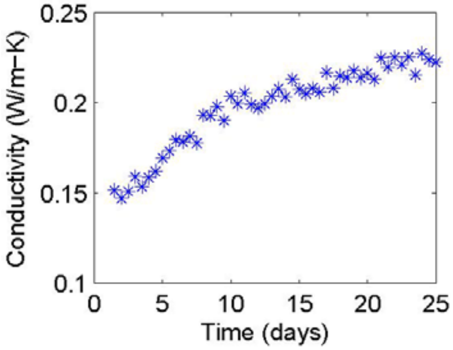

Figure 6: Experiment #1: Effective thermal conductivity

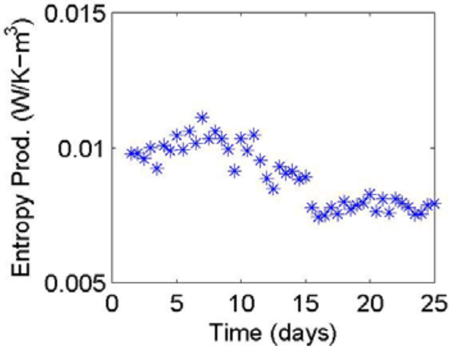

Figure 7: Experiment #1: Entropy production

The significant results from this experiment include the following:

- Since the air temperature was held constant and the base temperature was free to fluctuate, the formation of depth hoar corresponded to an overall cooling of the snow. The lowest thermocouple reading decreased more than the temperature of the top thermocouple resulting in a reduction in the temperature gradient.

- The reduction of the temperature gradient indicates that the formation of ice chains helped to dissipate the potential across the snow and move the system closer to equilibrium. This behavior is required by the second law of thermodynamics.

- The effective thermal conductivity increased with the formation of depth hoar. The rate of change began to flatten out slightly at approximately 10 days. Though a steady-state value was never reached, the decreasing rate of change indicated that the system was approaching an optimum configuration.

- The entropy production decreased as the system became more efficient. This is due to a greater portion of the heat flux moving through the ice network with the formation of chains. Since the heat could flow more easily, the force required (the temperature gradient) was reduced.

- The decrease in the overall temperature would normally cause the entropy production to increase for a given heat flow. However, the reduction in the gradient resulting from the increase in conductivity offset this effect.

3.2. Experiment #2: Closed system; Constant temperature gradient

A snow sample with a density of 120 kg/m^3 was subjected to a constant temperature gradient of 85 K/m (±3 K/m) for approximately 15 days. Figure 8 shows a snow sample of the end of the experiment on a 1 mm grid. The metamorphosed microstructure was similar in grain size and morphology to the depth hoar formed in experiment #1.

Figures 9 to 12 show the evolution of the temperature, heat flux, effective thermal conductivity, and entropy production, respectively. Fluctuations in the laboratory air temperature beginning on day 9 caused the variations observed in the data.

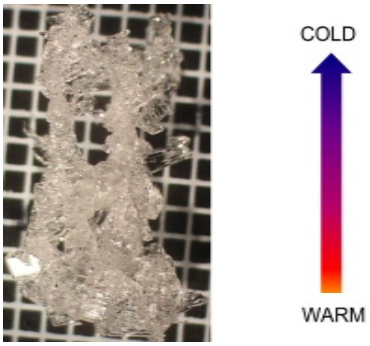

Figure 8: Experiment #2: Vertical chains of depth hoar on 1 mm grid

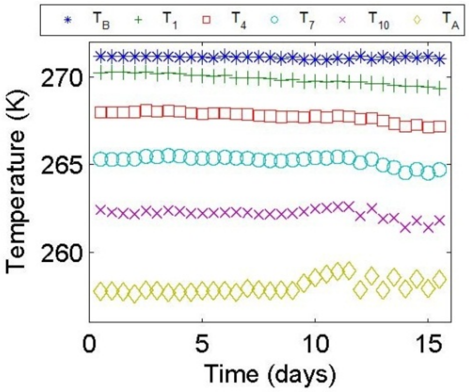

Figure 9: Experiment #2: Temperature (T_B - Base; T_1 – 1 cm; T_4 – 4 cm; T_7 – 7 cm; T_{10} – 10 cm; T_A - Air) Distances are measured from base of snow sample.

