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About the relationship between microstructural and effective properties of snow computed on 3D images: comparison with measurements and models

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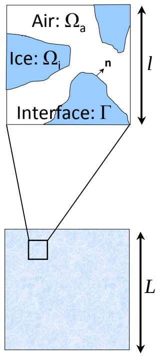
Context

The determination of accurate macroscopic effective properties of snow is critical for several topics related to cryospheric sciences such as climate modelling, hydrology or avalanche forecasting. Among different approaches, the upscaling methods allow to estimate the macroscopic behaviours from microscopic information of the medium, provided that the condition of separation of scales is satisfied. For 15 years, several X-ray tomographic acquisitions have been performed leading to a set of 3D images of snow representative of a wide range of snow types coming from cold-room experiments or field collections. In this work, we estimated some effective properties in the x-, y- and z-directions on 3D images using the upscalling method. The results are expressed as a function of density or time. The goal is:

- to compare our estimates of effective properties to models and measurements
- to study the link between the physical and microstructural properties
- to study the anisotropy of properties, thanks to computations in the three directions

Physical properties - Upscaling method



REV

Snow layer

HEAT TRANSFER

MASS TRANSFER

Microscopic physical description

$$\begin{aligned} \nabla \cdot (k_{\text{ice}} \nabla T_{\text{ice}}) &= 0 \quad \text{in } \Omega_{\text{ice}} \\ \nabla \cdot (k_{\text{air}} \nabla T_{\text{air}}) &= 0 \quad \text{in } \Omega_{\text{air}} \\ T_{\text{ice}} - T_{\text{air}} &= 0 \quad \text{on } \Gamma \\ (k_{\text{air}} \nabla T_{\text{air}} - k_{\text{ice}} \nabla T_{\text{ice}}) \cdot \mathbf{n} &= 0 \quad \text{on } \Gamma \end{aligned}$$

- k_{ice}, k_{air}: ice, and air thermal conductivities
- T_{ice}, T_{air}: ice and air temperatures

$$\begin{aligned} \mu \Delta \mathbf{v} - \nabla p &= \rho (\mathbf{v} \cdot \nabla) \mathbf{v} \quad \text{in } \Omega_{\text{air}} \\ \nabla \cdot \mathbf{v} &= 0 \quad \text{in } \Omega_{\text{air}} \\ \mathbf{v} &= \mathbf{0} \quad \text{on } \Gamma \end{aligned}$$

- v, p: velocity and pressure of air
- μ, ρ: dynamic viscosity and density of air

Condition $l << L$

Condition $l << L$
Small pore Reynolds number < O(1)

$$\nabla \cdot (\mathbf{k}_{\text{eff}} \nabla T) = 0$$

with $\boxed{\mathbf{k}_{\text{eff}}} = \frac{1}{|\Omega|} \left(\int_{\Omega_{\text{air}}} k_{\text{air}} (\nabla \mathbf{t}_{\text{air}} + \mathbf{I}) d\Omega + \int_{\Omega_{\text{ice}}} k_{\text{ice}} (\nabla \mathbf{t}_{\text{ice}} + \mathbf{I}) d\Omega \right)$

- **I**: identity tensor
- T: temperature
- $\mathbf{t}_{ice}$, $\mathbf{t}_{air}$: ice and air Ω-periodic vectors

$$\nabla \cdot \langle \mathbf{v} \rangle = 0$$

with $\langle \mathbf{v} \rangle = -\frac{\boxed{\mathbf{K}}}{\mu} \nabla p$

- <>: volume averaging

Macroscopic physical description

The above boundary value problems were solved over REVs extracted from the 3D images using *GeoDict* to estimate:

- Thermal conductivity tensor, $\boxed{\mathbf{k}_{\text{eff}}}$
- Intrinsic permeability tensor, $\boxed{\mathbf{K}}$
- Tortuosity tensor of air, $\boldsymbol{\tau}_{air}$ → same boundary problem than $\mathbf{k}_{eff}$ with k_{ice}=0 and k_{air} = 1

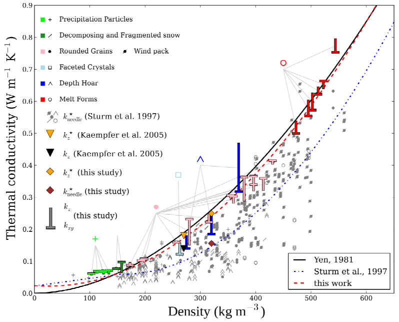
Non-diagonal terms of the tensors are negligible → we only considered the diagonal terms in the following.

Structural properties - Image analysis

The structural properties were computed on REVs extracted from 3D images:

- Density ρ_s – using a standard voxel counting algorithm.
- Specific surface area SSA and equivalent sphere radius r_{es} = 3 / (SSA x ρ_{ice}) - using a stereologic method (Flin et al., 2011).
- Correlation length, l_c – by fitting the two-point probability function by an exponential curve (Löwe et al., 2011).

Effective thermal conductivity vs density



Anisotropy

→ FC and DH: $k_z > k_{xy}$

→ some RG: $k_z < k_{xy}$

Comparisons with our estimates

→Fit of Yen 1981 (used in the Crocus snowpack model): good agreement

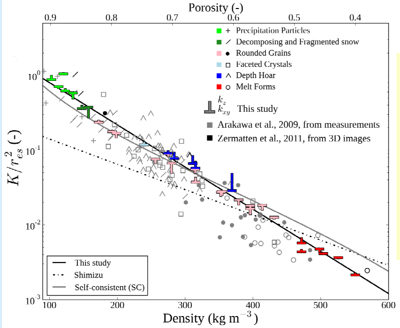
→Needle probe measurements of Sturm 1997: underestimation

Proposed fit (average value)

k_{eff} = 2.5x10$^{-6}\rho_s^2$ -1.23x10$^{-4}\rho_s$+0.024

Calonne et al., 2011

Permeability vs density



Anisotropy

→ similar to k_{eff}

Comparison with our estimates

→ In overall: good agreement

→ Fit of Shimizu: bias linked to the choice of the characteristic length

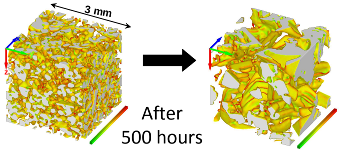
Proposed fit

K=(3±0.3)r_{es}^2exp((-0.013±0.0003)ρ_s)

Calonne et al., 2012

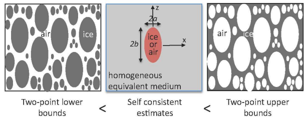
Time evolution of microstructure and effective properties during a temperature gradient metamorphism

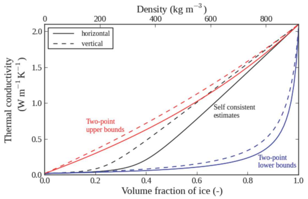
→ We used 3D images of snow obtained from a cold-room experiment of TG metamorphim of 43°C/m at –4°C.

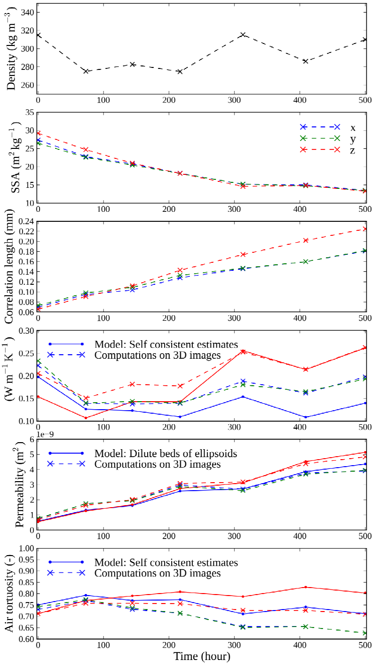


→ Our estimates of effective properties are compared to models based on ellipsoidal inclusions and which require basic information such as:

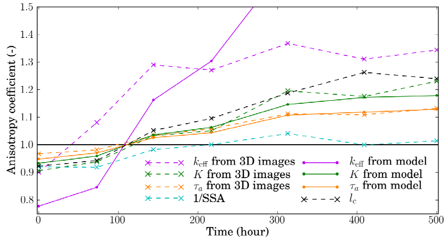
- density: ice/air proportion
- correlation lengths: ellipsoidal shape







$$\text{Anisotropy coefficient} = \frac{\text{vertical - value (z)}}{\text{horizontal - value (xy)}}$$



Time evolution

→ Physical properties evolve only because of the microstructure evolution (no change in density).

→ Development of anisotropic properties: z-values become higher with time.

Comparison with models

→ analytical models are of the same order of magnitude as our estimates.

→ analytical models reproduce roughly the anisotropy of properties.

Calonne et al., 2013

Conclusion

The upscaling method seems to be an interesting way to estimate macroscopic properties of snow from its microstructure. We highlight the following points:

- The strong link between the snow microstructure and its effective properties at macroscale.
- Analytical models, based on basic information, offer good estimates of properties and anisotropy coefficients of snow.
- The upscaling method provides the anisotropy of the properties, which is challenging to access by measurements.
- Concerning the parametrizations of k_{eff} and K vs density:
  - Shimizu's fit seems inappropriate
  - Yen's fit is in good agreement with our estimates of k_{eff}
  - Sturm's fit seems to systematically underestimate k_{eff}
- The underestimation of the thermal conductivity observed with the needle probe measurements requires further investigations.

This work offers new outlooks for accurate measurements and multiscale modeling of the snowpack.

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