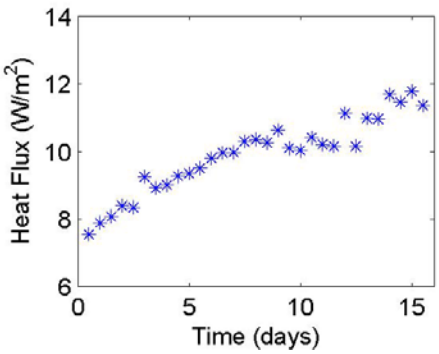

Figure 10: Experiment #2: Heat flux

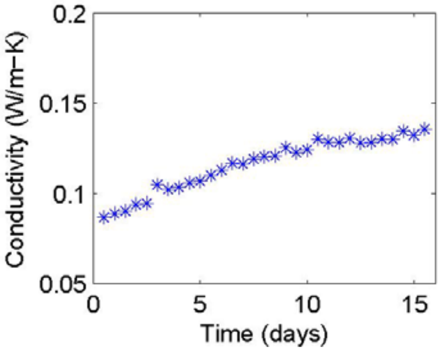

Figure 11: Experiment #2: Effective thermal conductivity

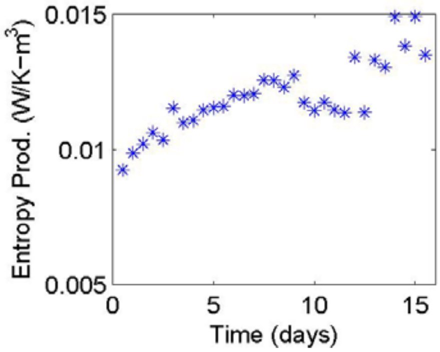

Figure 12: Experiment #2: Entropy production

The significant results from this experiment include the following:

- Even though the boundary temperatures were held constant, the lowest thermocouple within the snow (located 1 cm above the base) cooled slightly more than the top thermocouple. This resulted in a small decrease (~5 K/m) in the temperature gradient. For the sake of analysis, the gradient was assumed constant.

- Since the temperature gradient was held nearly constant, the system was not permitted to dissipate the potential. Instead, the formation of ice chains increased the heat flux in an effort to increase the entropy of the surroundings. This progression toward equilibrium is dictated by the second law.

- Similar to experiment #1, the effective conductivity increased with the formation of ice chains. This increase in efficiency allowed more heat to flow through the snow with the same forcing gradient.

- The increase in heat flux mentioned above resulted in a rise in the entropy production. Since the system could not dissipate the temperature gradient, it increased the entropy production in an attempt to move the surroundings closer to equilibrium. This behavior follows the second law as well as the observations of Mahulikar and Herwig (2004) described previously.

- At first glance, the increase in entropy production with the formation of ice chains appears to be contrary to the results of experiment #1. This occurred due to an increase in the heat flux through the snow at a constant temperature gradient. The heat flux increased because the snow became more efficient at transporting heat and the temperature gradient was fixed. If the gradient was reduced, the metamorphosed snow would produce less entropy for the same heat flow compared to the initial snow microstructure. This was the behavior observed in experiment #1.

3.3. <u>Experiment #3: Open system: constant temperature gradient</u>

A snow sample with density of 150 kg/m^3 was subjected to an initial temperature gradient of 110 K/m. Though the air and base plate temperatures were held constant, the temperatures at the top and bottom of the thermocouple array that were used in the calculations varied slightly. As the metamorphism progressed, the temperature near the base of the sample gradually decreased while the uppermost measurement remained relatively constant. The temperature gradient after 16 days was approximately 90 K/m as calculated using the top and bottom thermocouples on the array. Figure 13 shows the final morphology of the depth hoar with similar characteristics to experiments #1 and #2.

Figures 14 to 18 show the evolution of the temperatures, temperature gradient, heat flux, effective thermal conductivity, and entropy production, respectively.

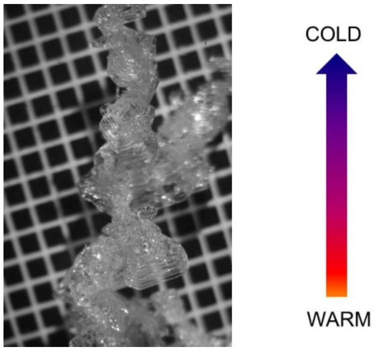

Figure 13: Experiment #3: Vertical chain of depth hoar on 1 mm grid

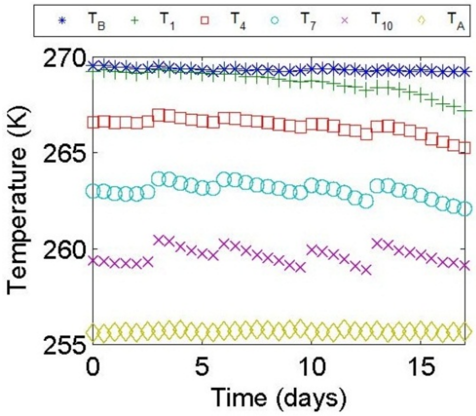

Figure 14: Experiment #3: Temperature (T_B - Base; T_1 – 1 cm; T_4 – 4 cm; T_7 – 7 cm; T_{10} – 10 cm; T_A - Air) Distances are measured from base of snow sample.

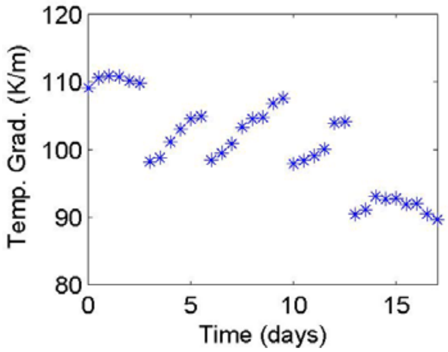

Figure 15: Experiment #3: Temperature gradient

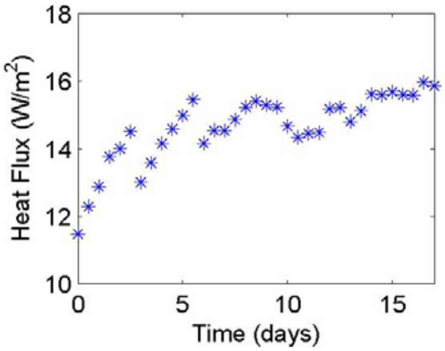

Figure 16: Experiment #3: Heat flux

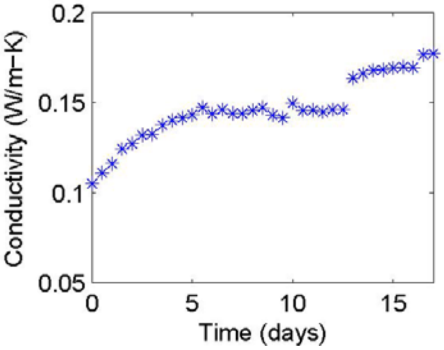

Figure 17: Experiment #3: Effective thermal conductivity

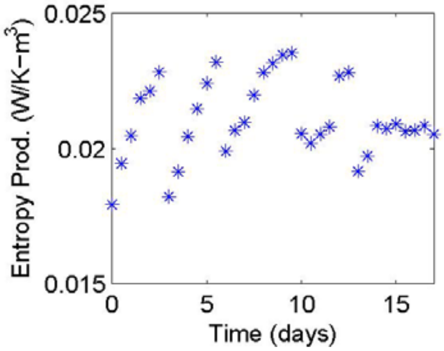

Figure 18: Experiment #3: Entropy Production

Because the top snow surface was exposed to the sub-saturated room air, ice sublimated at a rate of several mm per day. Snow was sifted onto the surface every few days in an effort to maintain a constant depth. This resulted in an immediate increase in the temperature from the insulation provided by the new snow. In the days between the additions of snow, the surface moved downward with the sublimation. Since the location of the uppermost thermocouple on the array remained stationary, the shrinking snow depth effectively cooled the thermocouple and caused the ramping seen in the data.

Experiment #3 exhibited behaviors similar to both of the other experiments. Pertinent results from these data include the following:

- The temperature of the lowest thermocouple decreased slightly while the uppermost thermocouple measurement remained nearly constant. This caused the temperature gradient and the overall temperature to decrease similarly to experiment #2.

- The heat flux increased similarly to experiment #1.

- The effective thermal conductivity increased similarly to both experiments #1 and #2.

- The simultaneous decrease in temperature gradient and increase in heat flux resulted in the entropy production fluctuating around a nearly constant value. This illustrates that if the constraints allow, a system will simultaneously dissipate the potential over the system and increase the entropy of the surroundings to facilitate equilibrium.

4. SUMMARY AND CONCLUSIONS

The likelihood of an avalanche increases dramatically when depth hoar develops early in the winter. The conditions that form this weak layer are not directly related to the physical forces imposed on the snow. Instead, the thermal environment is primarily responsible for the formation of ice chains in the direction of the heat flow. When overloaded, this morphology can propagate fractures over long distances eliminating the support for the overlying snow slab. These are the conditions that often result in deep slab avalanches.

Laboratory experiments were conducted to examine the influence of heat transfer in snow on the evolution of the morphology. The development of depth hoar chains corresponded to an increase in thermal conductivity in the direction of heat flow for all experiments. This increase in conductivity was dictated by the second law of thermodynamics. That is, the changes in the morphology all facilitated a progression toward equilibrium by reducing the temperature gradient, increasing the heat flux through the system to increase the entropy of the surroundings, or both.

Examination of snow metamorphism from the perspective of the second law of thermodynamics provides insight on the direction of the natural morphological evolution. Snow transforms into chains of depth hoar in the presence of a temperature gradient or imposed heat flow because it must in order to progress toward equilibrium. To behave in any other manner would violate the second law of thermodynamics.

5. ACKNOWLEDGEMENTS

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